ACTA TEHNICA CORVINIENSIS – Bulletin of Engineering
Tome VIII [2015] Fascicule 2 [April – June]
ISSN: 2067 – 3809

1. József FARKAS, 2. Károly JÁRMAI

OPTIMUM DESIGN OF WELDED STIFFENED PLATE STRUCTURE FOR A FIXED STORAGE TANK ROOF

1-2. University of Miskolc, H-3515 Miskolc, HUNGARY

Abstract: *The optimization problem of a welded fixed roof for a vertical storage tank is studied. The load from snow and from a 150 mm soil layer is considered. The roof is constructed from stiffened sectorial trapezoidal plate elements and radial beams. The stiffeners are of halved rolled I-section and the radial beams are constructed from rolled I-sections. To find the minimum cost solution the thickness of the base plate, the position, number and size of circumferential stiffeners, the size of radial beams as well as the number of sectors is varied. The distances of stiffeners are non-equidistant. In the cost function the cost of material, welding and painting is taken into account.*
Keywords: *welded fixed roof, optimization, vertical storage tank*

INTRODUCTION

In 1960 the first author has designed a roof structure for a series of storage tanks. The roofs constructed from welded stiffened plate sectorial elements have been suitable for carrying the load of a 150 mm soil layer used to decrease the evaporation loss of stored liquid (kerosene).

From this time the design of stiffened plates has been the main research theme for the first author. The problem of selecting the optimal number of stiffeners led to the structural optimization and the authors have worked out a lot of studies in the field of optimum design of metal structures.

Since the welding is an expensive technology, the decrease of cost of welded structures is an important task for designers. Therefore, our research group, based on international welding time data, has developed a suitable cost analysis. The adaptation and development of effective mathematical optimization methods made it possible to use an optimum design system for the economic (minimum cost) design of welded structures [1]-[4].

In the present study this economic design method is applied for a fixed storage tank roof constructed from stiffened plate sectorial elements and radial beams. In the optimization procedure the optimum values of the following structural characteristics are sought: number and size of radial rolled I-section-beams, the thickness and the transverse non-equidistant stiffening of the deck plate elements.

The roof is designed to carry the snow load as well as the load of 150 mm thick soil layer mentioned earlier. Since the deck plate sectorial elements are trapezoidal and the deck plate thickness should be constant, the transverse stiffening is designed as non-equidistant. The variable distance of stiffeners is calculated from the condition that the deck plate of given thickness should fulfil the bending stress constraint in each part between two stiffeners.

LOADS

Snow load is calculated according to Eurocode 1 [5]

$$s = \mu_1 C_e C_t s_k \qquad (1)$$

$\mu_1 = 0.8, C_e = C_t = 1, s_k = 1.25$ kN/m^2, thus $s = 0.8 \times 1.25 = 1.0$ kN/m^2.

Soil load: 150 mm thick layer of a humid light sand of bulk density 17 kN/m^3

$$p_s = 0.15 \times 17 = 2.55 \text{ kN/m}^2.$$

Snow and soil together $s + p_s = 3.55$ kN/m^2, multiplied by a safety factor of 1.5. $p_M = 5.325 \times 10^{-3}$ N/mm^2.

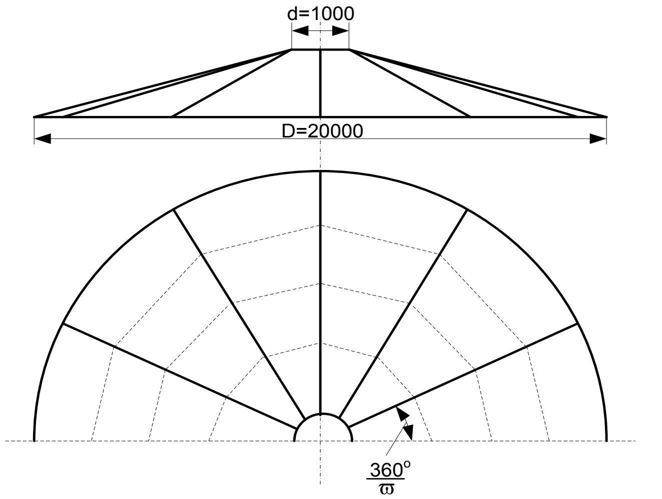

Figure 1. A fixed tank roof

Safety factor for the self mass of sectorial elements is 1.35, and for self mass of radial beams is 1.1.

NUMERICAL DATA

Storage tank diameter $D = 20$ m, inner ring beam diameter $d = 1.0$ m, roof angle $\alpha_0 = 15^0$.

