TAGUCHI’S ORTHOGONAL DESIGN BASED SOFT COMPUTING METHODOLOGY TO SOLVE CELL FORMATION PROBLEM ON PRODUCTION SHOP FLOOR

ABSTRACT:
The key problem in Cellular Manufacturing System (CMS) is to identify the machine cells and corresponding part families with an aim to curtail the intercell and intracell movement cost of the items. This paper demonstrates a state-of-the-art Soft-Computing based Simulated Annealing heuristic to cell formation problems in CMS. Thereafter Taguchi’s orthogonal design is utilized to solve the critical issues on the subject of parameters selection for the proposed heuristic which is further investigated on 20 widely practiced datasets obtained from related literature. It is shown to outperform the published methodologies such as ZODIAC, GRAFICS, MST, TSP-GA, GA and GP, producing 60% improved solutions.

KEYWORDS:
Group Technology, Cell Formation, Cellular Manufacturing, Clustering Approach, Simulated Annealing, Heuristic, Soft Computing

INTRODUCTION
Cellular manufacturing systems (CMS) exploit group technology (GT) as a manufacturing metaphysics which groups similar parts into part families depending on its manufacturing designs, features and geometric shapes and constructing machine cells to produce the part families [1]. Designing manufacturing cells has been named cell formation problem (CF/CFP), consists of the following procedures: usually similar parts are grouped into part families following their processing needs, and heterogeneous machines are grouped into machine cells and subsequently part families are designated to cells. The problem confronted in CMS is formation of such cells regardless of its category [2]. Not fundamentally the aforementioned steps are accomplished in precise sequences. Depending upon the requirements three solution methodologies are adopted: (i) part family identification and subsequent machine cell formation subject to the processing necessities of the part families, (ii) manufacturing cell formation by assembling heterogeneous machines and thereby the part families are assigned to cells, (iii) part families and machine cells are formed alongside. Numerous intelligent algorithms are widely practised in solving the CF problems owing to its NP-Complete nature [3].

Two diverse methodologies are realized in past literature in order to form machine-part cells, first is production flow analysis (PFA) which deals with processing necessities, operational sequences and operational time of the parts on the machines [4]. Another approach is the Part Coding Analysis (PCA) which utilizes predefined coding schemes to facilitate the process using several attributes of parts such as geometrical shapes, materials, design features and functional requirements etc. [5]. Various techniques are developed to solve manufacturing cell formation problems over the past four decades, these include similarity coefficient methods, clustering analysis, array based techniques, graph partitioning methods etc. The similarity coefficient approach was first suggested by McAuley [6]. The basis of similarity coefficient methods is to compute the similarity between each pair of machines and then to group the machines into cells based on their similarity measures. Few studies proposed to measure dissimilarity coefficients instead of similarity coefficient for machine-part grouping problems [7]. Clustering methods are categorized as hierarchical and non-hierarchical methods. Standard or typically designed clustering techniques could be utilized to build clusters of either components or machines. Most clustering based CF solution methodologies utilize machine-part incidence matrix, such as Single linkage clustering [6], Average linkage clustering algorithm [8]. Machine-part grouping problem is based on production flow analysis, in which the machine-part production cells are formed by permuting rows and
columns of the machine-part incidence matrix. Some of the methods are Rank Order Clustering (ROC) [9], Bond energy algorithm [10] etc. Dimopoulos and Mort [11] has proposed a hierarchical algorithm combined with genetic programming for efficient cell formation in CMS.

Array based methods consider the rows and columns of the machine-part incidence matrix as binary patterns and reconfigure them to obtain a block diagonal cluster formation. The ROC algorithm is the most familiar array-based technique for cell formation [9]. Substantial alterations and enhancements over ROC algorithm have been described by King and Nakornchai [12] and Chandrasekharan and Rajagopalan [13]. The direct clustering analysis has been stated by Chan and Milner [14], and bond energy analysis is performed by McCormick et al.[10].

