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ON H-TRICHOTOMY IN BANACH SPACES

Abstract:
In this paper we emphasize the notion of skew-evolution semiflows, considered a generalization of semigroups, evolution operators and skew-product semiflows, which arise in the stability theory. We define and characterize a particular case of trichotomy, called the H-trichotomy, which is useful in describing the behaviors of the solution of evolution equations. We emphasize the fact that the trichtomy, introduced in finite dimensions in [1] and [5], is a natural generalization of dichotomy. A similar concept for stability was studied for evolution operators in [2]. This paper considers also other asymptotic properties, as exponential growth and decay, stability and instability.
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INTRODUCTION

The concept of skew-evolution semiflows arises in the theory of evolution equations, which, as well as the theory of optimal control, is an important tool in describing processes derived from engineering or economics. The dynamical systems that study the real life phenomena are complex and the identification of appropriated mathematical models is difficult because in the case of systems described by partial differential equations the state space is often of infinite dimension. It is interesting to reconsider the definitions of asymptotic properties for differential equations by means of skew-evolution semiflows. In what follows, we will consider a more general case for asymptotic behaviors that not involves necessarily exponentials, but, instead, properly defined functions. Let us define the set Γ *of all continuous functions* $H:\mathbb{R}_+\to\mathbb{R}_+^*$. *We will denote by* Θ *the set of all functions* $f:\mathbb{R}_+\to\mathbb{R}_+$ *with the property that there exists a constant* $\mu\in\mathbb{R}$ *such that* $f(t)=e^{\mu t}$, $\forall t\geq 0$, *with the subsets* Θ_+ *and* Θ_-, *for positive, respectively negative values of* μ. *By* Ψ *is denoted the set of continuous functions* $h:\mathbb{R}_+\to[1,\infty)$ *defined such that, for all* $H\in\Gamma$, *there exist a function* $f\in\Theta$ *and a constant* $k>0$ *with the properties*

$$h(s)\leq kf(t-s)H(t),\ \forall t,s\geq 0 \ \text{ and}$$
$$h(2t)h(2s)\leq H(t+s),\ \forall t,s\geq 0.$$

Remark 1.1. *The set* Ψ *is not empty, as we can consider*

$$h(t)=f(t)=e^{\nu t} \text{ and } H(t)=e^{2\nu t},\ \nu>0,\ t\geq 0.$$

We will emphasize the notion of skew-evolution semiflows by means of evolution semiflows and evolution cocycles, as introduced by us in [4]. They naturally generalize notions as operators semigroups, evolution operators or skew-product semiflows. A skew-evolution semiflow depends on three variabiles, contrary to a skew-product semiflow, which depends only on two, and, hence, the study of asymptotic behaviors for skew-evolution semiflows in the nonuniform *setting arises as natural, relative to the third variabile. In this paper we will also consider the definitions and characterizations of some asymptotic properties, by means of the set of functions Θ, Γ and Ψ.*

SKEW-EVOLUTION SEMIFLOWS

Let us consider (X, d) a metric space, V a real or complex Banach space, V^* its topological dual and B(V) the family of linear V-valued bounded operators defined on V. The norm of vectors and operators is $\|\cdot\|$.

In what follows, we will denote Y = X× V and we will consider the set $T = \{(t,t_0) \in R \mid t \geq t_0 \geq 0\}$. By I is designed the identity operator on V.

Definition 2.1. *A mapping $\varphi : T \times X \to X$ with the properties:*

(s_1) $\varphi(t,t,x) = x$, $\forall (t,x) \in R_+ \times X$;

(s_2) $\varphi(t,s,\varphi(s,t_0,x)) = \varphi(t,t_0,x)$, $\forall (t,s),(s,t_0) \in T$, $\forall x \in X$ *is called evolution semiflow on X.*

Definition 2.2. *A mapping $\Phi : T \times X \to B(V)$ with the properties:*

(c_1) $\Phi(t,t,x) = I$, $\forall (t,x) \in R_+ \times X$;

(c_2) $\Phi(t,s,\varphi(s,t_0,x))\Phi(s,t_0,x) = \Phi(t,t_0,x)$, $\forall (t,s),(s,t_0) \in T$, $\forall x \in X$

is called evolution cocycle over the evolution semiflow φ.

