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DYNAMICS OF WORKING PROCESS OF FLAT SIEVES

Abstract:
The operation of separation of seeds is realized due to the vibration of sieve. The operation of separation is analyzed with the help of the particle model which executes vibration motions on a plane with friction. There are analyzed displacement regimes of particle by forward sliding and back sliding without detachment. Because of velocity discontinuity which appears as consequence of friction between particle and plan or of dropping on plan in the case of detachment, vibro-impact motion regimes appear. That is why, for the study of motion, there are applied the specific methods, concerning the vibro-impact regimes.

Keywords:
motion, flat sieves, dynamic model, sliding regimes

INTRODUCTION

Generally, the phenomenon of vibro-transfer is essentially influenced by the material behavior, characterized by composition, humidity, adherence, nature etc. In the first approximation, the experiences shown that the material can be schematized by a simple material particle which moves with friction on the vibrating surface (Figure 1).

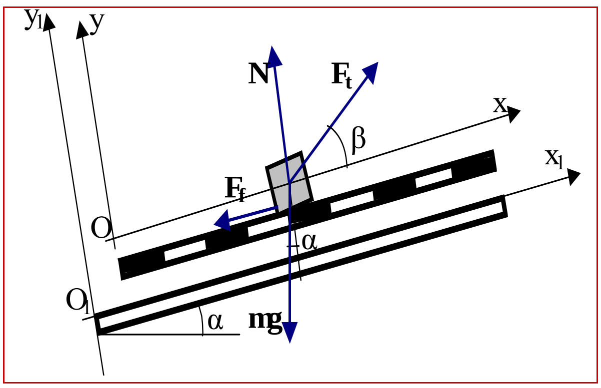

Figure 1. Dynamic model

The particle of mass m is supposed to be placed on the vibrating plan, inclined to the angle α, in relation to the horizontal surface. It is supposed that the vibrating plan executes a vibration translation motion on a direction which makes the angle β with the inclined plan and it has the amplitude r. Thus, a current point of the plan executes a vibration displacement, given by the law rsinψ, on a direction which makes the angle β with the inclined plan, where ψ=ωt. So, in relation to the fixed frame $O_1x_1y_1$, the coordinates of the point O, the origin of the mobile frame Oxy, bound to the inclined plan, (figure 1), at a certain moment are

$$x_0 = r\cos\beta\sin\psi$$

and

$$y_0 = r\sin\beta\sin\psi \qquad (1)$$

The differential equation of relative motion of particle of mass m has the form

$$m\bar{a}_r = \bar{F}_f + \bar{N} + m\bar{g} + \bar{F}_t \qquad (2)$$

Because the transport force of inertia is $\bar{F}_t = -m\bar{a}_t$, *where* a_t *is the acceleration of transport of particle, identical to the acceleration of the point O, the components of the transport force of inertia are*

$$F_{tx} = -m\ddot{x}_0 = mr\omega^2 \cos\beta \sin\psi\ , \qquad (3)$$

$$F_{ty} = -m\ddot{y}_0 = mr\omega^2 \sin\beta \sin\psi\ .$$

As consequence, the differential equation of relative motion (2) has the following projections on the axes of the frame Oxy:

$$m\ddot{x} = -\mu N sign\dot{x} + mr\omega^2 \cos\beta \sin\psi - mg\sin\alpha\ , \qquad (4)$$

$$m\ddot{y} = N + mr\omega^2 \sin\beta \sin\psi - mg\cos\alpha\ .$$

Taking into account that there are considered only the motions of particle, in contact to the plan, it must be put y=0, so that from the second equation (4), it results

$$N = m\left(g\cos\alpha - r\omega^2 \sin\beta \sin\psi\right). \qquad (5)$$

CHARACTERISTICS OF MOTION OF SLIDING

As a principle, the particle which is situated in the rest position, at a certain moment becomes to slide on the plan, forward or back.

For the beginning it is supposed that the particle executes a forward sliding motion in relation to the sieve. On the particle act the force of weight mg, the normal reaction N, and the force of friction F= μN; the motion of transport being a translation, the Coriolis force of inertia is null.

