ACTA TEHNICA CORVINIENSIS – Bulletin of Engineering
Tome VIII [2015] Fascicule 2 [April – June]
ISSN: 2067 – 3809

[1.] István ECSEDI, [2.] Ákos József LENGYEL

AN EQUILIBRIUM PROBLEM OF CURVED COMPOSITE BEAM WITH INTERLAYER SLIP

[1-2.] Institute of Applied Mechanics, University of Miskolc, H-3515 Miskolc-Egyetemváros, HUNGARY

Abstract: In this paper an equilibrium problem of two-layered curved composite beam with flexible shear connection is considered. Both end cross sections of the considered curved beam are radially guided. The applied load acts in radial direction. Three types of load are considered. In Example 1 the curved composite beam is partially loaded by uniform radial load. In Example 2 on the whole upper part of the curved beam is loaded by uniform radial load. The third example deals with the concentrated radial load. In all three cases the solution for radial displacement and cross sectional rotations are obtained by Fourier's method and by the application of derived formulae the slip and the normal force, the shear force and the bending moment are determined.

Keywords: curved beam, composite, interlayer slip

INTRODUCTION

Composite members have long been used in many civil engineering structures. In general they consist of two or more elements of the same of different materials connected by some means to form a single structural unit [5]. The problem of layered straight beam with imperfect shear connection has been studied for a long time. The first theory of this type of composite beams was developed by Newmark et al. [6]. The static analysis done by Newmark et al. [6] is based on the Euler-Bernoulli beam theory and become a basis of subsequent investigations of the layered beam with interlayer slip [7–10]. Above mentioned papers deal with straight layered beam. In [2] a two-layer ring with interlayer slip under the action of static load is analysed. In this paper we consider two-layered curved composite beam with imperfect shear connection whose deformation is in plane deformation. Our aim is to give the complete strength of materials solution of the equilibrium problem for curved composite beam with flexible shear connection shown in Figure 1. Both ends of curved composite beam are radially guided and the curvature is uniform. The formulation of the problem will be given in cylindrical coordinate system $Or\varphi z$. The plane $z=0$ is the plane of symmetry for the material, geometrical, loading and supporting conditions. Let

$$B_i=\left\{(r,\varphi,z)\,\middle|\,(r,z)\in A_i,\ \ 0\le\varphi\le 2\alpha\right\},\quad (i=1,2) \tag{1}$$

be, where A_i is the cross section of beam component B_i whose elastic material has Young modulus E_i $(i=1,2)$ according to Figure 1. The connection of beam component B_1 and B_2 at their common cylindrical boundary ∂B_{12}, which is given by next equations

$$r=c,\quad 0\le\varphi\le 2\alpha,\quad |z|\le\frac{t}{2}, \tag{2}$$

in radial direction is perfect, but in circumferential direction may be jump in the displacement field. This possible jump is called the interlayer slip. The applied radial load is f as shown in Figure 1. It is assumed that each curved layer separately follows the Euler-Bernoulli hypothesis and the load-slip relation for the flexible shear connection is linear. The paper presents solutions for radial displacement, slip, cross-sectional rotations and internal forces.

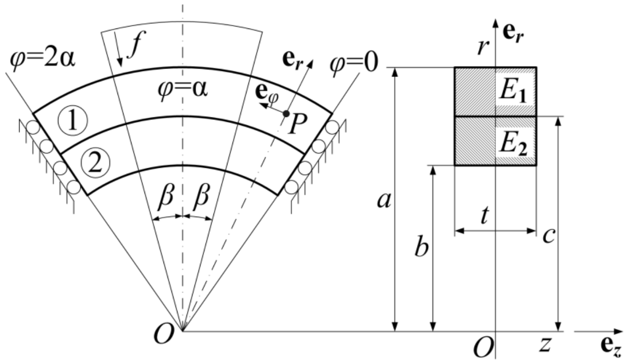

Figure 1. Curved composite beam

GOVERNING EQUATIONS

Denote the unit vectors of cylindrical coordinate system $Or\varphi z$ $\mathbf{e}_r$, $\mathbf{e}_\varphi$ and $\mathbf{e}_z$. The next displacement field will be used to describe the in-plane deformations of curved composite beam [2-4]

$$\mathbf{u}=u\mathbf{e}_r+v\mathbf{e}_\varphi+w\mathbf{e}_z,\quad u=U(\varphi),\quad w=0, \tag{3}$$

$$v(r,\varphi,z)=r\phi_i(\varphi)+\frac{\mathrm{d}U}{\mathrm{d}\varphi},\qquad (r,\varphi,z)\in B_i,\quad (i=1,2). \tag{4}$$