Length of a radial beam $L = 9500/\cos 15^0 = 9835$ mm. The characteristic sizes of a trapezoidal deck plate $x_A = 518$, $x_B = 10353$ mm. $\alpha = 180/\omega$, where $\omega = 10, 12, 14, 16$ is the number of sectors. The length of stiffeners is calculated for given ω: $y_i = x_i f_\omega$, where $f_\omega = 2\tan\alpha$.

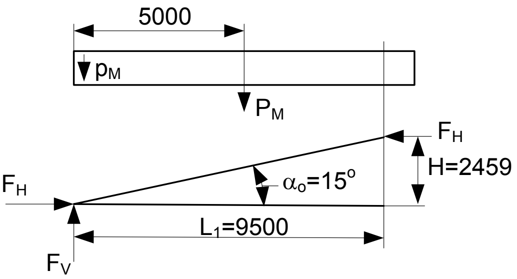

***Figure 2**. Forces from the roof load*

DESIGN OF SECTORIAL STIFFENED DECK PLATE ELEMENTS

Calculation of stiffener distances (x_{0i})

These distances are determined using the condition that the maximum normal stress due to bending in each plate element between stiffeners should not be larger than the yield stress. The maximum bending moment in a deck plate element is calculated approximately for a simply supported rectangular plate according to Timoshenko [6]

$$M_{i\max} = \beta_i p_M a_i^2 \tag{2}$$

where a_i is the smaller side length and β_i is given in function of $b_i / a_i \geq 1$ in Table 1.

***Table 1**. Bending moment factors*

b/a	1	1.1	1.2	1.3	1.4	1.5	1.6	
$10^4\beta$	479	554	627	694	755	812	862	
b/a	1.7	1.8	1.9	2.0	3.0	4.0	5.0	>5
$10^4\beta$	908	948	985	1017	1189	1235	1246	1250

The values of Table 1 are approximated by the following expressions

$$\beta_i = \beta_{\xi i} \text{ if } x_i - x_{i-1} \leq x_i f_\omega \text{ i.e. } x_i \leq \frac{x_{i-1}}{1 - f_\omega} \tag{3}$$

$$\beta_i = \beta_{\eta i} \text{ if } x_i - x_{i-1} > x_i f_\omega \tag{4}$$

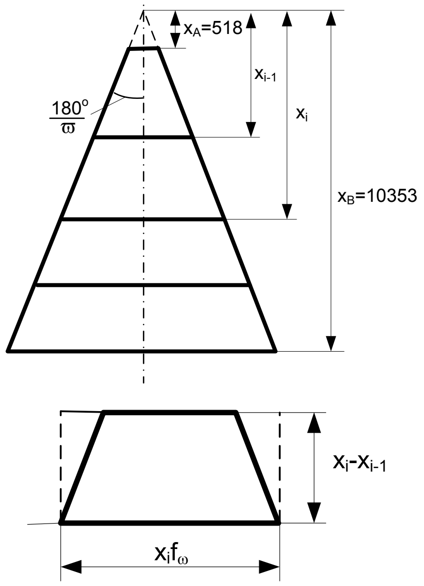

***Figure 3**. Stiffener distances and a part of the base plate*

$$\beta_{\xi i} = a_0 + b\xi_i + c\xi_i^2 + d\xi_i^3 + e\xi_i^4 \quad \xi_i = \frac{x_i f_\omega}{x_i - x_{i-1}} \tag{5}$$

$$\beta_{\eta i} = a_0 + b\eta_i + c\eta_i^2 + d\eta_i^3 + e\eta_i^4 \quad \eta_i = \frac{x_i - x_{i-1}}{x_i f_\omega} \tag{6}$$

a_0 = -0.08022658, b = 0.180443, c = -0.061636, d = 0.009575, e = -0.00056537

From equation

$$M_{i\max} = f_{y1} t^2 / 6 \tag{7}$$

t is the deck plate thickness, f_y = 235 MPa is the yield stress, f_{y1} = f_y/1.1 using equation (2).

$$r_i = \sqrt{\frac{t^2 f_{y1}}{6\beta_i p_M}} \tag{8}$$

and the sought stiffener distance is

$$x_{0i} = r_i + x_{i-1} \quad \text{if} \quad x_i \leq \frac{x_{i-1}}{1 - f_\omega} \tag{9}$$

$$x_{0i} = \frac{r_i}{f_\omega} \quad \text{if} \quad x_i > \frac{x_{i-1}}{1 - f_\omega} \tag{10}$$

The value of x_{0i} can be obtained by iteration with a MathCAD program.