If the expression (5) of the normal reaction N is introduced in the first differential equation in (4), it arrives at the following relation:

$$\ddot{x} = -\frac{g\sin(\alpha+\phi)}{\cos\phi} + \frac{r\omega^2\cos(\beta-\phi)}{\cos\phi}\sin\psi\ ,\ (\psi=\omega t). \qquad (6)$$

This relation represents the fundamental equation for the study of the forward motions of sliding on the vibrating sieve.

The beginning moment of the forward sliding is denoted by $t=t_1$ *and so,* $\psi_1 = \omega t_1$. *It can mention that this moment corresponds to the condition that the acceleration* $\ddot{x}$ *to be null.*

If the acceleration (6) is made equal to zero, it is obtained the following equation, in the initial moment of the forward motion of sliding:

$$\sin\psi_1 = \frac{g}{r\omega^2}\cdot\frac{\sin(\alpha+\phi)}{\cos(\beta-\phi)} \qquad (7)$$

Taking into account the relation (7), the fundamental equation (6) can be also written

$$\ddot{x} = r\omega^2\frac{\cos(\beta-\phi)}{\cos\phi}\left(\sin\psi - \sin\psi_1\right) \qquad (8)$$

The forward motion of sliding is characterized by $t > t_1$ *and so,* $\psi > \psi_1$.

Considering the function $\dot{x} = \dot{x}(t)$, *if* $\ddot{x} > 0$, *the function* $\dot{x}$ *is increasing. Thus, from the moment* $t = t_1$ *when the velocity is nullifying, i.e.* $\dot{x}(t_1) = 0$ *and* $\ddot{x} > 0$, *it results* $\dot{x} > 0$. *So, the forward motion of sliding takes place. Thus, from the relation (8), written for the moment given by* ψ_1 *it must be satisfied the inequality*

$$\sin\psi > \sin\psi_1. \qquad (9)$$

From the relation (7), written for ψ_1 *it can be supposed that* $\psi_1 \in \left(0;\frac{\pi}{2}\right)$ *which, in accordance to the inequality (9), leads to the condition* $\psi \in \left(\psi_1;\pi-\psi_1\right)$.

By integrating the differential equation of sliding motion which begins for $t=t_1$ *it is found*

$$\dot{x} = -r\omega\frac{\cos(\beta-\phi)}{\cos\phi}\begin{bmatrix}\cos\psi - \cos\psi_1 \\ +\sin\psi_1\cdot(\psi-\psi_1)\end{bmatrix} \qquad (10)$$

The forward regime of sliding stops at the moment $t = t_1^{/}$, *respectively the angle* $\psi = \psi_1^{/}$ *which corresponds to the nullifying of the relative velocity,* $\dot{x} = 0$. *So, by nullifying the expression of* $\dot{x}$, *it is deduced the equation*

$$\sin\psi_1 = \frac{\cos\psi_1^{/} - \cos\psi_1}{\psi_1 - \psi_1^{/}} \qquad (11)$$

This equation permits the calculus of the moment $t = t_1^{/}$, *corresponding to the cessation of sliding.*

The distance, covered in the case of the forward sliding is given by the integral

$$s_{1,2} = \int_{t_1}^{t_1'} \dot{x} dt \tag{12}$$

Taking into account the relation (10), after the effecting of calculus, the integral (12') becomes

$$s_1 = -\frac{r\cos(\beta-\phi)}{\cos\phi}\left[\begin{array}{l}\frac{(\psi_1'-\psi_1)^2}{2}\sin\psi_1\\ +\sin\psi_1' - \sin\psi_1 - (\psi_1'-\psi_1)\cos\psi_1\end{array}\right] \tag{13}$$