Graph Theoretic Approach depicts the machines as vertices and the similarity between machines as the weights on the arcs. An ideal seed non-hierarchical clustering algorithm for cellular manufacturing is proposed Chandrasekharan and Rajagopalan [15]. Srinivasan [16] implemented a method using minimum spanning tree (MST) for the machine-part cell formation problem. A polynomial-time algorithm based on a graph theoretic approach was developed by Veeramani and Mani [17]. In present article a novel approach has been developed using Median Linkage Clustering (MLC) algorithm to produce an initial feasible solution to CFP. Thereafter a Soft Computing based Simulated Annealing (SA) heuristic is adopted to enhance the quality of the initial solutions obtained. During past few decades Soft Computing techniques are exhaustively practiced by researchers in the vicinity of CMS. Tavakkoli-Moghaddam et al. [18] explained that dynamic condition of CFP becomes more complex and proposed Tabu Search (TS), Simulated Annealing (SA) and GA methods to solve this type of problems. Their study indicated that SA is better in terms of solution and complexity than TS, GA. Other authors [19] introduced an integer programming model for dynamic CFP and implemented SA algorithm to obtain the optimal solutions. Das et al.[20] proposed the multi-objective mixed integer-programming model for CMS design by minimizing machine operating and utilization cost and total material handling cost and maximizing system reliability. Lei and Wu [21] worked with multi-objective cell formation (CF) problem and proposed a Pareto-optimality based on multi-objective tabu search (MOTS) with different objectives: minimization of the weighted sum of inter cell and intra cell moves and minimization of the total cell load variation. A hybrid methodology based on Boltzmann function from simulated annealing and mutation operator from GA was proposed by Wu et al.[22] to optimize the initial cluster obtained from similarity coefficient method (SCM) and rank order clustering (ROC). Arkat et al.[23] developed a sequential model based on SA for large-scale problems and compared their method with GA. Ateme-Ngouema and Dao [24] investigated an Ant Algorithm based TS heuristic for cellular system design problem (CSDP) and the methodology proved to be much quicker than traditional methods when considering operational sequence, time and cost. Authors further proposed quantized Hopfield network for CFP to find optimal or near-optimal solution and TS was employed to improve the performance and the quality of solution of the network [25]. Durán et al.[26] reported a modified Particle Swarm algorithm with proportional likelihood instead of using velocity vector on CF problems where the objectives are the minimization of cell load variation and inter cellular parts movement and reported the stability of the method with low variability. A similar study was also performed by Anvari et al.[27] where a hybrid particle swarm optimization technique for CFP was reported. The initial solutions generated either randomly or using a diversification generation method and the technique also utilized mutation operator embedded in velocity update equation to avoid reaching local optimal solutions. Thereafter with due consideration, a wide variety of machine/part matrices were effectively solved by this approach. Interested readers could obtain an elaborated survey on Soft Computing based approaches in CMS proposed by Ghosh et al. [28].

**PROBLEM FORMULATION**

The cell formation problem in group technology begins with two fundamental tasks, namely, machine cell formation and part family identification. Similar machines are grouped to form machine cells and dedicated for the manufacture of one or more part families. In part family formation, parts with similar design features, attributes, shapes are grouped, so that the group of parts can be manufactured within a cell. Generally the cell formation problems are presented using a matrix namely machine-part incident matrix in which all the elements are either 0 or 1. Parts are arranged in columns and machines are in rows of the incidence matrix. An example matrix is presented in Figure 1.

In this matrix a 0 indicates no mapping or no processing and an 1 indicates mapping or processing. Therefore it depicts that machine 1 processes parts 2, 3, 5, machine 2 processes parts 1, 4, 5 and machine 3 processes parts 3, 5.

<table>
<thead>
<tr>
<th></th>
<th>p1</th>
<th>p2</th>
<th>p3</th>
<th>p4</th>
<th>p5</th>
<th>p6</th>
<th>p7</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>m2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>m3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>m4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>m5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 1. Machine-part incidence matrix of example dataset (5×7)

From this problem matrix a solution matrix could be obtained which presents the formation of cells as diagonal square boxes (Figure 2). Once some appropriate technique is employed to CFP this solution matrix is believed to be obtained. It can be interpreted that cell 1 contains [machines 1, 3 || | parts
and cell 2 contains \{machine 2 \mid \{parts 1, 4\}\}. An 1 outside the block means a part is processed through some machine which does not belong to any machine cell, therefore the intercellular move cost will be added. This element is known as an ‘exceptional element’ (EE) and a 0 inside a cell means lesser utilization of space and increased intracell move cost. It is known as a ‘void’. The objective of cell formation is to minimize the EEs and voids.