Application of the strain displacement relationships of the linearized theory of elasticity gives [1]

$$\varepsilon_r=\varepsilon_z=\gamma_{r\varphi}=\gamma_{\varphi z}=\gamma_{rz}=0, \tag{5}$$

$$\varepsilon_\varphi=\frac{1}{r}\left(\frac{\mathrm{d}^2U}{\mathrm{d}\varphi^2}+U\right)+\frac{\mathrm{d}\phi_i}{\mathrm{d}\varphi},\qquad (r,\varphi,z)\in B_i,\quad (i=1,2). \tag{6}$$

The strains given by Eqs. (5), (6) satisfy the requirements of the Euler-Bernoulli beam theory, only ε_φ is different from zero and all the shearing strains vanish. From the definition of interlayer slip s it follows that (Figure 1)

$$s(\varphi)=c\left[\phi_1(\varphi)-\phi_2(\varphi)\right]. \tag{7}$$

A detailed analysis gives the next expression for the interlayer shear force [2]

$$T(\varphi)=kc^2t\left[\phi_1(\varphi)-\phi_2(\phi)\right], \tag{8}$$

where k is the slip modulus, t is the thickness of cross section and the common cylindrical boundary of B_1 and B_2 is given by $r=c$ and $|z|\le t/2$. Application of the Hooke's law gives for normal stress σ_φ

$$\sigma_\varphi=E_i\left[\frac{1}{r}\left(\frac{\mathrm{d}^2U}{\mathrm{d}\varphi^2}+U\right)+\frac{\mathrm{d}\phi_i}{\mathrm{d}\varphi}\right], \tag{9}$$
$$(r,\varphi,z)\in B_i,\quad (i=1,2).$$

The internal forces are defined as

$$N_i=\int_{A_i}\sigma_\varphi \mathrm{d}A,\quad (i=1,2),\quad N=N_1+N_2, \tag{10}$$

$$M_i=\int_{A_i}r\sigma_\varphi \mathrm{d}A,\quad (i=1,2),\quad M=M_1+M_2. \tag{11}$$

The connection between the shear force S and normal force N is as follows [3, 4]

$$S(\varphi)=-\frac{\mathrm{d}N}{\mathrm{d}\varphi}. \tag{12}$$

Combination of Eq. (9) with Eqs. (10), (11) yields

$$N_i=\frac{A_iE_i}{R_i}W+A_iE_i\frac{\mathrm{d}\phi_i}{\mathrm{d}\varphi},\quad (i=1,2), \tag{13}$$

$$M_i=A_iE_iW+r_iA_iE_i\frac{\mathrm{d}\phi_i}{\mathrm{d}\varphi},\quad (i=1,2), \tag{14}$$

where

$$\frac{1}{R_i}=\frac{1}{A_i}\int_{A_i}\frac{\mathrm{d}A}{r},\quad r_i=\frac{1}{A_i}\int_{A_i}r\mathrm{d}A,\quad (i=1,2), \tag{15}$$

$$W(\varphi)=\frac{\mathrm{d}^2U}{\mathrm{d}\varphi^2}+U. \tag{16}$$

From Eqs. (10-14) it follows that

$$N=\frac{AE_0}{R}W+A_1E_1\frac{\mathrm{d}\phi_1}{\mathrm{d}\varphi}+A_2E_2\frac{\mathrm{d}\phi_2}{\mathrm{d}\varphi}, \tag{17}$$

$$S=-\left(\frac{AE_0}{R}\frac{\mathrm{d}W}{\mathrm{d}\varphi}+A_1E_1\frac{\mathrm{d}^2\phi_1}{\mathrm{d}\varphi^2}+A_2E_2\frac{\mathrm{d}^2\phi_2}{\mathrm{d}\varphi^2}\right), \tag{18}$$

$$M=AE_0W+r_1A_1E_1\frac{\mathrm{d}\phi_1}{\mathrm{d}\varphi}+r_2A_2E_2\frac{\mathrm{d}\phi_2}{\mathrm{d}\varphi}. \tag{19}$$

