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HEAT TRANSFER EFFECTS ON ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS FLUX

Abstract:
Theoretical solution of unsteady flow past an uniformly accelerated infinite vertical plate variable temperature and mass flux analyzed . The temperature from the plate to the fluid is raised to Tω and the species concentration is raised to constant rate. The dimensionless governing equations are solved using Laplace-transform technique. The velocity, temperature and concentration fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number and time. It was observed that the velocity increases with increasing values of thermal Grashof number or mass Grashof number. It was also observed that the velocity increases with decraesing values of the Schmidt number.

Keywords:
linearly, accelerated, vertical plate, variable temperature, mass flux

INTRODUCTION

Processes involving coupled heat and mass transfer occur frequently in nature. It occurs not only due to temperature difference, but also due to concentration difference or the combination of these two. Quite often, there exist certain industrial processes involving continuous surfaces that move steadily through an otherwise quiescent ambient environment for which a correct assessment of the axial temperature and concentration variation of the material are given relevant importance.

Gupta et al(1979) have studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis et al(1981). Raptis and Singh (1983) MHD free convection flow past an accelerated vertical plate. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar (1982). Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Basant Kumar Jha and Ravindra Prasad (1990) analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources . The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986).

It is proposed to study unsteady flow past an uniformly accelerated infinite vertical plate in the presence of variable temperature and mass flux. The dimensionless governing equations are solved using the Laplace-transform technique. Such a study found useful in hot extrusion of steel, the lamination and meltspinning processes in the extrusion of polymers.The solutions are in terms of exponential and complementary error function.

GOVERNING EQUATIONS

Here the unsteady flow of a viscous incompressible fluid past an uniformly accelerated vertical infinite plate with variable temperature and mass flux has been considered. The x-axis is taken along the plate in the vertically upward direction and the y *-axis is taken normal to the plate. At time* $t' \le 0$, *the plate and fluid are at the same temperature* T_∞ *and concentration* C'_∞. *At time* $t' > 0$, *the plate is accelerated with a velocity* $u = u_0 t'$ *in its own plane against gravitational field.The temperature from the plate to the fluid is maintained uniformly and the concentration level is raised at an uniform rate. Then under the usual Boussinesq's approximation the unsteady flow is governed by the following equations:*

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \qquad (3)$$

With the following initial and boundary conditions:

$$\begin{array}{lllll} & u=0, & T=T_\infty, & C'=C'_\infty & \text{forall } y,t'\le 0 \\ t'>0: & u=u_0 t', & T=T_\omega', & \dfrac{\partial C}{\partial y}=-\dfrac{j''}{D} & \text{at } y=0 \\ & u\to 0 & T\to T_\infty, & C'\to C'_\infty & \text{as } y\to\infty \end{array} \qquad (4)$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0},\ t = \frac{t' u_0^2}{\nu},\ Y = \frac{y u_0}{\nu},$$

$$\theta = \frac{T - T_\infty}{T_\omega - T_\infty},\ Gr = \frac{g\beta\nu(T_\omega - T_\infty)}{u_0^3},\ C = \frac{C' - C_\infty}{\left(\dfrac{j''\nu}{Du_0}\right)}, \qquad (5)$$

$$Gc = g\beta^*\left(\frac{j''\nu^2}{Du_0^4}\right),\ Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D}$$

in equations (1) to (4), lead to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \qquad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \qquad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \qquad (8)$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{lllll} & U=0, & \theta=0, & C=0 & \text{forall } Y,t\le 0 \\ t>0: & U=t, & \theta=t, & \dfrac{\partial C}{\partial y}=-1 & \text{at } Y=0 \\ & U\to 0, & \theta\to 0, & C\to 0 & \text{as } Y\to\infty \end{array} \qquad (9)$$

SOLUTION PROCEDURE

All the physical variables are defind in the nomenclature.The dimensionless governing equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[\begin{array}{l}(1 + 2\eta^2 \Pr)\ erfc\ (\eta\sqrt{\Pr}) \\ -\dfrac{2\eta}{\sqrt{\pi}}\sqrt{\Pr}\ \exp(-\eta^2 \Pr)\end{array}\right] \qquad (10)$$

$$C = 2\sqrt{t}\left[\frac{\exp(-\eta^2 Sc)}{\sqrt{\pi}\sqrt{Sc}} - \eta erfc(\eta\sqrt{Sc})\right]$$

$$U = t\left[(1+2\eta^2)erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right] \qquad (11)$$

