ACTA TEHNICA CORVINIENSIS – Bulletin of Engineering
Tome VIII [2015] Fascicule 1 [January – March]
ISSN: 2067 – 3809

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APPLICATION OF AMPLITUDE-FREQUENCY FORMULATION TO A NONLINEAR OSCILLATOR ARISING IN THE MICRO-ELECTRO-MECHANICAL SYSTEM (MEMS)

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Abstract: *Study the application of microelectromechanical system (MEMS) devices especially the electrically actuated MEMS devices which require few mechanical components and small voltage levels for actuation is continuously growing. The MEMS devices are widely used as capacitive accelerometer, capacitive sensor, switches and so on. In this paper, the energy balance method has been successfully used to study a nonlinear oscillator arising in the microbeam-based microelectromechanical system (MEMS). The nonlinear ODE equation is solved by a powerful mathematical tool, the amplitude-frequency formulation. The good agreement of results got from amplitude-frequency formulation with results from fourth-order RungeeKutta method indicates that the obtained period is of high accuracy.*
Keywords: *micro electro mechanical system (MEMS); amplitude-frequency formulation; nonlinear vibration; analytical methods*

INTRODUCTION

Study the application of microelectromechanical system (MEMS) devices especially the electrically actuated MEMS devices which require few mechanical components and small voltage levels for actuation is continuously growing. The MEMS devices are widely used as capacitive accelerometer [1], capacitive sensor [2], switches [3] and so on. Compared to the traditional mechanical systems, the MEMS devices are usually small, and their largest size will not exceed one centimeter, sometimes only in micron order. For this large surface-to-volume ratio, the integrated circuit (IC) technology in modern industry facilitates the fabrication of thousands of MEMS devices with increased reliability and reduced cost. However, electrostatic actuation, large deflections and damping caused by different sources give rise to nonlinear behavior. Nonlinearity in MEMS may cause some difficulties in computations. Until now, several techniques have been used to find numerical solutions, for example the shooting method [4] and energy balance method [5]. Although it is difficult to get analytic approximations for different phenomena in MEMS, there are some analytic techniques for nonlinear problems of MEMS. Recently, Fu et al. [5] investigated the nonlinear oscillation problem arising in the MEMS microbeam model by means of the energy balance method.

On the other hand, in the last decades, scientists have proposed and applied some analytical methods to nonlinear equations, such as variational iteration method [6-11], homotopy analysis method [12-13] and some other methods [14-19].

The main goal of this paper is to present an alternative approach, namely amplitude-frequency formulation [20], for constructing highly accurate analytical approximations to the nonlinear oscillation problem arising in the MEMS microbeam model.

PROBLEM DESCRIPTION

Figure 1 represents a fully clamped microbeam with uniform thickness h, length l, width b ($b>>5h$), effective modulus $= E/(1-\upsilon^2)$, Young's modulus E, Poisson's ratio υ and density ρ.

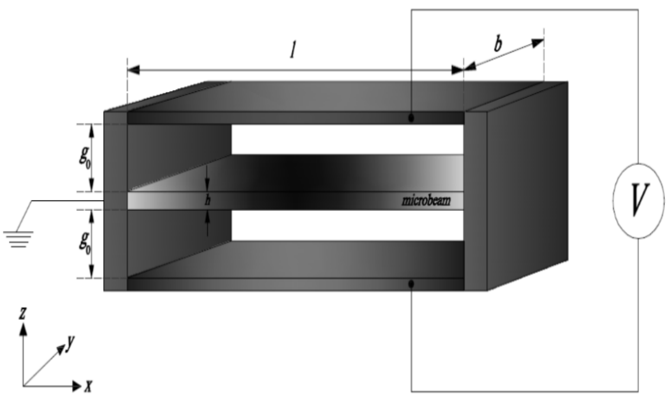

Figure 1. *Schematics of a double-sided driven clamped-clamped microbeam-based electromechanical resonator*

By applying the Galerkin Method and employing the classical beam theory and taking into account of the mid-plane stretching effect as well as the distributed electrostatic force, the dimensionless equation of motion for the microbeam is as follow:

$$(a_1u^4 + a_2u^2 + a_3)\ddot{u} + a_4u + a_5u^3 + a_6u^5 + a_7u^7 = 0, \quad u(0) = A, \ \dot{u}(0) = 0 \tag{1}$$

where u is the dimensionless deflection of the microbeam, a dot denotes the derivative with respect to the dimensionless time variable $t = \tau\sqrt{\bar{E}I / \rho bhl^4}$ with I and t being the second moment of area of the beam cross-section and time, respectively.