\begin{center}
\begin{tabular}{c|cccc}
  & p1 & p7 & p6 & p4 \\
\hline
m1  & 1 & 1 & 1 & 1 \\
m2  & 1 & 1 & 1 & 1 \\
m3  & 1 & 1 & 1 & 1 \\
m4  & 1 & 1 & 1 & 1 \\
\end{tabular}
\end{center}

Figure 2. Final block diagonal cell formation of example dataset (5×7)

To formulate the cell formation problem the following are considered,

\begin{itemize}
  \item i=1,…,M, denotes machine index
  \item j=1,…,P, denotes part index
  \item k=1,…,K, denotes cell index
\end{itemize}

The incidence matrix is \( A=[a_{ij}] \) demonstrates the mapping between machines and parts.

\[ a_{ik} = \begin{cases} 
1 & \text{if part } j \text{ processed by machine } i \\
0 & \text{otherwise} 
\end{cases} \]  \hspace{1cm} (1)

To measure the goodness of solutions, different performance measures have been proposed by researchers since past few decades. Various measures can be obtained from the critical survey of performance measures [29]. In this study grouping efficacy has been considered which is heavily utilized by other authors to measure the efficiency of their solutions [30] and it is given as:

\[ F = \frac{E - E_e}{E + E_v} \]  \hspace{1cm} (2)

Where: \( E = \) Total number of 1s; \( E_e = \) Total number of EEs; \( E_v = \) Total number of voids

The objective function which maximizes the efficiency is as follows:

\[ \text{Maximize } F = \frac{E_v}{E + E_v} \]  \hspace{1cm} (3)

subject to

\[ \sum_{k=1}^{K} x_{ik} = 1 \quad \text{for } i = 1, \ldots, M \]  \hspace{1cm} (4)

\[ \sum_{i=1}^{M} y_{jk} = 1 \quad \text{for } j = 1, \ldots, P \]  \hspace{1cm} (5)

\[ \sum_{k=1}^{K} y_{jk} \geq 1 \quad \text{for } k = 1, \ldots, K \]  \hspace{1cm} (6)

\[ \sum_{j=1}^{P} y_{jk} \geq 1 \quad \text{for } k = 1, \ldots, K \]  \hspace{1cm} (7)

\[ x_{ik} = 0 \text{ or } 1 \quad \text{for } i = 1, \ldots, M; k = 1, \ldots, K \]  \hspace{1cm} (8)

\[ y_{jk} = 0 \text{ or } 1 \quad \text{for } j = 1, \ldots, P; k = 1, \ldots, K \]  \hspace{1cm} (9)

where,

\[ y_{jk} = \begin{cases} 
1 & \text{if part } j \text{ is in cell } k \\
0 & \text{otherwise} 
\end{cases} \]  \hspace{1cm} (11)

\[ x_{ik} = \begin{cases} 
1 & \text{if machine } i \text{ is in block } k \\
0 & \text{otherwise} 
\end{cases} \]  \hspace{1cm} (12)

The function \( F \) is a fractional function in \( x \) and \( y \). The constraints (4) and (5) depict that each machine and each part is assigned to exactly one cell, respectively. Further constraints (6) and (7) demonstrate each cell contains at least one machine and one part respectively. Binary variables are expressed in (8) and (9). Constraints (6) and (7) ensure the elimination of empty cells, if any.

The objective of the proposed methodology is to maximize the value of function \( F \) (equation (3)). Therefore in order to achieve this objective, the number of Ees and voids are believed to be minimized. Thus equation (12) and (13) are utilized as the objective function of the proposed Soft Computing technique.

**SOLUTION METHODOLOGY**

The proposed methodology is developed using Soft Computing based SA algorithm to achieve optimal solutions. The SA algorithm simulates the physical annealing process, where particles of a solid arrange themselves into a thermal equilibrium. An introduction to SA can be found in the book by Aarts and Korst [31]. The general applications concerns combinatorial optimization problems of the following form where \( S \) is a finite set of feasible solutions.

\[ \min_{x \in S} g(x) \]  \hspace{1cm} (14)

The algorithm uses a pre-defined neighborhoods structure on ‘S’. A control parameter called temperature in analogy to the physical annealing process governs the search behavior. In each iteration, a neighbor solution \( y \) to the current solution \( x \) is computed. If \( y \) has a better objective function value than \( x \), the solution \( y \) is accepted, that is, the current solution \( x \) is replaced by \( y \). If, on the other hand, \( y \) does not have a better objective function value than \( x \), the solution \( y \) is only accepted with a certain probability depending on (i) the difference of the objective function values in \( x \) and \( y \), and (ii) the temperature parameter. The pseudocode below demonstrates SA procedure.

