PERFORMANCE OF A MAGNETIC FLUID BASED DOUBLE LAYERED ROUGH POROUS SLIDER BEARING CONSIDERING THE COMBINED POROUS STRUCTURES

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Abstract: Efforts have been made to obtain the analytical solution for pressure, load and friction for a magnetic fluid based double layered rough porous slider bearing. The permeability of the upper layer is based on the model of Kozeny-Carman while Irmay's model governs the permeability of the lower layer. Regarding roughness, the method adopted by Christensen and Tonder finds the application here in statistical averaging of the associated Reynolds equation. The magnetic field is taken oblique to the stator. The results are illustrated by graphical representations which show that the introduction of combined porous structure of the double layered results in an enhanced performance. The friction remains considerably reduced. The magnetization tries to compensate the adverse effect of roughness for a large range of combined porous structures.

Keywords: Slider Bearing, Magnetic Fluid, Porous structure, Roughness

INTRODUCTION
The contribution of surface roughness and properties of lubricant film on the load carrying capacity and friction is an important aspect in the analysis of slider bearings. Porous sliders are important in fluid cushioned moving pads. Applications of porous bearings in mounting horsepower motors include vacuum cleaners, water pumps, record players, tape recorders and generators. Traditionally, the analyses of porous slider bearings have been based upon the Darcy’s model, where Darcy’s equations were applied to guide fluid motion through the porous medium (Murti (1974), Srinivasan (1977), Verma (1978), Kumar (1980), Lin (2001), Khan et al. (2011)). Cusano (1972) obtained an analytical solution for the performance characteristics of a two layered porous bearing using the short bearing approximation. Bujurke (1992) investigated the influence of couple stresses on the dynamic properties of a double layered porous slider bearing. Later on, Rao et al (2013) presented an analysis of a long journal bearing with a double layer porous lubricant film using couple stress and Newtonian fluids. In all the above studies a porous layered lubricant film configuration increased the load carrying capacity and reduced the coefficient of friction in the bearing.

In most of the studies conventional lubricants were used. Use of magnetic fluid as a lubricant modifying the performance of the bearings has been very well established. The application of magnetic fluid as a lubricant was investigated by many authors (Agrawal (1986), Bhat and Deheri (1995), Odenbach (2004), Nada and Osman (2007), Urreta et al. (2009), Huang et al. (2011)). In all these studies it has been established that the performance of bearing system could be improved by using a magnetic fluid as the lubricant.

Surface roughness evaluation is very important for many fundamental problems such as friction, load carrying capacity, contact deformation, heat and electric current conditions, tightness of contact joints and positional accuracy. For this reason surface has been the subject of experimental and theoretical investigations for many decades. In literature, many investigations such as (Tzeng and Saibel (1967), Christensen and Tonder (1969a, 1969b, 1970), Prajapati (1991), Gupta and Deheri (1996)) accounting for surface roughness effect, have been proposed in order to seek a more realistic representation of bearing surfaces. Gururajan and
Prakash (2002) examined the effect of surface roughness in hydrodynamic narrow porous journal bearings operating under steady conditions. Deheri et al. (2004) analyzed the performance of longitudinally rough slider bearings with squeeze film formed by a magnetic fluid. Deheri et al. (2005) discussed the performance of transversely rough slider bearings with squeeze film formed by a magnetic fluid. The behavior of squeeze film between rough porous infinitely long parallel plates with porous matrix of variable film thickness was discussed by Patel et al. (2008). Patel et al. (2010) dealt with the performance of a magnetic fluid-based squeeze film between transversely rough triangular plates. Shimpi and Deheri (2011) analyzed the performance of a magnetic fluid-based squeeze film between porous infinitesimally long rough rectangular plates. Patel and Deheri (2012) investigated the performance of a ferrofluid lubricated rough porous inclined slider bearing considering slip velocity. All the above investigations indicate that although magnetization introduces a positive effect on the performance, the bearing suffers owing to transverse surface roughness. It was noticed that the performance of the bearing system could be made to improve by suitably choosing the magnetization parameter in the case of negatively skewed roughness, which became sharper with the variance. Recently, Patel and Deheri (2013) launched an analysis for the comparison of various porous structures on the performance of a magnetic fluid-based transversely rough short bearing. It was seen that the negatively skewed roughness induced an increase in load carrying capacity which could be canalized to compensate the adverse effect of porosity, at least in the case of Kozeny-Carman model and the effect of magnetization responding more in the case of Kozeny-Carman model as compared to Irmay’s model.

The objective of this study is to investigate the performance of a magnetic fluid-based double layered rough porous slider bearing considering the combined porous structures.