In Eq. (1), the physical parameters a_i (i = 1–7) are given by:

$$a_1 = \int_0^1 \phi^6 d\zeta, \tag{2}$$

$$a_2 = -2\int_0^1 \phi^4 d\zeta, \tag{3}$$

$$a_3 = \int_0^1 \phi^2 d\zeta, \tag{4}$$

$$a_4 = \int_0^1 (\phi''''\phi - N\phi''\phi - V^2\phi) d\zeta, \tag{5}$$

$$a_5 = -\int_0^1 (2\phi''''\phi^3 - 2N\phi''\phi^3 + \alpha\phi''\phi\int_0^1 (\phi')^2 d\zeta) d\zeta, \tag{6}$$

$$a_6 = \int_0^1 (\phi''''\phi^5 - N\phi''\phi^5 + 2\alpha\phi''\phi^3\int_0^1 (\phi')^2 d\zeta) d\zeta, \tag{7}$$

$$a_7 = -\int_0^1 (\alpha\phi''\phi^5\int_0^1 (\phi')^2 d\zeta) d\zeta. \tag{8}$$

which, the following nondimensional variables and parameters are introduced:

$$\alpha = \frac{6g_0^2}{h^2}, \ \zeta = \frac{x}{l}, \ N = \frac{\bar{N}l^2}{EI}, \ V^2 = \frac{24\varepsilon_0 l^4 \bar{V}^2}{\bar{E}h^3 g_0^3} \tag{9}$$

while a prime (') indicates the partial differentiation with respect to the coordinate variable x. The trial function is

$$\phi(\zeta) = 16\zeta^2 (1-\zeta)^2 .$$

The parameter N denotes the tensile or compressive axial load, g_0 is initial gap between the microbeam and the electrode, V the electrostatic load and ε_0 vacuum permittivity.

BASIC CONCEPT OF AMPLITUDE-FREQUENCY FORMULATION

For a generalized nonlinear oscillator in Eq. (10), we use two trial functions [21]:

$$u_1 = A\cos\omega_1 t, \quad \omega_1 = 1 \tag{18}$$

$$u_2 = A\cos\omega_2 t, \quad \omega_2 = \omega \tag{19}$$

where ω is assumed to be the frequency of the nonlinear oscillator, Eq. (10).

The residuals are:

$$R_1(t) = -\cos t + f(A\cos t) \tag{20}$$

and

$$R_2(t) = -\omega^2\cos(\omega t) + f(A\cos(\omega t)) \tag{21}$$

We introduce R_{11} and R_{22} defined as [14]:

$$R_{11}(t) = \frac{4}{T_1}\int_0^{\frac{T_1}{4}} R_1(t)\cos(t).dt, \quad T_1 = 2\pi \tag{22}$$

and

$$R_{22}(t) = \frac{4}{T_2}\int_0^{\frac{T_2}{4}} R_2(t)\cos(\omega t).dt, \quad T_2 = \frac{2\pi}{\omega} \tag{23}$$

Applying He's frequency-amplitude formulation, we have:

$$\omega^2 = \frac{\omega_1^2 R_{22}(t) - \omega_2^2 R_{11}(t)}{R_{22}(t) - R_{11}(t)} \tag{24}$$

where ω_1=1 and ω_2=ω.

APPLICATION OF PROPOSED METHOD

In order to assess the accuracy of He's amplitude-frequency formulation for solving nonlinear governing equation of motion and to compare it with the other solutions, in this section we consider this method.

According to He's amplitude-frequency formulation [20], we choose two trial functions u1(t)=Acost and u2(t)=Acosωt, where ω is assumed to be the frequency of the nonlinear oscillator Eq. (1). Substituting the trial functions into Eq. (9) results in, respectively, the following residuals

$$R_1(t) = -(a_1A^4\cos(t)^4 + a_2A^2\cos(t)^2 + a_3)\times A\cos(t) + a_4A\cos(t) + a_5A^3\cos(t)^3 + a_6A^5\cos(t)^5 + a_7A^7\cos(t)^7 \tag{25}$$

and

$$R_2(t) = -(a_1A^4\cos(\omega t)^4 + a_2A^2\cos(\omega t)^2 + a_3)\times A\cos(\omega t)^2\omega^2 + a_4A\cos(\omega t) + a_5A^3\cos(\omega t)^3 + a_6A^5\cos(\omega t)^5 + a_7A^7\cos(\omega t)^7 \tag{26}$$