**Pseudocode (SA)**

initialize;
repeat
  generate a candidate solution;
  evaluate the candidate;
  determine the current solution;
  reduce the temperature;
  until termination condition is met;
**Initial Solution Generation**

In this article Sorenson’s similarity coefficient method [32] is exploited which is further combined with median linkage clustering technique to form the
Machine cell. Median Linkage Clustering Algorithm is theoretically and mathematically simple algorithm practiced in hierarchical clustering analysis of data [33]. It delivers informative descriptions and visualization of possible data clustering structures. When there exists hierarchical relationship in data this approach can be more competent.

**Similarity Coefficient Method**

The Sorenson’s similarity coefficient metric is demonstrated as,

$$ S_{ij} = \frac{2a_{ij}}{a_{ii} + a_{jj}} $$

\( S_{ij} \) = Similarity between machine i and machine j,
\( a_{ij} \) = the number of parts processed by both machines i and j,
\( a_{ii} \) = the number of parts processed by machine i but not by machine j,
\( a_{jj} \) = the number of parts processed by machine j but not by machine i.

In order to facilitate the computation in Matlab the similarity matrix obtained using equation (15) further transformed into distance matrix using,

$$ d_{ij} = 1 - S_{ij} $$

Utilizing (16) dissimilarity relationship can be obtained between machines and an \( m \times m \) dissimilarity matrix can be obtained as depicted in Figure 3.

**Machine Group Formation**

The proposed hybrid technique takes the input of the similarity matrix obtained from the previous stage and produces dendrogram structure that links individual machines or subgroup of machines according to their values of similarity coefficients. Median linkage function is implemented on the basis of hierarchical cluster information. If machine cell r is formed combining cell p and q, and \( n_r \) is the number of machines in cell r, \( x_{ir} \) is the i\textsuperscript{th} machine of cell r, then median linkage is computed using the formula,

$$ d(r,z) = \frac{1}{2} \| x_r - x_z \|_2 $$

which is the Euclidean distance between the weighted centroids of two cells where,

$$ x_r = \frac{1}{n_r} (x_{1r} + x_{2r} + \ldots + x_{nr}) $$

The matrix \( Z \) is generated from this function is a \( (m-1) \times 3 \) matrix, where m is the number of machines in the original dataset. Columns of the matrix contain cluster indices linked in pairs to form a binary tree. The leaf nodes are numbered from 1 to m. Leaf nodes are the singleton clusters from which all higher clusters are built. Further the dendrogram can be obtained from the matrix which indicates a tree of potential solutions. The dendrogram is presented in Figure 4 which visualize the hierarchical cluster formation.

A threshold value is used for cutting the linkage matrix \( Z \) obtained from previous step into clusters. Clusters are formed when a node and all of its subnodes have inconsistent value less than the threshold value. All leaves at or below the node are grouped into a cluster. Output is a vector of size m containing the cluster assignments of each machine row.

**Part Group Formation**

In order to obtain the part family, similar steps are repeated on part-machine incidence matrix which is a transpose matrix of machine-part incidence matrix. The dendrogram could also be obtained for part cluster on the similar manner of achieving the machine dendrogram. Thereafter the vector containing the cluster assignments for parts is presented as,

$$ S_2 = [1, 2, 2, 2, 1] $$

Therefore the vector obtained could be presented as,

$$ S_1 = [1, 2, 2, 1] $$

Therefore the initial solution generation algorithm is explained as,

**Input:** Machine-part incidence matrix

**Step 1.** Procedure Similarity()

Step 1.1. Compute similarity values between pair of machines using equation (15)

Step 1.2. Compute the similarity matrix of the parts

Step 1.3. transform the similarity matrix into a distance matrix using equation (16)

Step 1.4. End

**Step 2.** Procedure MedianCluster()

Step 2.1. loop

Step 2.2. Compute the distance between two clusters using equation (17)

Step 2.3. Construct matrix \( Z \) of size \((m-1) \times 3\) to from the hierarchical tree structure