If in the relation (13) it is replaced $\sin\psi_1$ *given by the equation (11), then for the displacements with forward sliding, it can be written the relation*

$$s_1 = \frac{r\cos(\beta-\phi)}{\cos\phi}\cdot\Phi(\psi_1), \tag{14}$$

In an analogous way it is treated the case corresponding to the back motion of sliding. By back sliding it means the relative motion with friction of the material particle on the vibrating sieve, in the negative direction of the Ox axis, i.e. in opposite direction to the transporting one.
Taking into account that the force of friction is orientated in the positive direction of the Ox axis and projecting the differential equation of the relative motion, it is obtained

$$m\ddot{x} = \mu N - mg\sin\alpha + mr\omega^2\cos\beta\sin\psi \tag{15}$$

or, if it is taken into account the equation (2), it results

$$\ddot{x} = -\frac{g\sin(\alpha-\phi)}{\cos\phi} + r\omega^2\frac{\cos(\beta+\phi)}{\cos\phi}\sin\psi \tag{16}$$

This relation represents the fundamental equation for the study of the back motions of sliding on the vibrating sieve.
The back sliding begins at the moment $t=t_2$ *and* $\psi_2 = \omega t_2$, *when* $\ddot{x} = 0$.
From the expression (16) of the acceleration, made equal to zero, it is obtained the equation

$$\sin\psi_2 = \frac{g}{r\omega^2}\cdot\frac{\sin(\alpha-\phi)}{\cos(\beta+\phi)} \tag{17}$$

Taking into account the relation (17), the fundamental equation (16) can be also written as fallows:

$$\ddot{x} = r\omega^2\frac{\cos(\beta+\phi)}{\cos\phi}(\sin\psi - \sin\psi_2) \tag{18}$$

Because the back motion of sliding begins at the moment $t=t_2$ *and it correspunds to the interval* $t > t_2$, *respectively* $\psi > \psi_2$, *in the same way as in the previous case, it is considered the function* $\dot{x} = \dot{x}(t)$ *which, for* $\ddot{x} < 0$, *is a decreasing one. Thus, begining with the moment* $t=t_2$ *for wich* $\ddot{x}(t_2) = 0$, *the velocity* $\dot{x}$ *is negative (*$\dot{x} < 0$*), so that a back motion of sliding takes place.*
In accordance to the relation (18) and from the condition $\ddot{x} < 0$, *it results*

$$\sin\psi < \sin\psi_2. \tag{19}$$

Supposing ψ_2 *given by the relation (17) in the first quadrant, i.e.* $\psi_2 \in \left(0;\frac{\pi}{2}\right)$, *it results that the angle* ψ *must be in the interval* $\psi \in (\pi - \psi_2; 2\pi)$.
By the integration of the differential equation (18), it is obtained the expression of the velocity:

$$\dot{x} = -r\omega\frac{\cos(\beta+\phi)}{\cos\phi}\left[\begin{array}{l}\cos\psi - \cos\psi_2\\ +\sin\psi_2\cdot(\psi-\psi_2)\end{array}\right] \tag{20}$$

The end of duration of the back sliding is obtained by nullifying the expression of the velocity $\dot{x}$, *given by the relation (20). The final moment, denoted by* $t = t_2'$, *respectively the angle* $\psi = \psi_2'$, *is obtained by solving the transcedental equation*

$$\sin\psi_2 = \frac{\cos\psi_2' - \cos\psi_2}{\psi_2 - \psi_2'} \tag{21}$$

The distance, covered in the case of the back sliding, is given by the integral

$$s_2 = \int_{t_2}^{t_2'} \dot{x} dt \tag{22}$$

Taking into account the relation (20), after effecting the calculus, becomes

$$s_2 = -\frac{r\cos(\beta+\phi)}{\cos\phi}\left[\begin{array}{l}\frac{(\psi_2'-\psi_2)^2}{2}\sin\psi_2 + \sin\psi_2'\\ -\sin\psi_2 - (\psi_2'-\psi_2)\cos\psi_2\end{array}\right] \tag{23}$$

