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SPUR GEAR DRIVES WITH ASYMMETRIC INVOLUTE–LANTERN MESHING

ABSTRACT:
A new asymmetric meshing has been proposed, formed by spur gears with a very small number of teeth where the transverse contact ratio is bigger than one. The meshing in the general case where the gears have different number of teeth has been clarified. The main geometric parameters have been defined. The equations of the tooth profiles have been determined. Analytic dependencies for the geometric dimensions of the gears have been shown. The geometric parameters of the asymmetric gear drive have been found.

KEYWORDS:
Spur gear, Asymmetric teeth, Involute – lantern meshing, Pressure angle, Protruding profile, Concave profile

INTRODUCTION

The use of spur involute gears for continuous transmission of rotary motion is limited in cases where teeth number z of the pinion (the small gear) is very small (z = 4,3,2,1). Then the transverse contact ratio ε_α is smaller than one and after a specified gear pair goes out of meshing the next gear pair still has not meshed. Due to the small contact ratio the movement in the gear is interrupted. In practice this means that using the traditional involute meshing it is not possible to realize continuous transmission of motion from the one gear to the other one if gears have a very small teeth number.

If the gear meshing is involute and symmetric, in [Alipiev et al., 2009] it is proved that at $\varepsilon_\alpha > 1$ the smallest equal teeth number of the gears is e $z_1 = z_2 = 5$. This result, got in different ways, is well-known also by the publications of [Kotelnikov, 1973], [Bulgakov, 1995] and [Kapelevich et al., 2002]. When the asymmetric involute meshing is used [Alipiev, 2008] for the smallest equal teeth number of the gears, synthesized using the "method of the realized potential" [Alipiev, 2009] it is got $z_1 = z_2 = 4$. In this case the opposite lateral teeth profiles are drawn by different involute curves. Then the transverse contact ratio for the driving direction of movement is $\varepsilon_\alpha > 1$, and for the non-driving direction - $\hat{\varepsilon}_\alpha < 1$.

Gears of a small teeth number find practical application mainly in gear pumps, gear compressors, kinematic transmissions, realizing large tooth ratios, some types of elevating mechanisms etc. Under equal other conditions (dimensions, weight, width of gears), the small teeth number leads to an increase of the volume of tooth spaces and an increase of the efficiency of the gear pump. Besides, by decreasing the teeth number of the pinion and keeping the tooth space of the gear, its tooth ratio increases.

An object of survey in the present paper is the geometry of a new type of asymmetric meshing where the continuous transmission of motion $(\varepsilon_\alpha > 1)$ in the driving direction is realized by gears, having a very small teeth number.

GENERAL DATA FOR THE MESHING

The transverse contact ration in the driving direction of motion could be increased if an asymmetric meshing shown on Figure 1 [Alipiev, 2010] is used. In this case the asymmetric teeth of the meshed gears have a protruding and concave sides. Despite the geometric shape of each tooth of the generally different gears 1 and 2 is simply defined by three successively connected curves. The first curve, shaping the protruding (driving) tooth side is the involute curve ME $(M^/E^/)$. It is got as a trajectory of point K on the straight line AB, when this straight line rolls on the respective base circle of a radius r_{b1} or r_{b2}. The beginning of the first curve begins from the base circle, and its end ends in the point E $(E^/)$. Using the second curve is shaped the tooth crest in the area from point $(E$ $(E^/)$ to point N $(N^/)$. This curve is an arc of a circle (called lantern one) of a radius r_{p1} (r_{p2}), of a center C_1 (C_2) at a distance of r_{c1} (r_{c2}) from the gear center O_1 (O_2).

Despite the center C_1 (C_2) lies on the tangent to the base circle of the respective gear, descended from the end point E ($E^{/}$) of the involute curve.