Step 2.4. Construct the dendrogram from \( Z \)

Step 2.5. loop

Step 2.6. create machine cells for the minimum level of dissimilarity coefficient and obtain \( S_1 \)

Step 2.7. A=Transpose(A) and Go to step 1

Step 2.8. create part families for the minimum level of dissimilarity coefficient and obtain \( S_2 \)

Step 2.9. stop

Output: \( S_1 \) and \( S_2 \)

**The Proposed Soft Computing Technique**

This subsection describes the proposed SA algorithm in depth. In this algorithm, the number of cells resulting in the best solution is fixed initially. The initial input is a solution string to the problem in hand which is generated from Median Linkage technique. Therefore
the initial input string is \( S_0 \) which is obtained combining \( S_1 \) and \( S_2 \).

The size of the solution string is \( m\times p \), where \( m \) is the number of machines and \( p \) is the number of parts. Each bit of the string represents cell number of the corresponding machine or part (string indices). Therefore T2 states machine 1 is placed in cell 1, and machine 2, 3, 4, 5 are placed in cell 2, and part 1 and 7 are placed in cell 1, and part 2 to 6 are placed in cell 2.

To understand the goodness of the solution a performance evaluation criterion is assumed to be explained which is presented by equation (3). Therefore the objective is set to minimize the count of EEs and voids to improve the fitness of obtained solutions. Some symbolization used in the algorithm are introduced as,

- \( S_{\text{cur}} \rightarrow \) current solution
- \( S_i \rightarrow \) neighbourhood solution
- \( f(S_i) \rightarrow \) current fitness value
- \( f(S_i) > f(S_{\text{best}}) \rightarrow \) best fitness value

The steps of the proposed algorithm can be summarized as follows.

Step 1. Obtain an initial solution \( S_0 \) by using similarity coefficient and Median Linkage procedures.

Step 2. Evaluate \( S_0 \) and Calculate corresponding fitness value \( f_0 \):

\[ f_0 = f(S_0) \]

Step 3. Set \( f_{\text{best}} = f_0 \),

\[ S_{\text{best}} = S_0 = S_{\text{cur}} \]

Step 4. Initialize SA Heuristic and its parameters: \( T_{\text{init}}, T_{\text{final}}, a, M, \alpha, \text{iter} = 0, \text{count} = 0, \text{count1} = 0 \).

Step 5. If \( \text{count} < M \), then repeat Steps 5.1 to 5.9.

Step 5.1. Generate a new machine cell formation configuration by performing single-move (randomly selecting a part or machine and moving it to another cell).

Step 5.2. Read cell formation configuration from above steps and generate corresponding neighbourhood solution \( S_i \).

Step 5.3. If \( f(S_i) > f_{\text{best}} \), then \( S_{\text{best}} = S_i, S_{\text{cur}} = S_i, \text{count} = \text{count} + 1 \), go to Step 5.

Step 5.4. If \( f(S_i) = f_{\text{best}} \), then \( S_i = S_i, \text{count1} = \text{count1} + 1, \text{count} = \text{count} + 1 \), go to Step 5.

Step 5.5. Compute \( \delta = f(S_i) - f(S_{\text{cur}}) \). Obtain a random variable \( r \) in the range of \( U(0,1) \).

Step 5.6. If \( e^{\alpha \delta} > r \),

Step 5.7. set \( S_{\text{cur}} = S_i \),

Step 5.8. \( \text{count1} = 0 \),

Step 5.9. else \( \text{count1} = \text{count1} + 1 \).

Step 5.10. \( \text{iter} = \text{iter} + 1 \).

Step 5.11. until freezing temperature \( (T_{\text{final}}) \) is reached;

Step 5.12. reduce the temperature using \( T_{\text{final}} = \alpha \times T_{\text{iter}} \) function;

The SA procedure is repetitively employed until a solution is achieved which attains the highest fitness score. All the parameters and counters are initialized in step 4. A special move, namely single-move, is utilized in the proposed algorithm to guide the solution searching procedure. From the understanding of exhaustive testing, it is spotted that single move ordinarily leads to improved solutions effortlessly and competently. Thus single-move is practiced as a principle component for finding better neighborhood solution in step 5.1. The algorithm also verifies the number of instances when neighborhood solutions become static. If this number attains a pre-fixed constant value, the fitness value of current configuration is compared to the optimal solution obtained thus far to conclude whether to prolong the iterations or stop with the best solution achieved.