2. ANALYSIS

The geometry and configuration of the problem is shown in the Figure 1. All the assumptions of conventional lubrication theory are retained (Srinivasan (1977)). The porous regions are assumed to be homogeneous and isotropic and the lubricant is an incompressible fluid. The pressure in porous regions satisfies the Laplace equation.

A porous material is filled with globular particles (a mean particle size $D_c$), which is given in Figure 1A. The Kozeny-Carman equation is a well-known relation used in the field of fluid dynamics to calculate the pressure drop of a fluid flowing through a packed bed of solids. This formulation remains valid only for laminar flow. The Kozeny-Carman equation mimics some experimental trends and hence serves as a quality control tool for physical and digital experimental results. The Kozeny-Carman equation is very often presented as permeability versus porosity, pore size and turtuosity. Liu (2009) (Patel and Deheri (2013)), suggests that the Kozeny-Carman formulation turns in the relation

$$\phi_l = \frac{D_c^2 e^3}{180(1-e)^2}$$

where $e$ is the porosity.

In Figure 1B, the model consists of three sets of mutually orthogonal fissures (a mean solid size $D_s$) and assuming no loss of hydraulic gradient at the junctions, Irmay (1955) (Patel and Deheri (2014)) derived the permeability
where $m = (1 - e)$ and $e$ is the porosity.

It is assumed that the bearing surfaces are transversely rough. According to the stochastic model of Christensen and Tonder (1969a, 1969b, 1970), the thickness $h(x)$ of the lubricant film is taken as

$$h(x) = \bar{h}(x) + h_s$$  \hspace{1cm} (1)

where $\bar{h}(x)$ is the mean film thickness and $h_s$ is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. $h_s$ is governed by the probability density function

$$f(h_s) = \begin{cases} \frac{3\beta}{32c_1} \left(1 - \frac{h_s^2}{c^2}\right)^3 & , \text{ if } -c_s \leq h_s \leq c_s \\ 0 & , \text{ elsewhere} \end{cases}$$

wherein $c$ is the maximum deviation from the mean film thickness. The mean $\alpha$, the standard deviation $\sigma$ and the parameter $\epsilon$, which is the measure of symmetry of the random variable $h_s$, are defined and discussed in Christensen and Tonder (1969a, 1969b, 1970). The details can be culled from these investigations.

Introduction of these assumptions lead to a modified Reynolds equation for the film region

$$\frac{d}{dx} \left[ \left(h^3 + 12\phi_1 H_1\right) \frac{dh}{dx} - \frac{\mu \rho M^2}{2} \right] + \frac{d}{dx} \left[ \left(h^3 + 12\phi_1 H_1\right) \frac{dh}{dx} - \frac{\mu \rho M^2}{2} \right] = 6\mu U \frac{dh}{dx} + 12\mu \nu \frac{dh}{dx}$$ \hspace{1cm} (2)

Neglecting the side leakage effect and since there is no normal velocity i.e. $Vh=0$, equation (1) reduces to

$$\frac{d}{dx} \left[ \left(h^3 + 12\phi_1 H_1\right) \frac{dh}{dx} - \frac{\mu \rho M^2}{2} \right] = 6\mu U \frac{dh}{dx}$$ \hspace{1cm} (3)

The magnetic field is taken to be oblique to the stator as in Agrawal (1986). Prajapati (1995) investigated the effect of various forms of magnitude of the magnetic field. The magnitude of the magnetic field is taken as

$$M^2 = kx \left(1 - \frac{x}{L}\right)$$

where $k$ is a suitably chosen constant from dimensionless point of view so as to produce a required magnetic field of strength over $10^{-23}$ (Bhat and Deheri (1995)).

Under the usual assumptions of hydro magnetic lubrication (Bhat (2003); Prajapati (1995); Deheri, et. al. (2005)) and stochastically averaging (2) by the method of Christensen and Tonder, the Reynolds equation governing the pressure distribution is obtained as

$$\frac{d}{dx} \left[ \left(g(h) + 12\phi_1 H_1\right) \frac{dh}{dx} - \frac{\mu \rho M^2}{2} \right] = 6\mu U \frac{dh}{dx}$$ \hspace{1cm} (4)

where

$$g(h) = h^3 + 3h^2 \alpha + \frac{3\alpha^2 + \alpha^2}{h} + 3\alpha^2 + \alpha^2 + \alpha$$

while $\mu$ is the magnetic susceptibility, $\mu$ is the free space permeability, $\mu$ is the lubricant viscosity, $\phi_1$ is permeability of inner layer, $\phi_1$ is permeability of outer layer, $H_1$ is wall thickness of the inner layer, $H_2$ is wall thickness of the outer layer, $U$ is tangential velocity of the slider.