It should be noted that in this calculation the transverse bending moments are neglected but the plate elements are calculated as simply supported and it is also neglected that their edges are partially clamped.

Design of stiffeners

A stiffener is subject to a bending moment

$$M_{si\max} = p_M s_i x_i^2 f_\omega^2 / 8 \tag{11}$$

where $s_i = \dfrac{x_{i+1} - x_{i-1}}{2}$

and the effective plate width

$$s_{ei} = \left(\frac{1.8}{\beta_{0i}} - \frac{0.8}{\beta_{0i}^2} \right) s_i \tag{12}$$

where

$$\beta_{0i} = \frac{s_i}{t} \sqrt{\frac{f_y}{E}}, \quad \text{but } \beta_{0i} \geq 1 \tag{13}$$

E = 2.1x10^5 MPa is the elastic modulus.

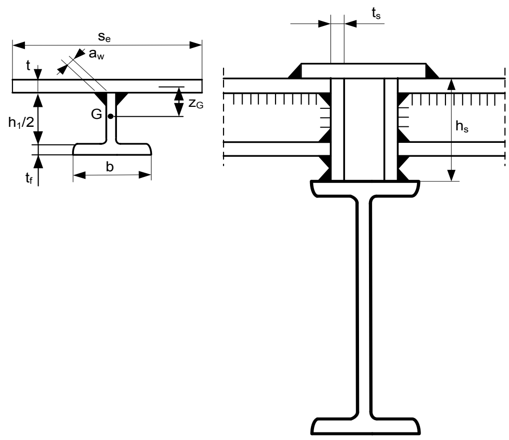

***Figure 4**. Cross-section of a stiffener and connection to the radial beam*

The required section modulus is given by

$$W_{0i} = \frac{M_{si\max}}{f_{y1}} \tag{14}$$

The cross-sectional area of a stiffener of halved rolled I-section and the effective plate part

$$A_{ei} = \frac{h_{1i}t_{wi}}{2} + b_i t_{fi} + s_{ei}t,\quad h_{1i} = h_i - 2t_{fi} \tag{15}$$

The distances of the gravity centres G_i

$$z_{Gi} = \frac{1}{A_{ei}}\left[\frac{h_{1i}t_{wi}}{2}\left(\frac{h_{1i}}{4} + \frac{t}{2}\right) + b_i t_{fi}\left(\frac{h_i + t - t_{fi}}{2}\right)\right] \tag{16}$$

and

$$z_{G1i} = \frac{h_i + t - t_{fi}}{2} - z_{Gi} \tag{17}$$

the moments of inertia

$$I_{yi} = s_{ei}tz_{Gi}^2 + \frac{h_{1i}^3 t_{wi}}{96} + \frac{h_{1i}t_{wi}}{2}\left(\frac{h_{1i}}{4} + \frac{t}{2} - z_{Gi}\right)^2 + b_i t_{fi}\left(\frac{h_i + t - t_{fi}}{2} - z_{Gi}\right)^2 \tag{18}$$

The section moduli are defined as

$$W_{yi} = I_{yi}/z_{0i} \tag{19}$$

where z_{0i} is the greater of z_{Gi} and z_{G1i}.

The required stiffener profile is selected from Table 2 to fulfil the stress constraint

$$W_{yi} \ge W_{0i}$$

Table 2. Ub profiles used for halved rolled i-section stiffeners

UB profile	h	b	t_w	t_f
152x89x16	152.4	88.7	4.5	7.7
168x102x19	177.8	101.2	4.8	7.9
203x133x26	203.2	133.2	5.7	7.8
254x102x25	257.2	101.9	6.0	8.4
305x102x28	308.7	101.8	6.0	8.8

Cost calculation for a sectorial stiffened plate element

The fabrication sequence has two parts:

(a) Welding of the base plate from 7 elements using SAW (Submerged Arc Welding) butt welding. The length of the plate (9835 mm) is divided into 7 parts welded together with 6 butt welds using SAW technology. The total length of welds is

$$L_{w1} = 30783 f_\omega \tag{20}$$

and the cost is calculated as

$$K_{w1} = k_w\left(\Theta_1\sqrt{7\rho V_1} + 1.3C_{w1}t^2 L_{w1}\right) \tag{21}$$

where $k_w = 1.0\$/\min, \Theta_1 = 2, \rho = 7.85x10^{-6}$ kg/mm³, $C_{w1} = 0.1559x10^{-3}$,

$$V_1 = \frac{10353 + 518}{2} 9835 f_\omega t = 53.4581x10^6 f_\omega t \tag{22}$$