Definition 2.3. *The mapping*

$C : T \times Y \to Y$, $C(t,s,x,v) = (\varphi(t,s,x),\Phi(t,s,x)v)$,

where φ is an evolution semiflow on X and the mapping Φ is an evolution cocycle over φ, is called skew-evolution semiflow on Y.

The next example emphasizes a skew-evolution semiflow generated by a system of differential equations.

Example 2.1. *Let us consider the system of differential equations*

$$\begin{cases} \dot{u} = (2t\sin t - 3)u \\ \dot{w} = (t\cos t + 2)w \\ \dot{z} = (2 - \cos t)z. \end{cases}$$

Let us define the spaces X = $\boldsymbol{R}_+$ and V = $\boldsymbol{R}^3$, which is endowed with the norm $\|v\|$ = $|v_1| + |v_2| + |v_3|$, where $v=(v_1, v_2, v_3) \in$ V.

The mapping

$$\varphi : T \times R_+ \to R_+,\ \varphi(t,s,x) = t - s + x$$

is an evolution semiflow on $\boldsymbol{R}_+$.

The mapping

$$\Phi : T \times X \to B(V),$$

$$\Phi(t,s,x)(v_1,v_2,v_3) = (U(t,s)v_1, W(t,s)v_2, Z(t,s)v_3),$$

where $U(t,s)=u(t)u^{-1}(s)$, $W(t,s)=w(t)w^{-1}(s)$, $Z(t,s)=z(t)z^{-1}(s)$, $\forall (t,s) \in T$ and u(t), w(t) and z(t), where $t \in \boldsymbol{R}_+$, are the solutions of the given system of differential equations, is an evolution cocycle over the evolution semiflow φ on the metric space $\boldsymbol{R}_+$. We obtain that C = (φ, Φ) is a skew-evolution.

The following asymptotic behaviors of skew-evolution semiflow are useful in characterizing the property of H-trichotomy, as well as their characterizations.

Definition 2.2. *A skew-evolution semiflow C = (φ, Φ) is said to have exponential growth if there exists a nondecreasing function $g : R_+ \to [1,\infty)$ with the property $\lim_{t\to\infty} g(t) = \infty$ such that:*

$$\|\Phi(t,t_0,x)v\| \leq g(t-s)\|\Phi(s,t_0,x)v\|,$$
$$\forall\ (t,s),(s,t_0) \in T,\ \forall\ (x,v) \in Y.$$

Proposition 2.1. *A skew-evolution semiflow C = (φ, Φ) has exponential growth if and only if there exist some constants $M \geq 1$ and $\omega > 0$ such that:*

$$\|\Phi(t,t_0,x)v\| \leq Me^{\omega(t-s)}\|\Phi(s,t_0,x)v\|,$$
$$\forall\ (t,s),(s,t_0) \in T,\ \forall\ (x,v) \in Y.$$

Proof. *Necessity. Let $t \geq s \geq t_0 \geq 0$ and n be the integer part of the real number t - s. We obtain successively*

$$\|\Phi(t,t_0,x)v\| \leq g(1)\|\Phi(t-1,t_0,x)v\|$$
$$\leq \ldots \leq [g(1)]^n \|\Phi(t-n,t_0,x)v\| \leq$$
$$\leq Me^{n\omega}\|\Phi(s,t_0,x)v\| \leq Me^{\omega(t-s)}\|\Phi(s,t_0,x)v\|,$$

for all $(t,s),(s,t_0) \in T$ and all $(x,v) \in Y$, where we have denoted M = g(1) > 1 and $\omega = \ln M > 0$.