If in the relation (23), it is replaced $\sin\psi_2$ *given by the equation (21), then for the displacements with back sliding, it can be written the relation*

$$s_2 = \frac{r\cos(\beta+\phi)}{\cos\phi}\cdot\Phi(\psi_2) \qquad (24)$$

If during the time $T = \frac{2\pi}{\omega}$, *the material particle moves by forward and back sliding, the advance in the positive direction of the axis* O_1x_1 *has the value*

$$s = s_1 - s_2 \qquad (25)$$

and the average velocity of particle is

$$v_m = (s_1 - s_2)\frac{1}{T} = (s_1 - s_2)\frac{\omega}{2\pi}. \qquad (26)$$

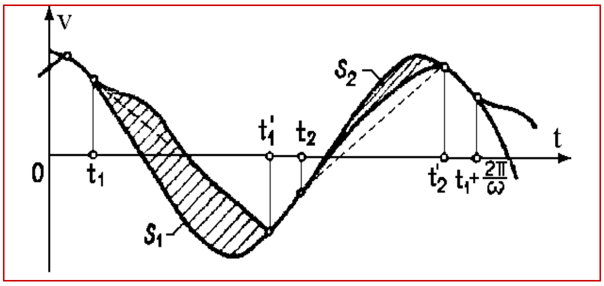

Figure 2. Absolute velocity, transport velocity and displacement

The graphical representations of the absolute velocity v, *transport velocity* v_t *and displacement* s_t *with sliding along the vibrating sieve, on which there are superposed the slips* s_1 *and* s_2, *are shown in Figure 2, for a cycle of vibration.*

CONCLUSION

All obtained results correspond to the case of sliding motion, without detachment, i.e. for $N > 0$. *In accordance to the relation (5), it results*

$$\sin\psi < \frac{g}{r\omega^2}\cdot\frac{\cos\alpha}{\sin\beta}. \qquad (27)$$

The analysis of possible motion regimes can be more easily made with the help of the kinematical index:

$$K = \frac{r\omega^2}{g} \qquad (28)$$

Thus, a condition for do not exist detachment, in accordance to the relation (5), is that the equation $N=0$ *do not have solution, that leads to the inequality*

$$K < \frac{\cos\alpha}{\sin\beta} \qquad (29)$$

Now, it is supposed the condition (27) as satisfied, so that all regimes of motion are with sliding, only. The characteristic indexes of forward and back motions of sliding are denoted by the parameters

$$K_{1,2} = \frac{\sin(\alpha \mp \phi)}{\cos(\beta \pm \phi)}. \qquad (30)$$

As consequence, the relation (7), with the notations (29), becomes

$$\sin\psi_{1,2} = \frac{K_{1,2}}{K} \qquad (31)$$

For the beginning, it is considered $K_1<K_2$. *Then, there are the following possible situations:*

a] $K_1<K<K_2$ *for which the angle* ψ_2 *can not exist, situation that corresponds to a sliding motion, forward only* (AI_t)*;*

b] $K<K_1<K_2$ *when no one of the angles of motion initiation is possible, that corresponds to the situation of rest (R);*

c] $K_1<K_2<K$, *situation when both types of sliding are possible* ($\psi_1<\psi_2$), *and the regime of motion is with forward and back sliding* (AI_t+AI_p).

For the situation when $K_1> K_2$ *the possible cases are as follows:*

a] $K_2<K<K_1$ *for which the moment* ψ_1 *does not exist, which shows that the only possible regime of motion is with back sliding* (AI_p)*;*

b] $K<K_2<K_1$, *where initial moments for motions with sliding do not exist, i.e. there is the rest, only (R);*

c] $K_2<K_1<K$ *where there are possible solutions for both initial moments* ($\psi_1 <\psi_2$) *and so, the regime of motion is with forward and back sliding* (AI_t+AI_p).

Finally, in accordance to the relations (31), it can be written

$$K_1 - K_2 = -\frac{\sin 2\phi\cos(\alpha+\beta)}{\cos(\beta+\phi)\cos(\beta-\phi)} \qquad (32)$$

The conditions $K_1<K_2$ are realized if $\cos(\alpha+\beta)>0$, that leads to $\beta<(\pi/2)-\alpha$. The other situation, $K_1>K_2$ can appear if $\cos(\alpha+\beta)<0$, i.e. only for $\beta>(\pi/2)-\alpha$.

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