(b) Welding of stiffeners to the base plate and to two edge radial plates to complete a sectorial plate element using fillet welds:

$$K_{w2} = k_w\left(\Theta_2\sqrt{(n_{st} + 3)\rho V_2} + \sum_i T_i + T_s\right) \tag{23}$$

where n_{st} is the number of stiffeners, $\Theta_2 = 3$,

$$V_2 = V_1 + V_s + \sum_i V_{sti} \tag{24}$$

the volume of the edge radial plates is

$$V_s = 2x9835 h_s t_s \sqrt{1 + 0.25 f_\omega^2} \tag{25}$$

$t_s = 6$ mm, h_s equals to the stiffener maximum height + 30 mm, the volume of a stiffener is

$$V_{sti} = A_{sti}x_i f_\omega, A_{sti} = \frac{h_{1i}t_{wi}}{2} + b_i t_{fi} \tag{26}$$

welding time for a stiffener is

$$T_i = 1.3C_{w2}a_w^2 2x_i f_\omega + 1.3C_{w3}a_w^2 2(2h_{1i} + 4b_i) \tag{27}$$

where $C_{w2} = 0.2349x10^{-3}, C_{w3} = 0.7889x10^{-3}$ constants for SAW and SMAW (Shielded Metal Arc Welding) fillet welds, respectively, $a_w = 3$ mm, the second part is multiplied by 2, since the welding position is mainly vertical.

The time of welding of the two edge radial plates to the base deck plate is

$$T_s = 1.3C_{w3}a_w^2 L_s, L_s = 2x9835\sqrt{1 + 0.25 f_\omega^2} \tag{28}$$

Material cost of a complete sectorial element is

$$K_{m1} = k_m \rho V_2, k_m = 1.0 \text{ \$/kg.} \tag{29}$$

The painting cost of a complete sectorial element is

$$K_{P1} = k_P S, k_P = 28.8x10^{-6} \text{ \$/mm}^2, \tag{30}$$

$$S = S_s + \sum_i S_{sti} + 2x53.4581x10^6 f_\omega \tag{31}$$

$$S_s = 2x9835 h_s\sqrt{1 + 0.25 f_\omega^2} \tag{32}$$

$$S_{sti} = (h_{1i} + 2b_i)x_i f_\omega \tag{33}$$

The total cost of a sectorial element is

$$K_s = K_{m1} + K_{w1} + K_{w2} + K_{P1} \tag{34}$$

Results of cost calculation for a sectorial element of ω = 12 show that the minimum cost corresponds to the thickness of t = 4 mm. Therefore the further calculations are performed for this thickness only. Table 3 shows the calculated stiffener distances and sizes for ω = 12 and t = 4 mm.

Table 3. Stiffener distances and sizes for ω = 12 and t = 4 mm

x_i mm	h mm
518	-
2197	152.4
3314	152.4
4299	152.4
5248	152.4
6184	152.4
7114	152.4
8041	152.4
8968	177.8
9600	177.8

The cost parts in $ for this sectorial element are as follows: $K_m = 1259$, $K_{w1} = 212$, $K_{w2} = 639$, $K_p = 2001$, the total cost for one element is $K_s = 4112$.

DESIGN OF RADIAL BEAMS

Radial beams of rolled I-section are subject to bending and compression. The load is calculated from snow and soil load (p_M), the mass of a sectorial element (q) and the self mass ($\rho_1 A_r$):

$$p = p_M + q + \rho_1 A_r,\quad q = \rho_1 V_2/L_1,$$
$$\rho_1 = 7.85x10^{-5} \text{ N/mm}^3, L_1 = 9500 \text{ mm.} \tag{35}$$

The maximum bending moment is

$$M_{r\max} = pL_1^2/8 \tag{36}$$

The compression force is

$$N_H = F_M\cos 15^0 + F_V\sin 15^0 \tag{37}$$

where

$$F_V = P_M = pL/2, L = 20000 \text{ mm,}$$

$$H = 9500\sin 15^0 = 2459\,mm \qquad (38)$$

$$F_H = \frac{1}{H}\left[F_V L_1 - P_M\left(\frac{L}{2} - \frac{d}{2}\right)\right] = 2.0333 P_M \qquad (39)$$

It should be noted that the load acting on the half tank side only causes smaller forces acting on radial beams.

