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[1.] R. MUTHUCUMARASWAMY, [2.] G. NAGARAJAN, [3.] V.S.A. SUBRAMANIAN

THERMAL RADIATION AND MHD EFFECTS ON FLOW PAST AN VERTICAL OSCILLATING PLATE WITH CHEMICAL REACTION OF FIRST ORDER

ABSTRACT:
Thermal radiation and first order chemical reaction effects on unsteady free convective flow of a viscous incompressible flow past an infinite isothermal vertical oscillating plate with mass transfer in the presence magnetic field is considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to T_w and the concentration level near the plate is also raised to C'_w . An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity, temperature and concentration are studied for different parameters like magnetic field parameter, phase angle, Schmidt number, chemical reaction parameter, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing magnetic field parameter or radiation parameter. It is also observed that the velocity increases with decreasing phase angle ωt .

KEYWORDS:
chemical reaction, radiation, oscillating, vertical plate, magnetic field

INTRODUCTION

Magnetoconvection plays an important role in agriculture,petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil,and in the assessment of aquifers,geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earths core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al (1979). MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al (1979a). The dimensionless governing equations were solved using Laplace transform technique.

The Effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre and Young (1958) have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das et al (1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al (1996). The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. England and Emery (1969) have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar(1996). The governing equations were solved analytically. Das et al(1999) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar (1979). The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar (1983). The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al. (1994).

However the combined study of MHD and thermal radiation effects on infinite oscillating isothermal vertical plate in the presence of chemical reaction of first order is not studied in the literature. It is proposed to study the chemical reaction effects on unsteady flow past infinite isothermal vertical oscillating plate, in the presence of magnetic field and thermal radiation. The dimensionless governing equations are tackled using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

BASIC EQUATIONS AND ANALYSIS

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t'>0$, the plate starts oscillating in its own plane with frequency ω' and the temperature of the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. The plate is also subjected to a uniform magnetic field of strength B_0. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following

$$\frac{\partial u}{\partial t'} = g\beta(T-T_\infty) + g\beta^*(C'-C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \qquad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_l C' \qquad (3)$$

In most cases of chemical reactions, the rate of reaction depends on the concentration of the species itself. A reaction is said to be of the order n, if the reaction rate if proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

With the following initial and boundary conditions:

$$\begin{array}{llll} t'\le 0: & u=0, & T=T_\infty, & C'=C'_\infty \quad for\ all\ y \\ t'>0: & u=u_0 cos\omega' t', & T=T_w, & C'=C'_w \quad at\ y=0 \\ & u=0, & T\to T_\infty, & C'\to C'_\infty \quad as\ y\to\infty \end{array} \qquad (4)$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\,\sigma\left(T_\infty^4 - T^4\right) \qquad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\,T - 3T_\infty^4 \qquad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3 (T_\infty - T) \qquad (7)$$

On introducing the following non-dimensional quantities:

$$\begin{gathered} U=\frac{u}{u_0},\ t=\frac{t' u_0^2}{\nu},\ Y=\frac{y u_0}{\nu},\ \theta=\frac{T-T_\infty}{T_w-T_\infty}, \\ Gr=\frac{g\beta\nu(T_w-T_\infty)}{u_0^3},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, \\ Gc=\frac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^3},\ \omega=\frac{\omega'\nu}{u_0^2}, \\ R=\frac{16a^*\nu^2\sigma T_\infty^3}{k u_0^2},\ Pr=\frac{\mu C_p}{k}, \\ Sc=\frac{\nu}{D},\ M=\frac{\sigma B_0^2\nu}{\rho u_0^2},\ K=\frac{\nu K_l}{u_0^2} \end{gathered} \qquad (8)$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - M\,U \qquad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\theta \qquad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - K\,C \qquad (11)$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llllll} & U=0, & \theta=0, & C=0, & for\ all & Y,t\le 0 \\ t>0: & U=\cos\omega t, & \theta=1, & C=1, & at & Y=0 \\ & U=0, & \theta\to 0, & C\to 0 & as & Y\to\infty \end{array} \qquad (12)$$