The relevant boundary conditions are

$$P(0) = P(L) = 0$$ \hspace{1cm} (5)

Introducing the non dimensional quantities

$$\bar{h} = \frac{h}{h_1} = \frac{h_2}{h_2} - (\bar{h}_2 - 1)X, \bar{P} = \frac{h_2}{\mu U L} \mu \bar{P} = \frac{h_2}{h_1}$$

$$\mu = \frac{\mu_0 \mu \bar{h}_1}{L} \frac{\bar{h}_1}{\bar{h}_2}$$

$$X = \frac{x}{L}, \bar{\sigma} = \frac{\sigma}{h_1}, \bar{\alpha} = \frac{\alpha}{h_1}$$

$$\bar{\rho} = \frac{\mu}{\mu_0} \frac{1}{h_1^3} \left[ \frac{180(1 - \alpha)^2}{h_1^2} + \frac{1}{(1 - m^3)(1 + m^3)} \right]$$

and solving equation (4) using boundary conditions (2) the dimensionless form of the pressure distribution is obtained as

$$P = \frac{\mu}{2} X \left(1 - X\right) + \frac{6}{(1 - \bar{h}_2)}$$

$$\left[ D \ln \bar{F} \left( \frac{J_1}{J_2} \right) + F \ln \left( \frac{\bar{F} F_1}{I_2 \bar{F}} \right) \right] + F \tan^{-1} \left( \frac{2h_2 - \bar{h}_2}{\sqrt{v_2 - J_2}} + c_2 (1 - \bar{h}_2) \right)$$ \hspace{1cm} (6)

where
The expression for non-dimensional load carrying capacity of the bearing system then turns out to be

\[
W = \frac{h^3 w}{\mu U B L^2} = \frac{k^*}{12} \left[\frac{6}{(1 - f^2)^2} + c^* (1 - f^2)^2\right]
\]

3. RESULTS AND DISCUSSION

One can easily see that equation (6) determines the non-dimensional pressure distribution while the dimensionless load carrying capacity is obtained from equation (7). It is clearly observed that the increase in pressure is

\[
F = f \left(\frac{\delta P}{2 \delta x} + \frac{1}{h}\right) dx = \frac{1}{(F_2 - 1)^2} \left[\frac{2}{2} \left(\frac{F_2 - 1}{2}\right) + \frac{1}{4} \left(L_3 - L_4\right)\right]
\]

The effect of double layer is to decrease the frictional force. For a conventional lubricant with smooth surfaces the present discussions reduces to the study of Srinivasan (1977). In addition, from the equation (8) one can easily notice that the combined effect of the two different porous structures is quite significant in reducing the friction.
It is manifest that the expression occurring in equation (7) is linear with respect to the magnetization parameter; as a result, an increase in magnetization would lead to increased load carrying capacity. Probably, this may be due to the fact that the effective viscosity of the lubricant gets increased due to the magnetization.

The fact that \( R \) reduces the load carrying capacity is exhibited in figures 2-6.

Figure 2: Variation of load carrying capacity with respect to \( R \) and \( \psi \).

Figure 3: Variation of load carrying capacity with respect to \( R \) and \( e \).

Figure 4: Variation of load carrying capacity with respect to \( R \) and \( \bar{e} \).

Figure 5: Variation of load carrying capacity with respect to \( R \) and \( \bar{e} \).
Figures 7-9 present the variation of load carrying capacity with respect to the combined porous structures parameter. In figure 8, it is appealing to note that even the porous structure increases the load carrying capacity in a limited way.

\[ \psi \]

![Figure 9: Variation of load carrying capacity with respect to \( \psi \) and \( \alpha \).](image)

\[ \varepsilon \]

![Figure 10: Variation of load carrying capacity with respect to \( e \) and \( \varepsilon \).](image)

\[ \delta \]

![Figure 11: Variation of load carrying capacity with respect to \( e \) and \( \delta \).](image)

The effect of porosity given in figures 10-12 indicates that the porosity follows the path of porous structure parameter in reducing the load carrying capacity. From figure 11, it is observed that to a limited extent the smaller values of standard deviation fail to prevent the increase in load carrying capacity due to small values of porosity parameter. In other words the combined effect of porosity and standard deviation is quite significant for small values of these two parameters as it can improve the performance.

\[ \alpha = -0.05 \]

![Figure 12: Variation of load carrying capacity with respect to \( e \) and \( \alpha \).](image)

\[ \delta = 0.14 \]

![Figure 13: Variation of load carrying capacity with respect to \( \bar{e} \) and \( \delta \).](image)

\[ \alpha = -0.05 \]

![Figure 14: Variation of load carrying capacity with respect to \( \bar{e} \) and \( \alpha \).](image)

The effect of skewness presented in figures 13-14 suggests that positively skewed roughness decreases the load carrying capacity while the load carrying capacity increases due to negatively skewed roughness. Similar is the trends of variance as far as load carrying capacity is concerned (Figure 15). Further, the combined effect of...
standard deviation and skewness introduces a positive effect for larger values of standard deviation in the case of negatively skewed roughness (Figure 13).