Stress constraint for bending and compression according to Eurocode 3 [7]

$$\frac{N_H}{\chi A_r f_{y1}} + k_{yy}\frac{M_{r\max}}{W_y f_{y1}} \le 1 \qquad (40)$$

where

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}, \phi = 0.5\left[1 + 0.21\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right] \qquad (41)$$

$$\bar{\lambda} = \frac{10353}{r\lambda_E}, \lambda_E = \pi\sqrt{\frac{E}{f_y}} = 93.9 \qquad (42)$$

r is the radius of gyration, A_r is the cross-sectional area,

$$k_{yy} = 0.95\left(1 + 0.6\bar{\lambda}\frac{N_H}{\chi A_r f_{y1}}\right) \qquad (43)$$

The suitable rolled I-profile is selected from an Arcelor product catalogue using the British UB profiles.

COST OF A RADIAL BEAM

Material cost

$$K_M = k_m \rho V_R, V_R A_r L_R, L_R = 9825\ mm, \qquad (44)$$

cost of welding to the inner ringbeam and to the tank shell

$$K_W = k_w\left[\Theta_2\sqrt{\rho V_R} + 1.3 C_{w3} a_w^2 2x2\left(2h_1 + 4b\right)\right] \qquad (45)$$

the factor of 2 is used since the welding is mainly vertical.

Cost of painting

$$K_P = k_P\left(2h_1 + 4b\right)L_R \qquad (46)$$

Total cost of a radial beam

$$K_R = K_M + K_W + K_P \qquad (47)$$

ADDITIONAL COST

Material, welding and painting of a deck plate of size 200x6x9825 connecting the sectorial elements as well as welding of the sectorial elements to the radial beam

$$K_A = k_m \rho V_A + 1.3 C_{w2} a_w^2 4 L_R k_w + k_P 200 L_R \qquad (48)$$

$$V_A = 200x6L_R \qquad (49)$$

Total cost of the whole roof structure

$$K = \omega\left(K_s + K_R + K_A\right) \qquad (50)$$

OPTIMIZATION RESULTS

Table 4 and 5 summarize the results (masses and costs) for different values of ω for a sector and for the whole roof

Table 4. Masses in kg and costs for a sector containing a sectorial element and a radial beam

ω	ρV_s	K_s $	ρV_R	K_R $
10	1600	5046	806	1352
12	1259	4112	729	1248
14	1072	3556	588	1078
16	927	3081	588	1078

It can be seen that ω = 14 and ω = 10 gives the minimum mass and minimum cost for the whole roof, respectively. It should be noted that the case of ω = 8 is unrealistic, since in that case the sectorial element has not a trapezoidal but a circular sector form, which needs also partial radial stiffeners beside of the circumferential ones and the cost increases.

Table 5. Masses in kg and costs for the whole roof

ω	ρV_{roof}	K_{roof} $
10	24060	66550
12	23856	67400
14	23240	68470
16	24240	70650

CONCLUSIONS

Minimum cost design of a fixed roof of a vertical steel storage tank is worked out for a numerical model structure. Load of snow and a soil layer is considered. The roof is constructed from sectorial stiffened plate elements and radial beams. The number of sectors is varied between 10 and 16. The sectorial elements are circumferential stiffened with halved rolled I-section stiffeners welded to the base plate. The non-equidistant distances of stiffeners are calculated so that the plate parts are equally stressed. The radial beams are constructed from rolled I-sections. The cost function contents the cost of material, welding and painting. The cost calculation shows that the minimum roof mass and cost corresponds to the number of sections of 14 and 10 respectively.

Acknowledgement

The research was supported by the Hungarian Scientific Research Fund OTKA T 109860 project and was partially carried out in the framework of the Center of Excellence of Innovative Engineering Design and Technologies at the University of Miskolc.

References

[1.] J. Farkas, Optimum design of metal structures, Chichester: Ellis Horwood, 1984.
[2.] J. Farkas and K. Jármai, Analysis and optimum design of metal structures, Rotterdam: Balkema, 1997.
[3.] J. Farkas and K. Jármai, Economic design of metal structures, Rotterdam: Millpress, 2003.
[4.] J. Farkas and K. Jármai, Design and optimization of metal structures, Chichester: Horwood Publishing Ltd., 2008.
[5.] Eurocode 1. Actions on structures. Part 1-3. General actions. Snow loads. 2003.
[6.] S. Timoshenko and S. Woinowsky-Krieger, Theory of plates and shells, New York-Toronto-London: McGraw Hill, 1959.
[7.] Eurocode 3. Design of steel structures. Part 1-1. General structural rules. 2002.

ACTA Technica CORVINIENSIS
BULLETIN OF ENGINEERING
ISSN:2067-3809