SOLUTION PROCEDURE

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{1}{2}\left[\begin{array}{l} \exp\left(2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{Pr}+\sqrt{at}\right) \\ +\exp\left(-2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{Pr}-\sqrt{at}\right) \end{array}\right] \qquad (13)$$

$$C = \frac{1}{2}\left[\begin{array}{l} \exp\left(2\eta\sqrt{KtSc}\right) erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right) \\ +\exp\left(-2\eta\sqrt{KtSc}\right) erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right) \end{array}\right] \qquad (14)$$

$$\begin{aligned} U = {} & \frac{\exp(i\omega t)}{4}\left[\begin{array}{l} \exp\left(2\eta\sqrt{(M+i\omega)t}\right)erfc\left(\eta+\sqrt{(M+i\omega)t}\right) \\ +\exp\left(-2\eta\sqrt{(M+i\omega)t}\right)erfc\left(\eta-\sqrt{(M+i\omega)t}\right) \end{array}\right] \\ & + \frac{\exp(-i\omega t)}{4}\left[\begin{array}{l} \exp\left(2\eta\sqrt{(M-i\omega)t}\right)erfc\left(\eta+\sqrt{(M-i\omega)t}\right) \\ +\exp\left(-2\eta\sqrt{(M-i\omega)t}\right)erfc\left(\eta-\sqrt{(M-i\omega)t}\right) \end{array}\right] \\ & + (d+e)\left[\begin{array}{l} \exp\left(2\eta\sqrt{Mt}\right).erfc\left(\eta+\sqrt{Mt}\right) \\ +\exp\left(-2\eta\sqrt{Mt}\right).erfc\left(\eta-\sqrt{Mt}\right) \end{array}\right] \\ & - d\exp(at)\Big[\exp\left(-2\eta\sqrt{(M+b)t}\right).erfc\left(\eta-\sqrt{(M+b)t}\right) \\ & \qquad + \exp\left(2\eta\sqrt{(M+b)t}\right).erfc\left(\eta+\sqrt{(M+b)t}\right)\Big] \end{aligned}$$

$$
\begin{aligned}
&- e\exp(bt)\left[\exp\left(-2\eta\sqrt{(M+c)t}\right).erfc\left(\eta-\sqrt{(M+c)t}\right)\right.\\
&\qquad\left.+\exp\left(2\eta\sqrt{(M+c)t}\right).erfc\left(\eta+\sqrt{(M+c)t}\right)\right]\\
&- d\begin{bmatrix}\exp\left(2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{at}\right)\\ +\exp\left(-2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{at}\right)\end{bmatrix}\\
&- d\exp(at)\left[\exp\left(-2\eta\sqrt{\Pr(a+b)t}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{(a+b)t}\right)\right.\\
&\qquad\left.+\exp\left(2\eta\sqrt{\Pr(a+b)t}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{(a+b)t}\right)\right]\\
&\qquad-\frac{d}{2}\begin{bmatrix}\exp\left(2\eta\sqrt{Kt\,Sc}\right).erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\\ +\exp\left(-2\eta\sqrt{Kt\,Sc}\right).erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\end{bmatrix}\\
&+ e\exp(bt)\left[\exp\left(-2\eta\sqrt{Sc(K+c)t}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{(K+c)t}\right)\right.\\
&\qquad\left.+\exp\left(2\eta\sqrt{Sc(K+c)t}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{(K+c)t}\right)\right]
\end{aligned}
\qquad (15)
$$

where, $a=\dfrac{R}{Pr}, b=\dfrac{M-R}{Pr-1}, c=\dfrac{M-K\,Sc}{Sc-1}, d=\dfrac{Gr}{2b(1-Pr)}$

and $e=\dfrac{Gc}{2c(1-Sc)}$, *where* $\eta=Y/2\sqrt{t}$ *and erfc is called complementary error function.*

RESULTS AND DISCUSSION

The numerical values of the velocity, temperature and concentration fields are computed for different parameters like Magnetic field parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The purpose of the calculations given here is to assess the effects of the parameters M, K, R, Gr, Gc *and* Sc *upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.*

Figure 1 illustrates the effect of the concentration profiles for different values of the chemical reaction parameter ($K = 0.2, 2, 5, 10$ *) at* $t = 0.4$. *The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the velocity increases with decreasing chemical reaction parameter.*

The temperature profiles are calculated for different values of thermal radiation parameter($R = 0.2, 2, 5, 10$ *) at time* $t = 0.4$ *and these are shown in figure 2. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.*