We introduce R_{11} and R_{22} defined as:

$$R_{11}(t) = \frac{4}{T_1}\int_0^{\frac{T_1}{4}} R_1(t)\cos(t).dt = \frac{1}{128}A(40a_6A^4 - 40a_1A^4 - 48a_2A^2 + 48a_5A^4 + 35a_7A^6 + 64a_4 - 64a_3), T_1 = 2\pi \tag{27}$$

and

$$R_{22}(t) = \frac{4}{T_2}\int_0^{\frac{T_2}{4}} R_2(t)\cos(\omega t).dt = \frac{1}{128}A(48a_5A^2 + 40a_6A^4 + 35a_7A^6 + 64a_4 - 64\omega^2a_3 - 40A^4\omega^2a_1 - 48A^2\omega^2a_2), \quad T_2 = \frac{2\pi}{\omega} \tag{28}$$

Applying He's frequency-amplitude formulation, we have:

$$\omega^2 = \frac{\omega_1^2 R_{22}(t) - \omega_2^2 R_{11}(t)}{R_{22}(t) - R_{11}(t)} \tag{29}$$

where ω_1=1 and ω_2=ω. Finally, the amplitude-depended frequency can be approximated as:

$$\omega_{AFF} = \frac{\sqrt{2}}{4}\frac{\sqrt{64a_4 + 48a_5A^2 + 40a_6A^4 + 35a_7A^6}}{\sqrt{5a_1A^4 + 6a_2A^2 + 8a_3}} \tag{30}$$

Its approximate solution reads

$$u(t) = A\cos\left(\frac{\sqrt{2}}{4}\frac{\sqrt{64a_4 + 48a_5A^2 + 40a_6A^4 + 35a_7A^6}}{\sqrt{5a_1A^4 + 6a_2A^2 + 8a_3}}t\right) \tag{31}$$

The comparison between energy balance method and fourth-order Rungee Kutta method is plotted in Figure 2. Herein the values of parameters are taken as N= 10 and α= 24.

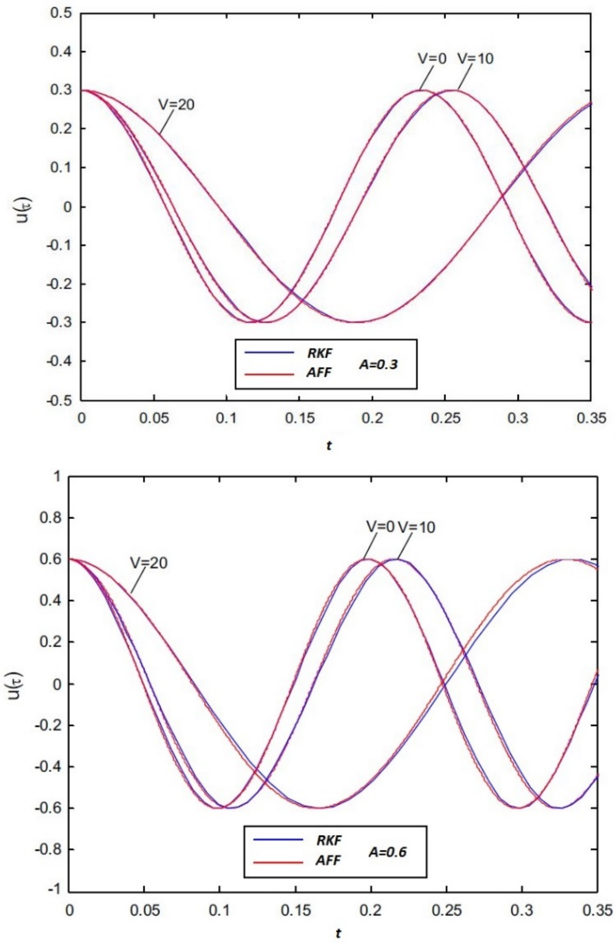

***Figure 2**. The comparison between energy balance method and the fourth-order Rungee Kutta method, solid lines: Rungee Kutta solutions and dashed lines: amplitude-frequency formulation.*

CONCLUSIONS

In this paper, the amplitude-frequency formulation has been successfully used to study a nonlinear oscillator arising in the microbeam-based MEMS where the midplane stretching effect and distributed electrostatic force are both considered. For the free vibration of a microbeam the governing equation is solved by a powerful mathematical tool, the amplitude-frequency formulation. The comparison of results got from amplitude-frequency formulation and fourth order Rungee Kutta method indicates that the obtained period is of high accuracy.

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ACTA Technica CORVINIENSIS
BULLETIN OF ENGINEERING
ISSN:2067-3809