Sufficiency. It is obtained immediately if we consider $g(u) = Me^{\omega u}$, $u \geq 0$.

Definition 2.3. *A skew-evolution semiflow C = (φ, Φ) is said to be with exponential decay if there exists a nondecreasing function $g : R_+ \to [1,\infty)$ with the property $\lim_{t\to\infty} g(t) = \infty$ such that:*

$$\|\Phi(s,t_0,x)v\| \le g(t-s)\|\Phi(t,t_0,x)v\|,$$
$$\forall\ (t,s),(s,t_0)\in T,\ \forall\ (x,v)\in Y.$$

Proposition 2.2. *A skew-evolution semiflow C = (φ,Φ) has exponential decay if and only if there exist some constants* $M \ge 1$ *and* $\omega > 0$ *such that:*

$$\|\Phi(s,t_0,x)v\| \le Me^{\omega(t-s)}\|\Phi(t,t_0,x)v\|,$$
$$\forall\ (t,s),(s,t_0)\in T,\ \forall\ (x,v)\in Y.$$

Proof. *Necessity. Let* $t \ge s \ge t_0 \ge 0$. *There exists a natural number n such that. We have following relations*

$$\|\Phi(s,t_0,x)v\| \le g(1)\|\Phi(s+1,t_0,x)v\| \le$$
$$\ldots \le [g(1)]^n\|\Phi(s+n,t_0,x)v\| \le$$
$$\le Me^{n\omega}\|\Phi(t,t_0,x)v\| \le Me^{\omega(t-s)}\|\Phi(t,t_0,x)v\|,$$

for all $(t,s),(s,t_0)\in T$ *and all* $(x,v)\in Y$, *where we have considered the constants M = g(1) > 1 and* $\omega = \ln M > 0$.
Sufficiency. It follows immediately for $g(u) = Me^{\omega u}$, $u \ge 0$

ON THE PROPERTY OF H-TRICHOTOMY

A general concept of exponential trichotomy is emphasized in this section.

Definition 3.1. *A mapping P:Y→ Y given by P(x,v)=(x, P(x)v), where P(x) is a projection on* Y_x *= {x} × V and x ∈ X, is called projector on Y.*

Definition 3.2. *A skew-evolution semiflow C = (φ,Φ) is said to be H-trichotomic if there exist some mappings* N_1, N_2, N_3: $R_+ \to R_+^*$ *and three projectors families* $\{P_k\}_{k\in\{1,2,3\}}$ *such that following conditions hold:*

(t_1) for each projector P_k, *k∈ {1,2,3}, the relation*

$$P(\varphi(t,s,x))\Phi(t,s,x) = \Phi(t,s,x)P(x)$$

holds for all $(t,s)\in T$ *and all x ∈ X;*
(t_2) for all x ∈ X, the projections $P_1(x)$, $P_2(x)$ *and* $P_3(x)$ *satisfy the conditions*

$$P_1(x)+P_2(x)+P_3(x)=I \text{ and } P_i(x)P_j(x)=0, \text{ for all } i, j \in \{1,2,3\},\ i \ne j;$$

(t_3) following inequalities

$$(t_3^1)\quad \begin{aligned} &H(t)\|\Phi(t,t_0,x)P_1(x)v\| \\ &\le N_1(s)\|\Phi(s,t_0,x)P_1(x)v\|; \end{aligned}$$

$$(t_3^2)\quad \begin{aligned} &H(s)\|\Phi(s,t_0,x)P_2(x)v\| \\ &\le N_2(t)\|\Phi(t,t_0,x)P_2(x)v\|; \end{aligned}$$

$$(t_3^3)\quad \begin{aligned} &\|\Phi(t,t_0,x)P_3(x)v\| \\ &\le N_3(s)H(t)\|\Phi(s,t_0,x)P_3(x)v\| \end{aligned}$$

and

$$\|\Phi(s,t_0,x)P_3(x)v\| \le N_3(t)H(s)\|\Phi(t,t_0,x)P_3(x)v\|,$$

hold for all $(t,s),(s,t_0)\in T$, *for all* $(x,v)\in Y$ *and all H∈ Γ.*

Remark 3.1. *In the particular case H(t) =* $e^{\nu t}$, *t ≥ 0, ν > 0, the exponential trichotomy for skew-evolution semiflows, defined and characterized by us in [3] for evolution operators, is obtained in a nonuniform setting.*