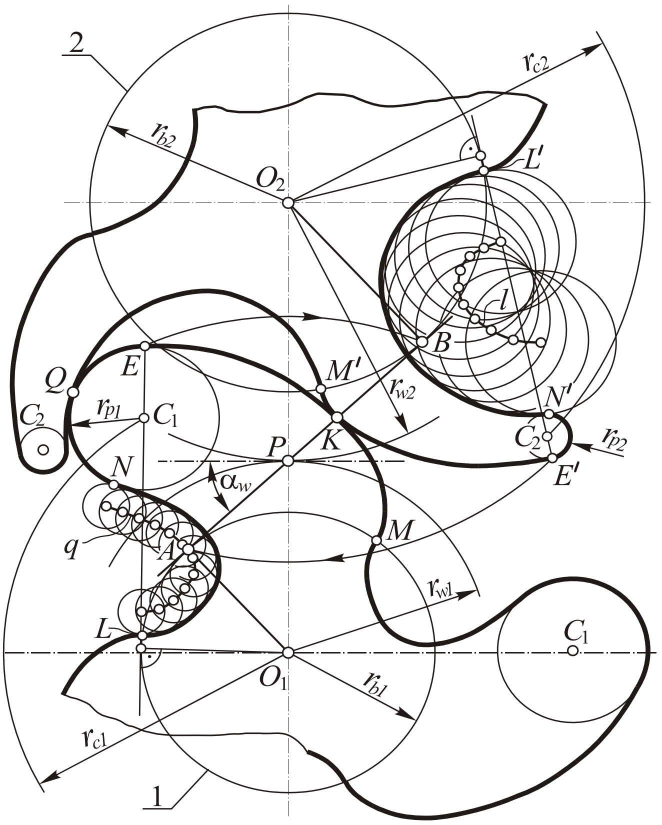

Figure 1. Formation of involute-lantern meshing

The third curve NL ($N^{/}L^{/}$) shapes the concave side of the asymmetric tooth. At gear 1 this curve NL is got as a wrapping curve of the relative places which the tooth crest gets (the lantern circle of a radius r_{p2}) from gear 2 in the plane of gear 1. Analogously the curve $N^{/}L^{/}$ of the tooth of gear 2 is got as a wrapping curve of the lantern circle of a radius r_{p1} of gear 1. The end point L ($L^{/}$) of the third curve lies on the respective base circle r_{b1} (r_{b2}) and appears as an initial point for the involute profile of the next neighbouring tooth.

The provision of maximum overlap by meshing of the involute profiles is got due to their contact along the whole line of action AB. For this purpose the areas of the involute teeth profiles are chosen in this way that the кръговите trajectories EB and $E^{/}A$ of their end points E and $E^{/}$ cross the end points B and A of the action line, and their initial points M and $M^{/}$ lie on the corresponding base circles.

The proposed gear meshing is called an „involute-lantern meshing”, because in the driving direction of motion the conjugate profilers are the meshed involute curves ME and $M^{/}E^{/}$, and in the opposite direction mesh the arc EN ($E^{/}N^{/}$, respectively) of the lantern circle of the one gear of the concave profile $L^{/}N^{/}$ (LN, respectively) of the asymmetric tooth of the other gear. In other words, in the one direction of movement the meshing is involute and in the other one - lantern. Hence the lantern meshing is corrected since the centers C_1 and C_2 of the lanterns do not lie on the respective centroids of the gears when realizing the gear meshing, defined as pitch circles of radii r_{w1} and r_{w2}. In contrast to the traditional lantern meshing where the tooth profile of the one gear is a circle and of the other one - an equidistant curve of a epicycloid, in the proposed meshing the non-involute teeth profile is a combination of two connected curves (concave profile and an arc of a circle).

EQUATION OF THE TEETH PROFILES

The geometry of asymmetric gear profiles is fully determined if the following independent values (Figure 1) are specified: teeth number z_1 and z_2 of both gears; the radii of the pitch circles r_{w1} and r_{w1}; the radii of the lantern circles r_{p1} and r_{p1}; the radii r_{c1} and r_{c1}, on which are placed the centers of the lanterns; the pressure angle of the involute meshing α_w.