$$+\frac{Grt^2}{6(\Pr-1)}\left[\begin{array}{l}(3+12\eta^2+4\eta^4)erfc(\eta) \\ -\dfrac{\eta}{\sqrt{\pi}}(10+4\eta^2)\exp(-\eta^2) \\ -(3+12\eta^2\Pr+4\eta^4(\Pr)^2) \\ erfc(\eta\sqrt{\Pr})+\dfrac{\eta\sqrt{\Pr}}{\sqrt{\pi}}(10 \\ +4\eta^2\Pr)\exp(-\eta^2\Pr)\end{array}\right]$$

$$(12)$$

$$+\frac{Gct\sqrt{t}}{3(Sc-1)\sqrt{Sc}}\left[\begin{array}{l}\dfrac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) \\ -\dfrac{4}{\sqrt{\pi}}(1+\eta^2 Sc)\exp(-\eta^2 Sc) \\ -\eta(6+4\eta^2)erfc(\eta) \\ +\eta\sqrt{Sc}(6+4\eta^2 Sc)erfc(\eta\sqrt{Sc})\end{array}\right]$$

where , $\eta = \dfrac{Y}{2\sqrt{t}}$.

RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr *and* t *upon the nature of the flow and transport. The value of the Schmidt number* Sc *is taken to be* 0.6 *which corresponds to water-vapor. Also, the* *values of Prandtl number* Pr *are chosen such that they represent air* ($Pr = 0.71$) *and water (Pr=7.0). The numerical values of the velocity,tamperature and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.*

The velocity for different values of the Schmidt number ($Sc = 0.16, 0.3, 0, 6, 2.01$), $Gr = Gc = 5$ *and Pr=0.71 time* $t = 0.4$ *are shown in figure 1 in the presence of air. The trend shows that the velocity increases with decreasing Schmidt number. It was observed that the relative variation of the velocity with the magnitude of the Schmidt number.*

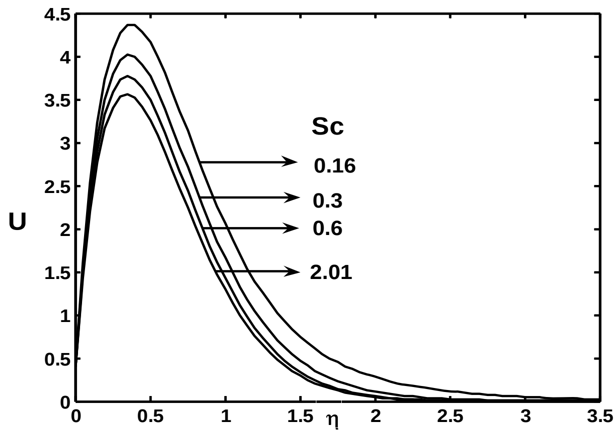

Figure 1. Velocity profiles for different values of Sc.

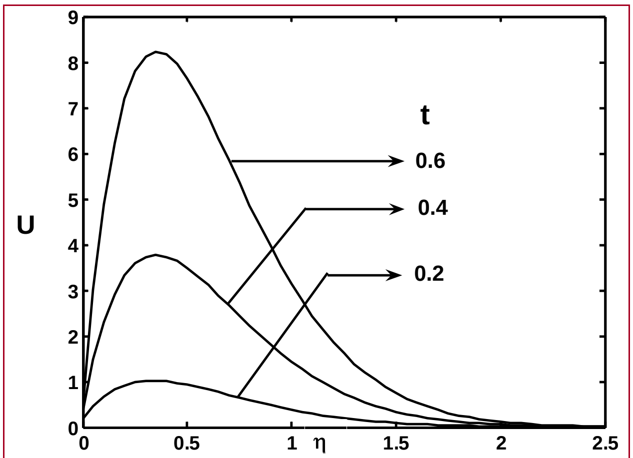

Figure 2. Velocity profiles for different values of t.