Remark 3.2. *(i) A projector P on Y with property (t_1) is also called invariant relative to the skew-evolution semiflow C = (φ, Φ);*
(ii) If three projectors families $\{P_k\}_{k\in\{1,2,3\}}$ *satisfy relations (t_1) and (t_2) of Definition 3.2, they are usually said to be compatible with the skew-evolution semiflow C.*
In what follows, we will denote a skew-evolution semiflow $C_k = (\varphi, \Phi_k)$, *k∈{1,2,3}, where* $\Phi_k(t,s,x)v = \Phi(t,s,x)P_k(x)v$, $(t,s)\in T$, $(x,v)\in Y$.

Example 3.1. *Let us consider the skew-evolution semiflow given in Example 2.1.*
We obtain for the evolution cocycle $\Phi : T\times X \to B(V)$ *following relations*

$$\Phi(t,s,x)(v_1,v_2,v_3) =$$
$$= (e^{2t\cos t - 2s\cos s - 2\sin t + 2\sin s - 3t + 3s}v_1,$$
$$e^{t\sin t - s\sin t + \cos t - \cos s + 2t - 2s}v_2,$$
$$e^{-\sin t + \sin s + 2t - 2s}v_3)$$

Let us define the projectors $P_1(x,v)=(v_1,0,0)$, $P_2(x,v)=(0,v_2,0)$ *and* $P_3(x,v)=(0,0,v_3)$. *As following relation holds*

$$2t\cos t - 2s\cos s - 2\sin t + 2\sin s - 3t + 3s \le -t + 5s + 4,\ \forall\ (t,s)\in T,$$

we have that

$$H_1(t)\|\Phi(t,s,x)P_1(x)v\| \leq N_1(s)|v_1|,$$
$\forall$ *(t, s, x, v)* $\in$ *T*$\times$ *Y,*

where we have denoted $H_1(t) = e^t$ *and* $N_1(s) = e^{5s+4}$.

According to the inequality

t sin t – s sin s + cos t - cos s + 2t - 2s $\geq$ *t – 3s - 2,* $\forall$ *(t, s)* $\in$ *T,*

it follows that

$$N_2(t)\|\Phi(t,s,x)P_2(x)v\| \geq H_2(s)|v_2|,$$
(t, s, x, v) $\in$ *T*$\times$ *Y,*

where we have considered $H_2(s) = e^{-3s}$ *and* $N_2(t) = e^{-t+2}$.

Also, as

- sin t + sin s + 2t - 2s $\leq$ *2t – s + 1,* $\forall$ *(t, s)* $\in$ *T,*

we have

$$\|\Phi(t,s,x)P_3(x)v\| \leq N_3(s)H_3(t)|v_3|$$
$\forall$ *(t, s, x, v)* $\in$ *T*$\times$ *Y*

and, as

- sin t + sin s + 2t - 2s $\geq$ *t - 2s - 1,* $\forall$ *(t, s)* $\in$ *T,*

we obtain

$$N_3(t)H_3(s)\|\Phi(t,s,x)P_3(x)v\| \geq |v_3|,$$
$\forall$ *(t, s, x, v)* $\in$ *T*$\times$ *Y,*

where, in both cases, we have denoted $H_3(u) = e^{2u}$ *and* $N_3(u) = e^{-u+1}$.

As a remark, we can consider, without any loss of generality, the function denoted H(t) = min{$H_1(t)$, $H_2(t)$, $H_3(t)$}, $t \geq 0$.