Here, we introduce A, E_0, R which are defined as

$$A=A_1+A_2,\quad E_0=\frac{E_1A_1+E_2A_2}{A}, \tag{20}$$
$$\frac{AE_0}{R}=\frac{A_1E_1}{R_1}+\frac{A_2E_2}{R_2}.$$

The next equations of equilibrium will be used [2, 3]

$$\frac{\mathrm{d}^2N}{\mathrm{d}\varphi^2}+N-f_r=0, \tag{21}$$

$$\frac{\mathrm{d}M}{\mathrm{d}\varphi}=0, \tag{22}$$

$$\frac{\mathrm{d}M_1}{\mathrm{d}\varphi}-K(\phi_1-\phi_2)=0. \tag{23}$$

In Eq. (23)

$$K=ktc^3. \tag{24}$$

We note, the unit of k is force/(length)[3] and the unit of K is (force)(length). Detailed forms of Eqs. (21-23) are as follows

$$\frac{AE_0}{R}\left(\frac{\mathrm{d}^2W}{\mathrm{d}\varphi^2}+W\right)+A_1E_1\left(\frac{\mathrm{d}^3\phi_1}{\mathrm{d}\varphi^3}+\frac{\mathrm{d}\phi_1}{\mathrm{d}\varphi}\right)+ \tag{25}$$
$$+A_2E_2\left(\frac{\mathrm{d}^3\phi_2}{\mathrm{d}\varphi^3}+\frac{\mathrm{d}\phi_2}{\mathrm{d}\varphi}\right)-f_r=0,$$

$$AE_0\frac{\mathrm{d}W}{\mathrm{d}\varphi}+r_1A_1E_1\frac{\mathrm{d}^2\phi_1}{\mathrm{d}\varphi^2}+r_2A_2E_2\frac{\mathrm{d}^2\phi_2}{\mathrm{d}\varphi^2}=0, \tag{26}$$

$$A_1E_1\frac{\mathrm{d}W}{\mathrm{d}\varphi}+r_1A_1E_1\frac{\mathrm{d}^2\phi_1}{\mathrm{d}\varphi^2}-K(\phi_1-\phi_2)=0. \tag{27}$$

In the present problem the boundary conditions can be formulated as

$$\phi_1(0)=0,\quad S(0)=0,\quad \left.\frac{\mathrm{d}U}{\mathrm{d}\varphi}\right|_{\varphi=0}=0, \tag{28}$$

$$\phi_1(2\alpha)=0,\quad S(2\alpha)=0,\quad \left.\frac{\mathrm{d}U}{\mathrm{d}\varphi}\right|_{\varphi=2\alpha}=0. \tag{29}$$

SOLUTION BY FOURIER SERIES EXPANSION

We will use the next representation by Fourier series of applied radial load which is given by as (Figure 1)

$$f_r(\varphi)=-f\left[H(\varphi-\alpha+\beta)-H(\varphi-\alpha-\beta)\right], \tag{30}$$

$$f_r(\varphi)=f_0+\sum_{j=0}^{\infty}f_j\cos\frac{j\pi}{\alpha}\varphi, \tag{31}$$

where $f=\text{constant}$ is the applied radial load, H is the Heaviside function and

$$f_0=-f\frac{\beta}{\alpha},\quad f_j=-f\frac{2\cos j\pi\sin\frac{j\pi\beta}{\alpha}}{j\pi}, \tag{32}$$
$$(j=1,2,\ldots).$$

We look for the solution of considered equilibrium problem for $U=U(\varphi),\ \phi_i=\phi_i(\varphi),\ (i=1,2)$ as

$$U(\varphi)=U_0+\sum_{j=1}^{\infty}U_j\cos\frac{j\pi}{\alpha}\varphi, \tag{33}$$

$$\phi_i(\varphi)=\sum_{j=1}^{\infty}\phi_{ij}\sin\frac{j\pi}{\alpha}\varphi,\quad (i=1,2). \tag{34}$$