In order to provide expression in scale of all geometric dimensions, coming from the experience in the theory of the traditional involute meshing, eit is more rational to use another set of independent values, including: the gearing module m; teeth number z_1 and z_2; the coefficients r^*_{p1} and r^*_{p2} of the radii of the lantern circles, the coefficients λ_1 and λ_2 of the shape of the concave teeth profiles; the angle of involute meshing α_w. In this set of parameters, z_1, z_2, r^*_{p1}, r^*_{p2}, λ_1 and λ_2 cre dimensionless values, and the module m is a scale factor. The relation between the real linear dimensions and the corresponding dimensionless values from the above mentioned two sets is defined by the following equations:

$$r_{w1} = m\,z_1/2\,, \qquad r_{w2} = m\,z_2/2\,, \tag{1}$$

$$r_{p1} = m\,r^*_{p1}\,, \qquad r_{p2} = m\,r^*_{p2}\,, \tag{2}$$

$$r_{c1} = \lambda_1 r_{w1} = m\lambda_1 z_1/2\,, \qquad r_{c2} = \lambda_2 r_{w2} = m\lambda_2 z_2/2\,. \tag{3}$$

In the proposed meshing, as it was already mentioned, the opposite lateral teeth profiles are drawn by two different curves. Hence by the curves, shaping the protruding teeth side, a continuous motion in the driving direction of motion is being transmitted, and by the other curves if being shaped the concave teeth side.

- Protruded profiles. For their drawing an involute curve is being used. In the co-ordinate system XOY (Figure 2) the parametric equation of the involute ME is of the type

$$\left.\begin{aligned} X_i &= -r_i \sin\delta_i \\ Y_i &= r_i \cos\delta_i \end{aligned}\right\} \tag{4}$$

where r_i is the polar radius of the current point i of the curve, and δ_i - its polar angle.

From the theory of involute meshing [Litvin, 1968] it is known that

$$r_i = \frac{r_b}{\cos\alpha_i}\ , \qquad (5)$$

$$r_b = 0{,}5 m\, z \cos\alpha_w\,. \qquad (6)$$

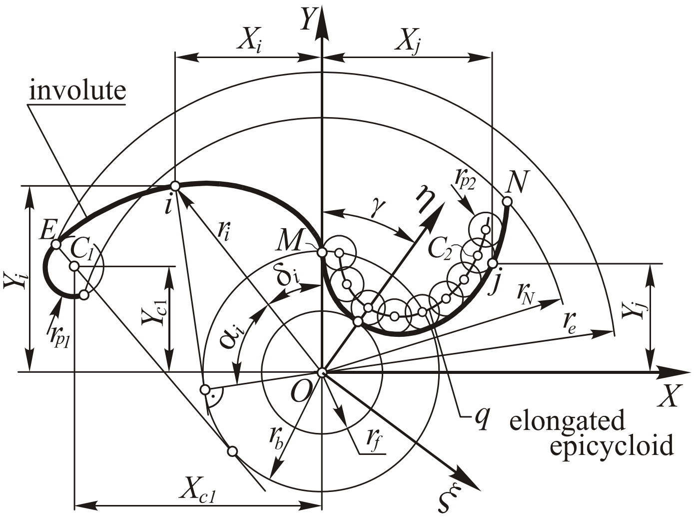

Figure 2. Geometry of teeth profiles

Taking into consideration equations (5) and (6) for the parametric equations of the involute curve finally is got and α_i is the angular para-

$$\left.\begin{aligned} X_i &= -0{,}5\, m\, z \cos\alpha_w \sin\delta_i / \cos\alpha_i = X_i(\alpha_i) \\ Y_i &= 0{,}5\, m\, z \cos\alpha_w \cos\delta_i / \cos\alpha_i = Y_i(\alpha_i) \end{aligned}\right\} \quad (7)$$

where $\delta_i = \mathrm{inv}\,\alpha_i = \tan\alpha_i - \alpha_i$,

meter of the curve. At $z = z_1$ with equations (7) are got the coordinates of the profile points ME of the gear 1, and at $z = z_2$ - the profile $M'E'$ (Figure 1) of the gear 2.