The velocity profiles for different phase angles ($\omega t = 0, \pi/6, \pi/3, \pi/2$ *),* $R=10, M=0.2, K=2, Gr=Gc=2$ *and* $t = 0.2$ *are shown in figure 3. It is observed that the velocity increases with decreasing phase angle* ωt. *Figure 4. demonstrates the effects of the magnetic field parameter on the velocity when (* $M = 0.2, 2, 5$ *),* $\omega t=\pi/6, Gr=Gc=5, R=10, K=5, Pr=0.71$ *and* $t = 0.6$. *It is observed that the velocity increases with decreasing magnetic field parameter.*

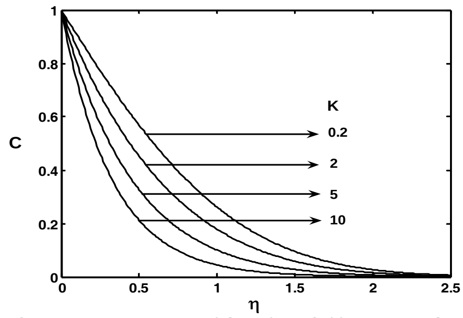

Fig. 1: Concentration profiles for different values of K

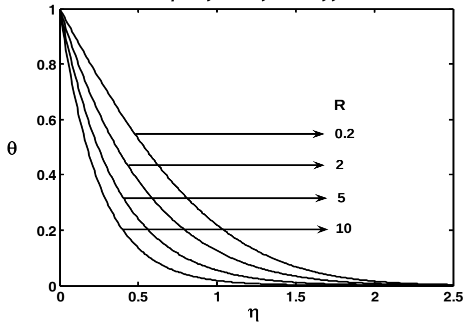

Fig. 2: Temperature profiles for different values of R

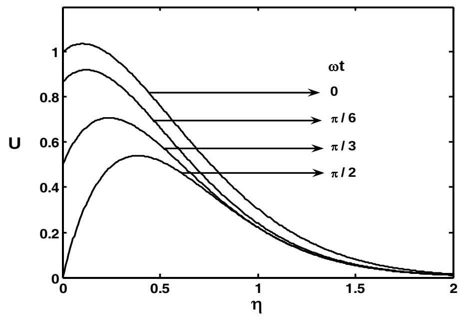

Fig. 3: Velocity profiles for different values of ωt

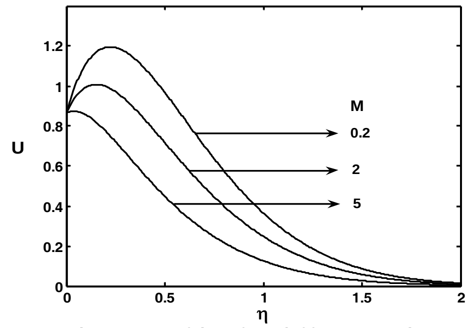

Fig. 4: Velocity profiles for different values of M

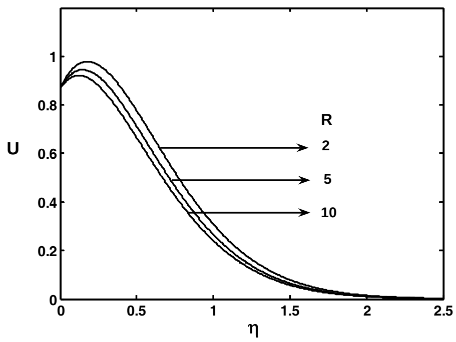

Fig. 5: Velocity profiles for different values of R

The effect of velocity for different values of the radiation parameter ($R = 2, 5, 10$*),* $\omega t = \pi/6$, $K = 2$, $M = 0.2$, $Gr = Gc = 2$ *and* $t = 0.6$ *are shown in figure 5. The trend shows that the velocity increases* with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation. Figure 6 illustrates the effect of the velocity for different values of the reaction parameter ($K = 0.5,5,15$), $\omega t = \pi/4$, $R = 10$, $M = 0.2$, $Gr = Gc = 2$ and $t = 0.6$. The trend shows that the velocity increases with decreasing chemical reaction parameter.