It follows that the skew-evolution semiflow C = (φ, Φ) is H-trichotomic.

The next main result of this paper can be considered as an integral characterization for the concept of H-trichotomy.

Theorem 3.1. *Let H$\in$ Γ and h$\in$ Ψ. A skew-evolution semiflow C = (φ, Φ) is H-trichotomic if and only if there exist some mappings M_1, M_2, M_3:* $R_+ \to R_+^*$, *some functions* $f_1, f_2 \in \Theta$ *and three projectors families $\{P_k\}_{k \in \{1,2,3\}}$ compatible with C such that the skew-evolution semiflow C_1 has exponential growth and the skew-evolution semiflow C_2 has exponential decay and such that following conditions hold:*

$$(i)\ \frac{1}{H(t)}\int_{t_0}^{t} h(\tau)\|\Phi_1(t,\tau,x)^* v^*\|d\tau \leq M_1(t_0)\|P_1(x)v^*\|;$$

$$(ii)\ h(t_0)\int_{t_0}^{t}\frac{1}{H(\tau)}\|\Phi_2(\tau,t_0,x)v\|d\tau \leq M_2(t)\|\Phi_2(t,t_0,x)v\|;$$

$$(iii)\ \int_{s}^{t} f_1(\tau-s)\|\Phi_3(\tau,t_0,x)v\|d\tau \leq M_3(t_0)\|\Phi_3(s,t_0,x)v\|;$$

$$(iv)\ \int_{s}^{t} f_2(t-\tau)\|\Phi_3(\tau,t_0,x)v\|d\tau \leq M_3(t_0)\|\Phi_3(t,t_0,x)v\|,$$

for all (t,s), (s,t_0) $\in$ *T and all (x,v)* $\in$ *Y,* $v^* \in V^*$ *with* $\|v^*\| \leq 1$.

Proof. *Necessity. As the skew-evolution semiflow C is H-trichotomic, it implies that the relations (t_3) of Definition 3.2 hold.*

(i) There exist a function $f \in \Theta_-$ *and a constant k > 0 with the property*
$h(s) \leq kf(t-s)H(t)$, $\forall t \geq s \geq 0$. *Let us denote* $f(t) = e^{-\nu t}$, $\nu > 0$. *We obtain*

$$\|\Phi_1(t,t_0,x)v\| \leq \frac{N_1(s)}{H(t)}\|\Phi_1(s,t_0,x)v\|$$
$$\leq \overline{M}_1(s)e^{-\nu(t-s)}\|\Phi_1(s,t_0,x)v\|$$

for all $(t,s),(s,t_0) \in T$ *and for all* $(x,v) \in Y$, *where we have considered the function* $\overline{M}_1 : R_+ \to R_+$, $\overline{M}_1(u) = k\frac{N_1(u)}{h(u)}$.

We obtain further

$$\frac{1}{H(t)}\int_{t_0}^{t} h(\tau)\|\Phi_1(t,\tau,x)^* v^*\|d\tau$$
$$\leq k\int_{t_0}^{t} e^{-\nu(t-\tau)}\|\Phi_1(t,\tau,x)^* v^*\|d\tau$$
$$\leq M_1(t_0)\|P_1(x)v^*\|$$

where we have denoted $M_1(u) = k\nu^{-1}\overline{M}_1(u)$, $u \geq 0$.

(ii) There exist a function $f \in \Theta_-$ *and a constant k > 0 with the property*
$h(t_0) \leq kf(s-t_0)H(s)$, $\forall s \geq t_0 \geq 0$. *Let us consider* $f(t) = e^{-\nu t}$, $\nu > 0$. *We have*