- Concave profiles. The concave profile of the asymmetric teeth is got in the following way. To the plane of the circle r_{w2} of gear 2 (Figure 1) the point C_2 is immovably connected. When this circle rolls without friction on the circle r_{w1} point C_2 draws the curve q in the plane of gear 1. Analogously the point C_1 of the plane of the circle r_{w1} draws the curve l in the plane of gear 2.

By formation of the lantern meshing as theoretical teeth profiles could be assumed:

a) point C_2 of gear 2, b) the curve q of gear 1. The equations of the curve q in the coordinate system $\xi O \eta$ shown in Figure 2 are written in the following way:

$$\left.\begin{aligned} \xi_{qj} &= \frac{m}{2}[(z_1+z_2)\sin\varphi_j - \lambda_2 z_2 \sin(\frac{z_1}{z_2}\varphi_j + \varphi_j)] \\ \eta_{qj} &= \frac{m}{2}[(z_1+z_2)\cos\varphi_j - \lambda_2 z_2 \cos(\frac{z_1}{z_2}\varphi_j + \varphi_j)] \end{aligned}\right\} \quad (8)$$

As by drawing the curve q the rolling circles r_{w1} and r_{w2} (Figure 1) contact externally, and the drawing point C_2 lies outside the circle r_{w1}, the got curve q is an elongated epicycloid.

In fact with gearings instead of the theoretical profiles are used their equidistant curves (equally spaced curves on the profile normals): a) the circle of a radius r_{p2} for gear 2, b) the curve MN for gear 1. The equation of the curve MN in the same coordinate system $\xi O \eta$ in this case is written as follows:

$$\left.\begin{aligned} \xi_j &= \frac{m}{2}\{(z_1+z_2)\sin\varphi_j - \lambda_2 z_2 \sin(\frac{z_1}{z_2}\varphi_j + \varphi_j) - \\ &\quad - \frac{2r^*_{p2}\,[\lambda_2 \sin(\frac{z_1}{z_2}\varphi_j + \varphi_j) - \sin\varphi_j\,]}{\sqrt{1 - 2\lambda_2 \cos\frac{z_1}{z_2}\varphi_j + \lambda_2^2}}\} \\ \eta_j &= \frac{m}{2}\{(z_1+z_2)\cos\varphi_j - \lambda_2 z_2 \cos(\frac{z_1}{z_2}\varphi_j + \varphi_j) - \\ &\quad - \frac{2r^*_{p2}\,[\lambda_2 \cos(\frac{z_1}{z_2}\varphi_j + \varphi_j) - \cos\varphi_j\,]}{\sqrt{1 - 2\lambda_2 \cos\frac{z_1}{z_2}\varphi_j + \lambda_2^2}}\} \end{aligned}\right\} \quad (9)$$

Out of the way of formation of the concave profile MN of the tooth of gear 1 it follows that equations (9) are equations of an equidistant curve of an elongated epicycloid.

Analogously are found the equations of the concave profile of gear 2. In this case the equations are got directly from equations (9), after replacing z_1 with z_2, z_2 with z_1, λ_2 with λ_1 and r^*_{p2} with r^*_{p1}.

In order to find the coordinates of the concave profile in the same coordinate system, in which the protruded profile is specified, it is necessary that the curve MN rotates at an angle γ. Then after the respective transformation between the coordinate systems $\xi O \eta$ and XOY finally the following equations for MN in XOY are got

$$\left.\begin{aligned} X_j &= \xi_j \cos\gamma + \eta_j \sin\gamma = X_j(\varphi_j) \\ Y_j &= -\xi_j \sin\gamma + \eta_j \cos\gamma = Y_j(\varphi_j) \end{aligned}\right\} \quad (10)$$

In which the coordinates ξ_j and η_j are preliminary defined from equations (9). The value of the angle γ is found in a numerical way. For the purpose from equation

$$\xi_j^2 + \eta_j^2 = r_b^2\,. \qquad (11)$$

Taking into consideration equations (6) and (9), the parameter φ_j for the coordinates of point M is defined, afterwards from equation

$$\gamma = \mathrm{arctg}(\xi_j / \eta_j) \qquad (12)$$

the angle γ is got.