**EXPERIMENTAL VERIFICATIONS**

In order to apply the proposed Soft Computing technique as a solution methodology to solve the CF problem, the effects of changing the values of the various parameters are studied. Determining the optimal set of parameters is crucial in this regards. Therefore in this article the Taguchi’s orthogonal design method is employed to determine the optimal values of the parameters.

**Taguchi Method for Parameters Selection**

The parameters are Initial temperature \( (T_{\text{init}}) \), temperature reducing factor \( (a) \) and Markov chain length \( (M) \). Other parameter such as final temperature \( (T_{\text{final}}) \) is taken as constant value = 0.00000001 initially. The parameters are termed as factors, and each factor has three discrete levels (Table 1). Hence an L9 orthogonal array is used, and this recommends that 9 sets of Taguchi experiments are prerequisite and the results are evaluated by using an analysis of variance (ANOVA) technique. The parameter settings for each experiment are shown in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( M )</th>
<th>( a )</th>
<th>( T_{\text{init}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>0.75</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.85</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>0.95</td>
<td>30</td>
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</table>

Table 2. Levels of parameters tested

<table>
<thead>
<tr>
<th>Experiments</th>
<th>( T_{\text{init}} )</th>
<th>( a )</th>
<th>( M )</th>
<th>( f_{\text{responses}} )</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.75</td>
<td>20</td>
<td>0.636364</td>
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<td>2</td>
<td>10</td>
<td>0.85</td>
<td>30</td>
<td>0.680000</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0.95</td>
<td>40</td>
<td>0.695652</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.75</td>
<td>40</td>
<td>0.650000</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.85</td>
<td>40</td>
<td>0.695652</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>0.95</td>
<td>40</td>
<td>0.695652</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>0.75</td>
<td>40</td>
<td>0.680000</td>
</tr>
<tr>
<td>8</td>
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<td>0.85</td>
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<td>0.695652</td>
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<tr>
<td>9</td>
<td>30</td>
<td>0.95</td>
<td>30</td>
<td>0.695652</td>
</tr>
</tbody>
</table>

Table 3. presents the results of the corresponding ANOVA analysis with S/N ratio (Larger-the-better). In Table 3, the variance ratios (F ratios) of the factors are determined. A test of significance at 95% confidence level is employed to spot the significance of these factors. The P values of the factors \( T_{\text{init}} \), \( a \), \( M \) are investigated and value of \( a \) is seen to be less than the critical level with degrees of freedom at \((2, 8)\). This suggests that \( a \) is the most significant factor in the proposed approach. The response table (Table 4) depicts the average of each response characteristic for each level of each of the factors. The table includes ranks based on Delta (\( \delta \)) statistics, which compare the relative magnitude of effects. The Delta statistic states the difference between the largest and the smallest average for each factor. Ranks are assigned.
In present study, the ranks indicate that temperature reducing factor (a) has the greatest influence. Markov chain length (M) has the next greatest influence, followed by Initial temperature (Tinit). The objective of the simulated annealing is to maximize the value of fitness function of equation (3), therefore factor levels should be fixed in such a way that the highest objective value could be achieved. The level averages in the response table and the main effects plot of Figure 5 show that the optimal solution is obtained when Tinit, a and M are set to 30, 0.95, 40 respectively.

Table 3. ANOVA table

<table>
<thead>
<tr>
<th>Factors</th>
<th>Degrees of Freedom</th>
<th>Factor Sum of squares</th>
<th>Mean Square (Variance)</th>
<th>F Ratio (Variance)</th>
<th>p Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tinit</td>
<td>2</td>
<td>0.000729</td>
<td>0.000364</td>
<td>2.67</td>
<td>0.273</td>
</tr>
<tr>
<td>a</td>
<td>2</td>
<td>0.004258</td>
<td>0.000219</td>
<td>15.58</td>
<td>0.060</td>
</tr>
<tr>
<td>M</td>
<td>2</td>
<td>0.000847</td>
<td>0.000424</td>
<td>3.10</td>
<td>0.244</td>
</tr>
<tr>
<td>Error</td>
<td>2</td>
<td>0.000273</td>
<td>0.000137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this study, the ranks indicate that temperature reducing factor (a) has the greatest influence. Markov chain length (M) has the next greatest influence, followed by Initial temperature (Tinit). The objective of the simulated annealing is to maximize the value of fitness function of equation (3), therefore factor levels should be fixed in such a way that the highest objective value could be achieved. The level averages in the response table and the main effects plot of Figure 5 show that the optimal solution is obtained when Tinit, a and M are set to 30, 0.95, 40 respectively.