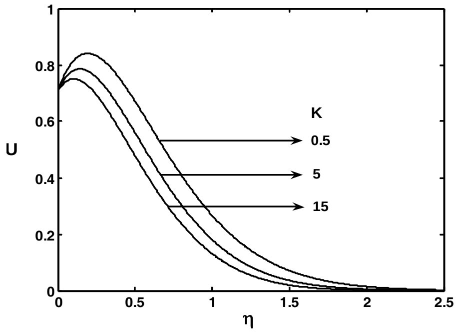

Fig. 6: Velocity profiles for different values of K

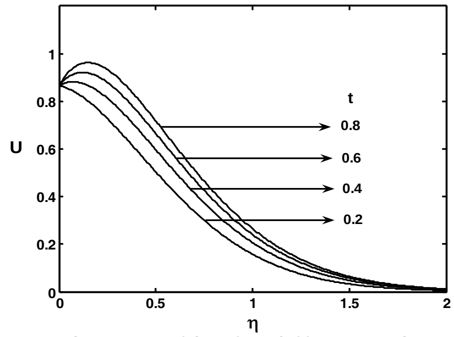

Fig. 7: Velocity profiles for different values of t

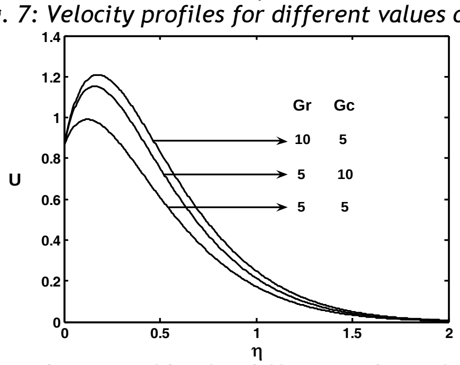

Fig. 8: Velocity profiles for different values of Gr, Gc

The effect of velocity profiles for different time ($t = 0.2,0.4,0.6,0.8$), $R = 10$, $M = 0.2$, $K = 2$, $Gr = Gc = 2$ and $\omega t = \pi/6$ are shown in Figure 7. In this case, the velocity increases gradually with respect to time t. The velocity profiles for different thermal Grashof number ($Gr = 5,10$), mass Grashof number ($Gc = 5,10$), $\omega t = \pi/6$, $K = 15$, $R = 5$, $M = 2$ and time $t = 0.8$ are shown in Figure 8. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.

CONCLUSION

The study of MHD and thermal radiation effects on flow past an oscillating infinite isothermal vertical plate, in the presence of chemical reaction of first order. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like ωt, M, R, K, Gr, Gc, Sc and t are studied. The study concludes that the velocity increases with decreasing phase angle ωt, magnetic field parameter M and chemical reaction parameter K. The trend is just reversed with respect to time t. As expected, the plate concentration increases with decreasing chemical reaction parameter.

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NOMENCLATURE

C' - species concentration in the fluid
C - dimensionless concentration
C_p - specific heat at constant pressure
D - mass diffusion coefficient
G_c - mass Grashof number
G_r - thermal Grashof number
g - accelerrated due to gravity
k - thermal conductivity
M - Magnetic field parameter
Pr - Prandtl number
R - Radiation parameter
Sc - Schmidt number
T - temperature of the fluid near the plate
t' - time
t - dimensionless time
u - velocity of the fluid in the x-direction
u_0 - velocity of the plate
U - dimensionless velocity
x - spatial coordinate along the plate
y' - coordinate axis normal to the plate m
y - dimensionless coordinate axis normal to the plate
β - volumetric coefficient of thermal expansion
β^* - volumetric coefficient of expansion with concentration
μ - cofficient of viscosity
υ - kinematic viscosity
ρ - density of the fluid
τ - dimensionless skin-friction kg.
θ - dimmensionless temperature
η - similarity parameter
erfc - complementary error function

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AUTHORS & AFFILIATION

[1.] *R. MUTHUCUMARASWAMY,*
[2.] *G. NAGARAJAN,*
[3.] *V.S.A. SUBRAMANIAN*

[1] *DEPARTMENT OF APPLIED MATHEMATICS, SRI VENKATESWARA COLLEGE OF ENGINEERING, SRIPERUMBUDUR 602 105, INDIA*
[2] *DEPARTMENT OF MATHEMATICS, PANIMALAR ENGINEERING COLLEGE, NAZARATHPET, POONAMALLEE,CHENNAI 602103, INDIA*
[3] *DEPARTMENT OF MATHEMATICS, ARUMUGAM PILLAI SEETHAIAMMAL COLLEGE, THIRUPPATTUR- 630211, INDIA*

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