BASIC GEOMETRICAL DEPENDENCES

The basic geometrical dimensions of gears and the parameters of the involute gearing are defined using Figure 3. In the same figure are shown also the lines of action, got as a geometric place of the contact points of asymmetric gear profiles in the still plane.

The straight line AB, as it was mentioned, is the line of action between the involute profiles. Its slope is defined by the pressure angle α_w, and its end points A and B coincide with the contact points of the straight line $N-N$ and the base circles r_{b1} and r_{b2}. In the proposed gearing the actual line AB of the involute meshing coincides with the theoretical line of action, by reason of that it has a maximum length. The contact of the concave profiles with the teeth crests (engaged in the lantern meshing) is realized over two lines of action – AQ and BR. The line of action AQ corresponds to the contact points between the concave tooth profile of gear 1 and the lantern circle (of a radius r_{p2}) of gear 2. Analogously, along the line of action BR contact the concave profile of gear 2 with the lantern circle (of a radius r_{p1}) of gear 1. The end points Q and R of the lines of lantern action are defined from the place of the boundary points N_1 and N_2.

- Pitch circles. By realizing the gear meshing the pitch circles, as centroids in the gearing, roll one over another without sliding. The diameters d_{w1} and d_{w2} of these circlesa are defined by the equations

$$d_{w1}=2r_{w1}=m\,z_1\,,\qquad d_{w2}=2r_{w2}=m\,z_2\,. \quad (13)$$

- Line of involute meshing. Its length l_{AB} is equal to the straight line AB. Taking into consideration the rectangular triangles PAO_1, PBO_2 and equations (13), for the straight line of action the following formula is got

$$l_{AB}=l_{AP}+l_{BP}=0{,}5\,m(z_1+z_2)\sin\alpha_w\,. \quad (14)$$

- Pressure angles in the end points of the involute curves. As it was already mentioned the place of the end points e_1 and e_2 of the involute profiles are defined so that the circles of radii r_{e1} and r_{e2} should cross the end points B and A of the line of lantern action. In this case from Figure 3 it is directly seen that $\overline{AB}=\overline{e_1b_1}=\overline{e_2b_2}$. Then from the rectangular triangles $e_1b_1O_1$, $e_2b_2O_2$ and equations (14) and (6) for the pressure angles α_{e1} and α_{e2} of the end involute points the following formulas are got

$$\mathrm{tg}\alpha_{e1}=\frac{l_{AB}}{r_{b1}}=\frac{z_1+z_2}{z_1}\,\mathrm{tg}\alpha_w\,,\ \mathrm{tg}\alpha_{e2}=\frac{l_{AB}}{r_{b2}}=\frac{z_1+z_2}{z_2}\,\mathrm{tg}\alpha_w\,. \quad (15)$$

- Radii of the end points of the involutes. From the triangles $e_1b_1O_1$, $e_2b_2O_2$ it follows that

$$r_{e1}=\sqrt{r_{b1}^2+l_{AB}^2}\,,\qquad r_{e2}=\sqrt{r_{b2}^2+l_{AB}^2}\,. \quad (16)$$

- Radii of the centers of the lantern circles. The centers C_1 and C_2 of the lantern circles lie on the normal of the involute curves, dropped from their end points e_1 and e_2. In order to define their position it is necessary preliminary to find the radii of the lantern circles. The calculation of r_{p1} and r_{p2} is done by numerical method, providing the simultaneous of the lantern circle with the concave and protruded tooth profile.

The value of the radii r_{c1} and r_{c2} to the centers of the lantern circles are defined by triangles $C_1b_1O_1$ and $C_2b_2O_2$, whence

$$r_{c1}=\sqrt{r_{b1}^2+(l_{AB}-r_{p1})^2}\,,\ r_{c2}=\sqrt{r_{b2}^2+(l_{AB}-r_{p2})^2}\,. \quad (17)$$

- Addendum circles. They are defined as distances from the center of the corresponding gear to its most distant tooth point. In the discussed case the radii r_{a1} and r_{a2} of the addendum circles are equal to the corresponding straight lines O_1a_1 and O_2a_2, i.e.