Table 4. Response table

<table>
<thead>
<tr>
<th>Levels</th>
<th>Tinit</th>
<th>a</th>
<th>M</th>
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Figure 5. Main effects plot

SA Maximum and Average Fitness curve

Figure 6. Convergence curve obtained from the heuristic approach for problem #1

Convergence Analysis

Convergence analysis is almost equivalent for all the problem datasets. Problem #1 of size 5×7 is selected as an example to demonstrate the convergence curve during iterations of the proposed metaheuristic technique as presented in Figure 6. For the first iteration the fitness score attained a value of 68. Since the computer program is designed to maximize the fitness function with the iteration counts therefore at 13th iteration it attained the value of 69.56, an increase of 2.3%. The final optimal solution is obtained with the fitness score of 69.56. Based on the experimentation for all the datasets reported in this article, it is observed that the fitness score is increased during the iteration. After reaching the best fitness score at some iteration and thereafter the fitness score continues to remain constant even if the number of iterations is increased. Since the proposed metaheuristic algorithm gives the same pattern of convergence for all the tested problems therefore the convergence property is established. For the 5×7 problem the proposed approach is executed for 45 iterations and took 1.7797 CPU seconds to attain the best solution which proves its computational efficiency.

Computational Results

The proposed method is tested with a set of 20 problems that have been published in the past literature and have been widely used in many comparative studies. All the data sets were transcribed from the original articles to avoid the inconsistencies in data. The sources of the datasets are shown in Table 5. The proposed method is simulated with Multivariate Statistical Analysis Toolbox and Matlab 7.1 and tested on a laptop with a 2.1 GHz processor and 2GB of RAM. Comparisons of the proposed SA method against other algorithms from the literature are given in Table 6. These algorithms include ZODIAC [34], GRAFICS [35], TSP based Genetic Algorithm [36], Genetic Algorithm [37], MST [16], GP [11]. For the problems solved with the proposed method to obtain optimal solution, the grouping efficacy value is improved or identical in all instances. This observation indicates that this technique is efficient and less complex because of its minimalism in simulation. All the solutions are obtained with negligible computational time (< 15 sec. for the largest datasets tested). Therefore this technique is highly comparable with complex soft computing techniques such as genetic algorithms (GA), evolutionary techniques, GA-TSP, Genetic Programming (GP) etc.
Figure 7 portrays the substantial improvement achieved by the proposed SA heuristic while comparing with other techniques. The above SA heuristic is shown to outperform the standard techniques in 12 instances, and equal in 8 instances, which further illustrates 60% improved result which is significant in terms of solution quality, time and space complexities.

Table 5. Source of Datasets

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*SAH: SA heuristic

**CONCLUSION**

This study portrays a Soft Computing based Simulated Annealing heuristic to construct manufacturing cells on production shop floor. The initial feasible solution to the proposed technique is obtained using Sorenson’s similarity coefficient method and median linkage clustering technique. This article also exploits Taguchi’s orthogonal design approach to select optimal set of parameters to the SA heuristic which is a crucial factor influencing the performance of the proposed technique. Computational results presented in previous section demonstrate that the proposed technique not only outperforms the standard techniques, but also several other well-known soft computing based cell formation solution methodologies such as genetic algorithms, GA-TSP, GP etc. from the literature. The proposed method attains 60% improved solutions by consuming lesser computational time and resources than that of the traditional complex soft computing based methodologies.

It is also shown that the proposed hybrid technique performs at least as well as, and often better than, some of the best algorithms for the cell formation on all problems tested.

Further work can be done by utilizing this technique in more complex cell formation problem which deals with ratio data for production volume, lot sizes, operational time, worker assignment and other multi-objective factors, often known as generalized cell formation problems.

**REFERENCES**

[17.] Veeramani, D. & Mani, K. (1996). A polynomial-time algorithm for optimal clustering in a special class of (0, 1)-

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**Table 5. Source of Datasets**

**Table 6. Computational Result**

**REFERENCES**