Figure 15: Variation of load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\sigma}$.

The variation of friction presented in figures 16-20 makes it clear that $\bar{\sigma}$ reduces the friction in a good way. In figure 20, the friction increases negligibly due to combined effect of $\bar{\sigma}$ and $\bar{\sigma}$, up to 0.003.

Figure 16: Variation of Friction with respect to $h_2$ and $\psi$.

Figure 17: Variation of Friction with respect to $h_2$ and $e$.

Figure 18: Variation of Friction with respect to $h_2$ and $\varepsilon$.

Figure 19: Variation of Friction with respect to $h_2$ and $\alpha$.

Figure 20: Variation of Friction with respect to $h_2$ and $\beta$.

Figure 21: Variation of Friction with respect to $\psi$ and $\varepsilon$. 
Likewise, the porous structure brings down the friction which can be seen from figures 21-23. From figure 22 it is found that porous structure introduces a small increase in the friction as well. Analogously, the porosity parameter reduces the friction in a rapid way which is manifest in figures 24-26. For smaller values of standard deviation and porosity parameter the friction increases and it decreases significantly afterwards (Figure 25).

The effect of positive skewness is to decrease the friction which gets further decreased due to positive variance which can be seen from figures 27-28.

Figure 22: Variation of Friction with respect to $\psi$ and $\overline{\psi}$.

Figure 23: Variation of Friction with respect to $\psi$ and $\overline{\psi}$.

Figure 24: Variation of Friction with respect to $e$ and $\overline{e}$.

Figure 25: Variation of Friction with respect to $\sigma$ and $\overline{\sigma}$.

Figure 26: Variation of Friction with respect to $e$ and $\overline{e}$.

Figure 27: Variation of Friction with respect to $\sigma$ and $\overline{\sigma}$.

Figure 28: Variation of Friction with respect to $\overline{e}$ and $\overline{\sigma}$.
Figure 29 presents the variation of friction with respect to the standard deviation. Of course, magnetization increases the friction. It is found that there is a negligible increase in friction due to magnetization.

A close scrutiny of some of the graphs suggest that the combined porous structure stands to increase the load carrying capacity and consequently resulting in an enhanced performance as the friction is sufficiently reduced at least in the case of negatively skewed roughness. Even the combined positive effect of variance(-ve) and negatively skewed roughness goes a long way in enhancing the performance because the standard deviation has a mild positive effect for lower to moderate values of combined porosity. The reduction in load carrying capacity owing to $\alpha$ can be minimized by the positive effect of magnetization for a large range of combined porous structures, irrespective of roughness. However, this reduction gets enhanced especially, when the negatively skewed roughness is involved. Some of the results presented here establish that the positive effect of magnetization gets enhanced when a double layered porosity is considered. This makes the situation more appealing as the layers are of different porous structures. The effect of negatively skewed roughness becomes more in the case of a double layered porous bearing as compared to that of a single layered bearing system.

4. VALIDATION

The conclusions are validated by giving a comparison of the load and friction of this investigation with some results from already published works. It is notice that load carrying capacity goes up at least by 1%. The results presented here jell well with that of the published article. However, here the load is substantially increased to the magnetization, in spite the adverse effect of roughness. (I- the load carrying capacity of this manuscript, II- the load carrying capacity of Srinivasan (1977))

**Table 1:** Variation of load carrying capacity with respect to $\alpha$ and $\psi$.

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**Table 2:** Variation of load carrying capacity with respect to $\xi$ and $\epsilon$.

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**Table 3:** Variation of friction with respect to $\alpha$ and $\psi$.

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5. CONCLUSION
It is established that this type of bearing system dealing with combined effect of double layered different porous structures may provide a better bearing design. However, to derive an all round improved performance of the bearing system, the roughness aspect must be given due consideration, even if suitable magnetic strength is in place. The use of different porous structures for different layer provides a better scope for minimizing the friction. At the same time the friction is considerably reduced here except in the case of magnetization, when the friction is marginally increased.

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