These functions satisfy all boundary conditions formulated by Eqs. (28), (29). Substitution Eqs. (31), (33), (34) into Eqs. (21-23) leads to the next system of equations

$$U_0=-f\frac{R}{AE_0}\frac{\beta}{\alpha}, \tag{35}$$

$$\mathbf{A}_j\mathbf{x}_j=\mathbf{b}_j,\quad \mathbf{A}_j=\left[a_{mnj}\right], \tag{36}$$
$$\mathbf{x}_j=\left[U_j,\phi_{1j},\phi_{2j}\right]^{\mathrm{T}},\quad \mathbf{b}_j=\left[f_j,0,0\right]^{\mathrm{T}},$$

$$a_{11j} = \frac{AE_0}{R}\left[\left(\frac{j\pi}{\alpha}\right)^2 - 1\right]^2, \qquad (37)$$

$$a_{12j} = \frac{A_1E_1 j\pi}{\alpha}\left[1-\left(\frac{j\pi}{\alpha}\right)^2\right], \qquad (38)$$

$$a_{13j} = \frac{A_2E_2 j\pi}{\alpha}\left[1-\left(\frac{j\pi}{\alpha}\right)^2\right], \qquad (39)$$

$$a_{21j} = \frac{AE_0}{R} j\pi\left[\left(\frac{j\pi}{\alpha}\right)^2 - 1\right], \qquad (40)$$

$$a_{22j} = -r_1 A_1 E_1\left(\frac{j\pi}{\alpha}\right)^2, \qquad (41)$$

$$a_{23j} = -r_2 A_2 E_2\left(\frac{j\pi}{\alpha}\right)^2, \qquad (42)$$

$$a_{31j} = \frac{A_1E_1 j\pi}{\alpha}\left[\left(\frac{j\pi}{\alpha}\right)^2 - 1\right], \qquad (43)$$

$$a_{32j} = -r_1 A_1 E_1\left(\frac{j\pi}{\alpha}\right)^2 - K, \qquad (44)$$

$$a_{33j} = K, \quad (j = 1, 2, \ldots). \qquad (45)$$

From the solution of system of linear equation (36) we obtain the expressions of deflection $U(\varphi)$, and cross-sectional rotations $\phi_1(\varphi)$, $\phi_2(\varphi)$. Applications of formulae (10-14) give the expressions of internal forces and couples.

EXAMPLES

Example 1

The next data are used in Example 1: $\alpha = \frac{\pi}{4}$, $\beta = \frac{\pi}{16}$, $f = 1\,[\mathrm{N}]$, $a = 0.04\,[\mathrm{m}]$, $b = 0.02\,[\mathrm{m}]$, $c = 0.03\,[\mathrm{m}]$, $E_1 = 10^{12}\,[\mathrm{Pa}]$, $E_2 = 8\times10^3\,[\mathrm{Pa}]$, $k = 80\times10^{10}\,[\mathrm{N/m^3}]$. *Figure 2 shows the deflection and the graph of slip function is shown in Figure 3.*

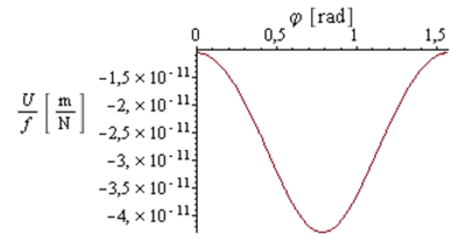

Figure 2. *Plot of the deflection function*

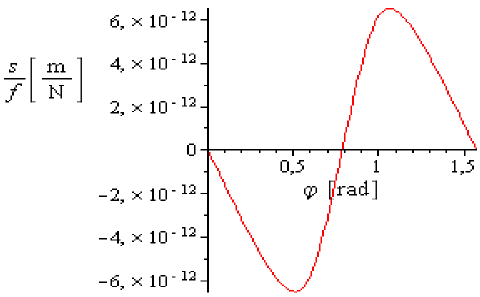

Figure 3. *Plot of slip function*

The graphs of internal forces N, S and bending moment M are presented in Figures 4, 5, 6.