The effect of velocity for different ($t = 0.2, 0.4, 0.6$), $Gr = 5$, $Gc = 5$ *and Pr=0.71 are studied and presented in figure 2. It was observed that the velocity increases with increasing values of* t *. Figure 3. demonstrates the effects of different thermal Grashof number* ($Gr = 2,5$) *and mass Grashof number* ($Gc = 2,5$) *and Pr=0.71 on the velocity at time* $t = 0.4$ *. It was observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.*

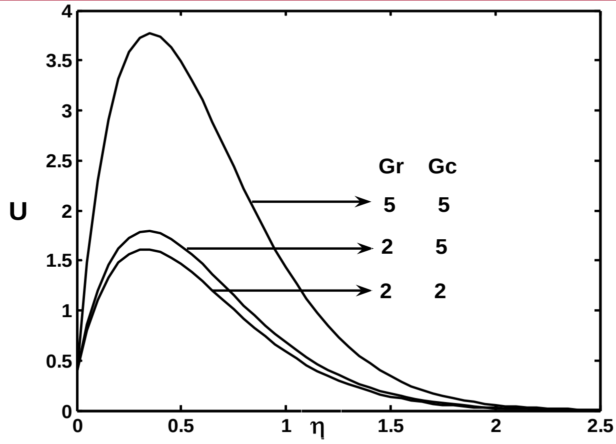

Figure 3. Velocity profiles for different values of Gr,Gc.

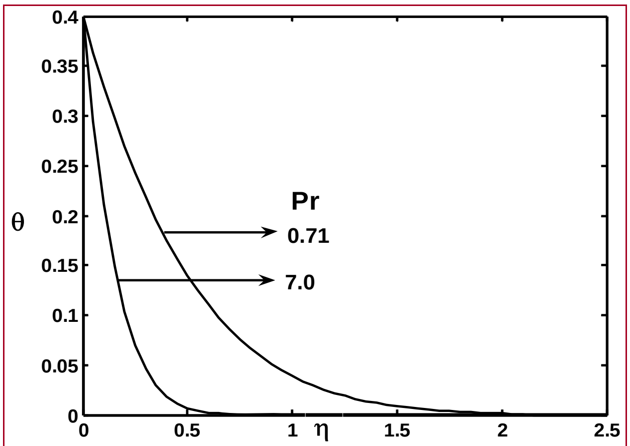

Figure 4. Temperature profiles for different values of Pr.

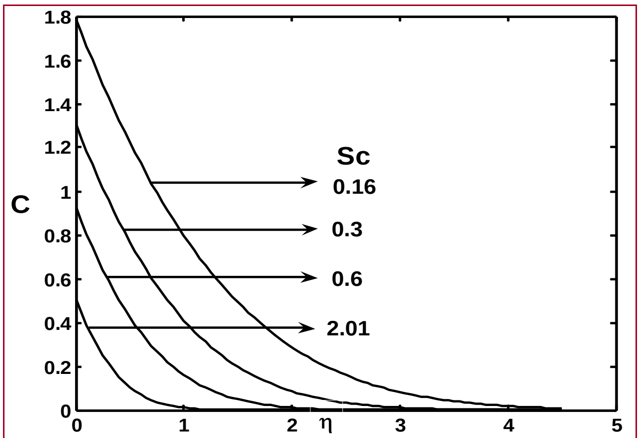

Figure 5. Concentration profiles for different values of Sc.

The temperature profiles are calculated for water and air from equation (10) and these are shown in Figure 4. at time t=0.4. The effect of the Prandtl number plays an important role in temperature field. It was observed thet the temperature increases with decreasing Prandtl number. This shows that the heat transfer is more in air than in water.

Figure 5 represents the effect of concentration profiles at time $t = 0.4$ for different Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It was observed that the wall concentration increases with decreasing values of the Schmidt number.

CONCLUSION

Theoretical syudy of unsteady flow past an uniformly accelerated infinite vertical plate in the presence of variable temperature and mass flux have been analyzed. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, mass Grashof number, Schmidt number and t are studied. The conclusion of the study are as follows:

- *The velocity increases with increasing values of Gr, Gc and t.*
- *The transient velocity increases with decreasing Schmidt number Sc.*
- *The wall concentration increases with decreasing Schmidt number.*

NOMENCLATURE, GREEK SYMBOLS, SUBSCRIPTS

A constant
C' species concentration in the fluid
C dimensionless concentration
C_p specific heat at constant pressure
D mass diffusion coefficient
G_c mass Grashof number
G_r thermal Grashof number
g accelerrated due to gravity
k thermal conductivity
Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate
t' time
t dimensionless time
u velocity of the fluid in the x-direction
u_0 velocity of the plate
U dimensionless velocity
x spatial coordinate along the plate
y' coordinate axis normal to the plate m
y dimensionless coordinate axis normal to the plate
β volumetric coefficient of thermal expansion
β^* volumetric coefficient of expansion with concentration
μ cofficient of viscosity
υ kinematic viscosity
ρ density of the fluid
τ dimensionless skin-friction kg.
θ dimmensionless temperature
η similarity parameter
erfc complementary error function

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