$$\|\Phi_2(s,t_0,x)v\| \leq \frac{N_2(t)}{H(s)}\|\Phi_2(t,t_0,x)v\|$$
$$\leq k\frac{N_2(t)}{h(t_0)}e^{-\nu(s-t_0)}\|\Phi_2(t,t_0,x)v\| \leq$$
$$\leq k\frac{N_2(t)}{h(t_0)}e^{\nu t}e^{-\nu(t-s)}e^{-\nu(2s-t_0)}\|\Phi_2(t,t_0,x)v\|$$
$$\leq \overline{M}_2(t)e^{-\nu(t-s)}\|\Phi_2(t,t_0,x)v\|$$

for all $(t,s),(s,t_0)\in T$ *and for all* $(x,v)\in Y$, *where we have denoted the function* $\overline{M}_2 : R_+ \to R_+$, $\overline{M}_2(u)=kN_2(u)e^{\nu u}$.

(iii) and (iv) are obtained by a similar argumentation, according to Proposition 2.1 and Proposition 2.2.

Sufficiency. *(i) Let* $t\geq t_0+1$ *and* $s\in[t_0,t_0+1]$. *As* $H\in\Gamma$ *and* $h\in\Psi$, *there exists a constant* $\alpha>0$ *such that* $h(s)\leq e^{-\alpha(t-s)}H(t)$, *for all* $(t,s)\in T$. *Then, as the skew-evolution semiflow* C_1 *has exponential growth, according to Proposition 2.1, there exist some constants* $M\geq 1$ *and* $\omega>0$ *such that following relations hold*

$$e^{-(\alpha+\omega)}\left|\left\langle v^*, e^{\alpha(t-t_0)}\Phi_1(t,t_0,x)v\right\rangle\right| =$$

$$e^{-(\alpha+\omega)}\int_{t_0}^{t_0+1}\left|\left\langle \Phi_1(t,\tau,x)^* v^*, e^{\alpha(t-t_0)}\Phi_1(\tau,t_0,x)v\right\rangle\right| d\tau\leq$$

$$\leq \int_{t_0}^{t_0+1} e^{\alpha(t-\tau)}\left\|\Phi_1(t,\tau,\varphi(\tau,t_0,x))^* v^*\right\| e^{-\omega(\tau-t_0)}\left\|\Phi_1(\tau,t_0,x)v\right\| d\tau\leq$$

$$\leq M\|v\|\int_{t_0}^{t} e^{\alpha(t-\tau)}\left\|\Phi_1(t,\tau,\varphi(\tau,t_0,x))^* v^*\right\| d\tau$$

$$\leq MN_1(t_0)\|P_1(x)v\|\left\|P_1(x)v^*\right\|$$

By taking supremum relative to $\|v^*\|\leq 1$, *we have*

$$\left\|\Phi_1(t,t_0,x)v\right\|\leq M_1(t_0)e^{-\alpha(t-t_0)}\left\|P_1(x)v\right\|,$$

for all $t\geq t_0+1$ *and all* $(x,v)\in Y$, *where* $M_1(u)=MN(u)e^{\alpha+\omega}$, $u\geq 0$.

On the other hand, for $t\in[t_0,t_0+1]$ *and* $(x,v)\in Y$, *we obtain*

$$\left\|\Phi_1(t,t_0,x)v\right\|\leq Me^{\omega(t-t_0)}\|v\|\leq \hat{M}e^{-\alpha(t-t_0)}\|v\|,$$

where we have denoted $\hat{M}=Me^{\alpha+\omega}$. *Hence, it follows that*

$$\left\|\Phi_1(t,t_0,x)v\right\|\leq[M_1(t_0)+\hat{M}]e^{-\alpha(t-t_0)}\|v\|,$$

for all $(t,t_0)\in T$ *and for all* $(x,v)\in Y$.

Further, if we consider

$$H(u)=f(u) \text{ and } N_1(u)=[M_1(u)+\hat{M}]f(u),$$

where $f(u)=e^{\nu u}\in\Theta_+$ *and* $u\geq 0$, *we obtain relation* (t_3^1).