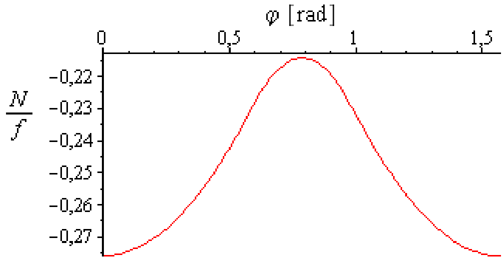

Figure 4. *Plot of the normal force function*

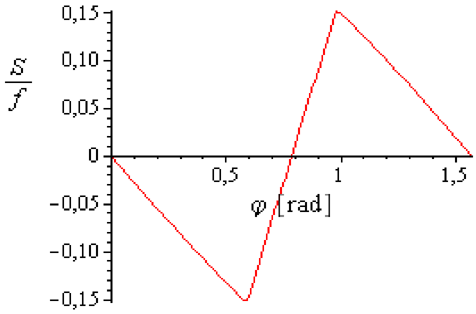

Figure 5. *Plot of the shear force function*

Example 2

In Example 2 the same data are used as in Example 1 except β, which is here $\beta = \frac{\pi}{4}$ (Figure 7). In this case we have

$$\frac{U}{f} = -\frac{R}{AE_0} = -1.041279\times10^{-10}\,[\mathrm{m/N}],\ \phi_1 = \phi_2 = 0,\ N = f,$$
$$S = 0,\ \frac{M}{f} = R = -0.033737\,[\mathrm{m}].$$

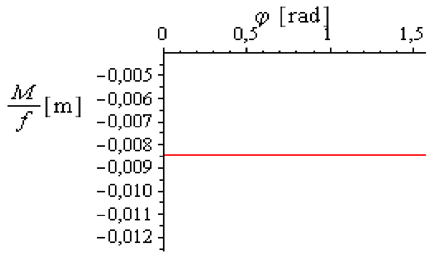

Figure 6. *Plot of bending moment*

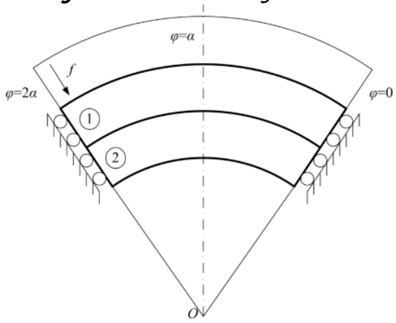

Figure 7. *The case of $\beta = \alpha$*

Example 3

Example 3 deals with the case of concentrated load applied at $\varphi = \alpha$ as shown in Figure 8. From equations of the third Section (Solution by Fourier series expansion) we obtain formulae concerned to the case of concentrated load by next limit calculation $\beta \to 0$ and

$f \to \infty$ such that $F = 2\beta f$ is a given finite value. The results of computations are shown in Figures 9-13. In Figures 9 and 10 the deflection function and the slip function are shown, the internal forces N and S are shown in Figures 11 and 12 and the graph of bending moment is presented in Figure 13.