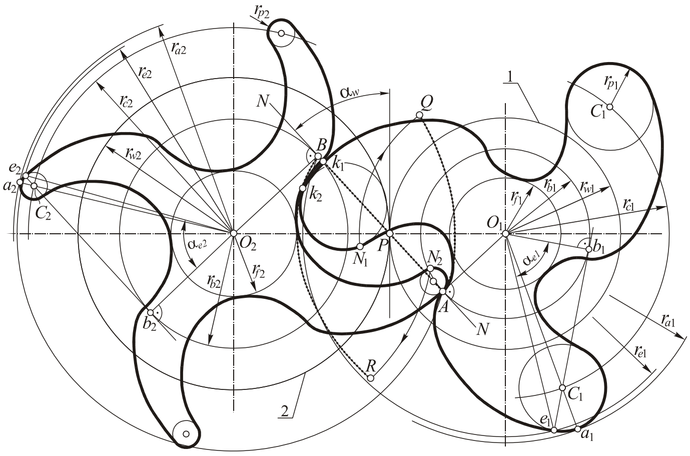

Figure 3. Asymmetric involute-lantern meshing $z_1=3$, $z_2=4$

$$r_{a1}=r_{c1}+r_{p1}\,,\qquad r_{a2}=r_{c2}+r_{p2}\,. \quad (18)$$

- Internal circles. They are defined by the formulas

$$r_{f1}=a_w-r_{a2}\,,\qquad r_{f2}=a_w-r_{a1}\,, \quad (19)$$

where $a_w=r_{w1}+r_{w2}=0{,}5\,m(z_1+z_2)$

is the centre distance of the gearing.

- Transverse contact ratio for the involute meshing. As the contact between the involute profiles is realized along the whole line of action, for the transverse contact ratio the following formula is effective

$$\varepsilon_\alpha=(z_1+z_2)\,\mathrm{tg}\,\alpha_w/2\pi\,. \quad (20)$$

CONCLUSION

With the proposed asymmetric meshing is overcome the shortcoming of the involute meshing, related to the impossibility the meshed gears to have a small teeth number.

In the present paper is shown that in case asymmetric involute-lantern meshing is used, with elongated involute profiles, there appears the possibility to provide continuous transmission of the motion in the driving direction by a very small teeth number.

REFERENCES

[1.] *Alipiev O.: Spur gearing with internal gearing. Patent application 110302/06.01.2009. Bulgaria*

[2.] *Alipiev O.: Geometric design of involute spur gear drives with symmetric and asymmetric teeth using the Realized Potential Method , Mechanism and Machine Theory, "Elsevier", Vol. 46, № 1, 2011, p. 10-32,* http://www.sciencedirect.com/science/journal/0094114X

[3.] *Alipiev O.: Geometric calculation of involute spur gears defined with generalized basic rack. Theory of Mechanisms and Machines, 6 (2008) 2, Russia, pp.60-70,* http://tmm.spbstu.ru/12/alipiyev_12.pdf

[4.] *Vulgakov E.: Theory of involute gears. Mashinostroyenie, Moscow,1995, p.320*

[5.] *Kotelnikov V.: The smallest number of spur external teeth, cut with a non-standard rack-type cutter. Works of universities - Mashinostroyenie, 6, (1973), Russia p. 52-56*

[6.] *F.L. Litvin, A. Fuentes, Gear geometry and applied theory, Cambridge University Press, Cambridge, 2004, p.782*

[7.] *Alipiev O., S. Antonov, T. Grozeva, D. Zafi-rov. Minimum numbers of teeth in symmetric and asymmetric involute spur gearings of a teeth ratio equal to one. 3rd International conference "Power transmissions'09", Kallithea - Greece, 2009, p. 51-58*

[8.] *Kapelevich A., Kleiss R. Direct Gear Design for Spur and Helical Gears, Gear Technology, 10/11 (2002) p. 29-35*

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