(ii) We have considered $H\in\Gamma$ *and* $h\in\Psi$, *hence there exists a constant* $\beta>0$ *such that* $h(s)\leq e^{-\beta(t-s)}H(t)$, *for all* $(t,s)\in T$. *As the skew-evolution semiflow* C_2 *has exponential growth, according to Definition 2.3, there exists a nondecreasing function* $g:\mathbf{R}_+\to[1,\infty)$ *with the property* $\lim\limits_{t\to\infty} g(t)=\infty$ *such that*

$$\left\|\Phi_2(s,t_0,x)v\right\|\leq g(t-s)\left\|\Phi_2(t,t_0,x)v\right\|,$$

$$\forall\ (t,s),(s,t_0)\in T,\ \forall\ (x,v)\in Y.$$

We will denote

$$K=\int_0^1 e^{-\beta\tau}g(\tau)d\tau.$$

We obtain successively following relations

$$K\|P_2(x)v\|=\int_{t_0}^{t_0+1} e^{-\beta(\tau-t_0)}g(\tau-t_0)\left\|\Phi_2(t_0,t_0,x)v\right\|d\tau\leq$$

$$\leq\int_{t_0}^{t_0+1} e^{-\beta(\tau-t_0)}\left\|\Phi_2(\tau,t_0,x)v\right\|d\tau$$

$$\leq M_2(t)e^{\beta(t-t_0)}\left\|\Phi_2(t,t_0,x)v\right\|$$

for all $(t,t_0)\in T$ *and for all* $(x,v)\in Y$. *According to Definition 2.2, this relation is equivalent with*

$$\left\|\Phi_2(s,t_0x)v\right\|\leq\frac{1}{K}M_2(t)e^{\beta(t-s)}\left\|\Phi_2(t,t_0,x)v\right\|,$$

for all $(t,s),(s,t_0)\in T$ *and for all* $(x,v)\in Y$.

If we take

$$H(u)=f(u) \text{ and } N_2(u)=M_2(u)f(u),$$

for $f(u)=e^{-\nu u}\in\Theta_-$ *and* $u\geq 0$, *relation* (t_3^2) *is obtained.*

(iii) and (iv) can simmilarly be proved.

CONCLUSION

In the last decades, a great progress concerning the study of asymptotic behaviors for evolution equations can be observed. The possibility of reducing the nonautonomous case in the study of evolutionary families or skew-product flows to the autonomous case of evolution semigroups on Banach spaces is considered an important way toward interesting applications. The study of the asymptotic behavior of linear skew-product semiflows has been used in the theory of evolution equations in infinite dimensional spaces. The approach from the point of view of asymptotic properties for the evolution *semigroup associated to the linear skew-product semiflows was essential. Instead, in our study we have considered more general characterizations for the asymptotic properties of the solutions of evolution equations, described by means of skew-evolution semiflows, which generalize the above notions. Also, the approach was not restrained by considering in the definitions exponentials. As a remark, in Definition 3.2 we have the definitions for H-stability, H-instability, H-growth and H-decay, characterized, respectively, by Theorem 3.1, which extends toward applications in engineering and economics the study of evolution equations.*

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REFERENCES

[1] S. ELAYDI, O. HAJEK, *Exponential trichotomy of differential systems*, J. Math. Anal. Appl., 129 (1988) 362-374.

[2] M. MEGAN, *On H-stability of evolution operators*, Preprint Series in Mathematics, West University of Timisoara, **66** (1995) 1-7.

[3] M. MEGAN, C. STOICA, *On uniform exponential trichotomy of evolution operators in Banach spaces*, Integral Equations Operator Theory, 60, No. 4 (2008) 499-506.

[4] M. MEGAN, C. STOICA, *Exponential instability of skew-evolution semiflows in Banach spaces*, Studia Univ. Babes-Bolyai Math., LIII, No. 1 (2008) 17-24.

[5] R.J. SACKER, G.R. SELL, *Existence of dichotomies and invariant splittings for linear differential systems III*, J. Differential Equations, **22** (1976) 497-522.

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