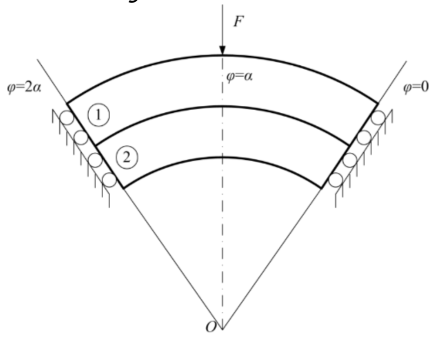

Figure 8. *The case ofconcentrated load* $(\beta \to 0, f \to \infty)$

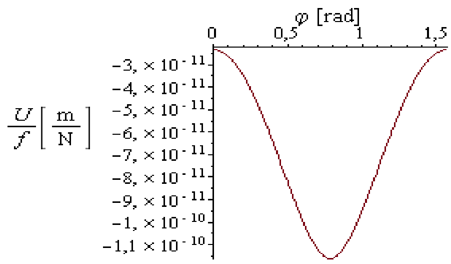

Figure 9. *The plot of* U *for* $\beta \to 0, f \to \infty$

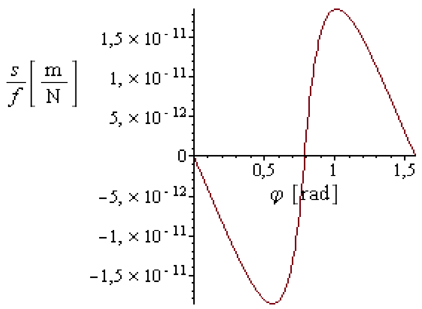

Figure 10. *The plot of* s *for* $\beta \to 0, f \to \infty$

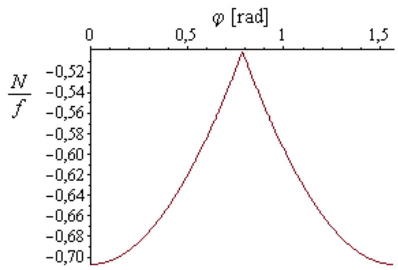

Figure 11. *The plot of* N *for* $\beta \to 0, f \to \infty$

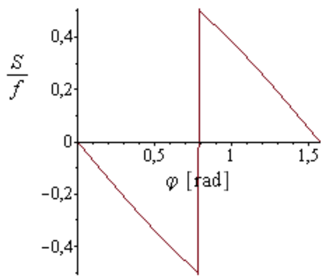

Figure 12. *The plot of* S *for* $\beta \to 0, f \to \infty$

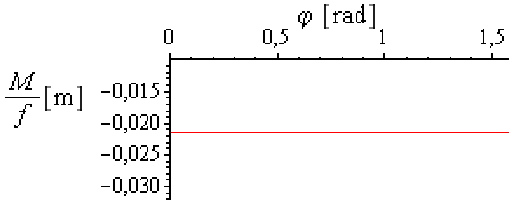

Figure 13. *The plot of* M *for* $\beta \to 0, f \to \infty$

CONCLUSIONS

Paper presents the solution of a static problem of a two-layered composite curved beam with flexible shear connection for radial displacement, slip, normal force, shear force and bending moment. The applied load acts in radial direction and the end cross sections of curved beam are radially guided. The presented analytical, solution can be used as benchmark solution to check the validity of the different numerical methods, such as finite differences and finite element method.

Acknowledgements

This research was (partially) carried out in the framework of the Center of Excellence of Innovative Engineering Design and Technologies at the University of Miskolc.

REFERENCES

[1] Sokolnikoff, I. S.: Mathematical Theory of Elasticity. McGraw-Hill, New York, 1956.
[2] Ecsedi, I., Dluhi, K.: Analysis of curved composite beams with interlayer slip. GÉP Vol. LIV, Issue 10-11, 24-27, 2003, (In Hungarian).
[3] Ecsedi, I., Dluhi, K.: A linear model for the static and dynamic analysis of non-homogeneous curved beams. Appl. Math. Modelling Vol. 29, 1211-1231, 2005.
[4] Ecsedi, I., Dluhi, K.: A model for the analysis of curved composite beams. Euro. Conf. on Num. Meth. and Comp. Mech. Miskolc, 15-19. July 2002.
[5] Ranzi, G.: Partial Interaction Analysis of Composite Beams. VDM Verlag, Saarbrücken, 2009.
[6] Newmark, N. M., Siess, C. P., Viest, I. M.: Test and analysis of composite beam with incomplete interaction. Proceedings of the Society for Experimental Stress Analysis. Vol. 9, 75-92., 1951.
[7] Girhammar, U. A., Gopu, V. K. A.: Composite beam-columns with interlayer slip – Exact analysis. ASCE Journal of Structural Engineering. Vol. 119, 1265-1282., 1993.
[8] Goodman, J. R., Popov, E. P.: Layered beam systems with interlayer slip. Wood Science. Vol. 1, 148-158., 1969.
[9] Girhammar, U.: A simplified analysis method for composite beams with interlayer slip. Int. Journ. of Mech. Science. Vol. 49, 515-530., 2007.
[10]Murakami, H.: A laminated beam theory with interlayer slip. Journ. of Appl. Mech. Vol. 51, 551-558.

ACTA Technica CORVINIENSIS
BULLETIN OF ENGINEERING
ISSN:2067-3809