Abstract: The optimization problem of a welded fixed roof for a vertical storage tank is studied. The load from snow and from a 150 mm soil layer is considered. The roof is constructed from stiffened sectorial trapezoidal plate elements and radial beams. The stiffeners are of halved rolled I-section and the radial beams are constructed from rolled I-sections. To find the minimum cost solution the thickness of the base plate, the position, number and size of circumferential stiffeners, the size of radial beams as well as the number of sectors is varied. The distances of stiffeners are non-equidistant. In the cost function the cost of material, welding and painting is taken into account.

Keywords: welded fixed roof, optimization, vertical storage tank

INTRODUCTION

In 1960 the first author has designed a roof structure for a series of storage tanks. The roofs constructed from welded stiffened plate sectorial elements have been suitable for carrying the load of a 150 mm soil layer used to decrease the evaporation loss of stored liquid (kerosene).

From this time the design of stiffened plates has been the main research theme for the first author. The problem of selecting the optimal number of stiffeners led to the structural optimization and the authors have worked out a lot of studies in the field of optimum design of metal structures.

Since the welding is an expensive technology, the decrease of cost of welded structures is an important task for designers. Therefore, our research group, based on international welding time data, has developed a suitable cost analysis. The adaptation and development of effective mathematical optimization methods made it possible to use an optimum design system for the economic (minimum cost) design of welded structures [1]-[4].

In the present study this economic design method is applied for a fixed storage tank roof constructed from stiffened plate sectorial elements and radial beams. In the optimization procedure the optimum values of the following structural characteristics are sought: number and size of radial rolled I-section-beams, the thickness and the transverse non-equidistant stiffening of the deck plate elements.

The roof is designed to carry the snow load as well as the load of 150 mm thick soil layer mentioned earlier. Since the deck plate sectorial elements are trapezoidal and the deck plate thickness should be constant, the transverse stiffening is designed as non-equidistant. The variable distance of stiffeners is calculated from the condition that the deck plate of given thickness should fulfil the bending stress constraint in each part between two stiffeners.

LOADS

Snow load is calculated according to Eurocode 1 [5]

\[ s = \mu_0 C_v C_r s_h \]  (1)

\[ \mu_0 = 0.8, C_v = C_r = 1, s_h = 1.25 \text{ kN/m}^2, \text{thus} s = 0.8 \times 1.25 = 1.0 \text{ kN/m}^2. \]

Soil load: 150 mm thick layer of a humid light sand of bulk density 17 kN/m³

\[ p_s = 0.15 \times 17 = 2.55 \text{ kN/m}^2. \]

Snow and soil together \( s + p_s = 3.55 \text{ kN/m}^2 \), multiplied by a safety factor of 1.5. \( p_M = 5.325 \times 10^{-3} \text{ N/mm}^2 \).

Safety factor for the self mass of sectorial elements is 1.35, and for self mass of radial beams is 1.1.

NUMERICAL DATA

Storage tank diameter \( D = 20 \text{ m} \), inner ring beam diameter \( d = 1.0 \text{ m} \), roof angle \( \alpha_0 = 15^\circ \).

Length of a radial beam \( L = 9500/\cos 15^\circ = 9835 \text{ mm} \). The characteristic sizes of a trapezoidal deck plate \( x_1 = 518, x_0 = 10353 \text{ mm} \). \( \alpha = 180/\omega \), where \( \omega = 10, 12, 14, 16 \) is the number of sectors.

The length of stiffeners is calculated for given \( \omega; y_i = x_{f_i} \), where \( f_i = 2\tan \alpha \).
DESIGN OF SECTORIAL STIFFENED DECK PLATE ELEMENTS

Calculation of stiffener distances \( x_0 \)

These distances are determined using the condition that the maximum normal stress due to bending in each plate element between stiffeners should not be larger than the yield stress. The maximum bending moment in a deck plate element is calculated approximately for a simply supported rectangular plate according to Timoshenko [6]

\[
M_{i\text{max}} = \beta_i a_i^2
\]  

where \( a_i \) is the smaller side length and \( \beta_i \) is given in function of \( b_i / a_i \geq 1 \) in Table 1.

### Table 1. Bending moment factors

<table>
<thead>
<tr>
<th>b/a</th>
<th>1</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>10( \beta )</td>
<td>479</td>
<td>554</td>
<td>627</td>
<td>694</td>
<td>755</td>
<td>812</td>
<td>882</td>
</tr>
<tr>
<td>b/a</td>
<td>1.7</td>
<td>1.8</td>
<td>1.9</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
</tr>
<tr>
<td>10( \beta )</td>
<td>908</td>
<td>948</td>
<td>985</td>
<td>1017</td>
<td>1189</td>
<td>1235</td>
<td>1246</td>
</tr>
</tbody>
</table>

The values of Table 1 are approximated by the following expressions

\[
\beta_i = \beta_{i0} \quad \text{if} \quad x_i - x_{i-1} \leq x_i f_y \quad \text{i.e.} \quad x_i \leq \frac{x_{i-1}}{1 - f_y / f_y^{\ast}}
\]

\[
\beta_i = \beta_{i0} \quad \text{if} \quad x_i - x_{i-1} > x_i f_y
\]

The required section modulus is given by

\[
W_{i0} = \frac{M_{i\text{max}}}{f_y}
\]  

\[ a_0 = -0.08022658, \quad b = 0.180443, \quad c = -0.061636, \quad d = 0.009575, \quad e = -0.00056537 \]

From equation

\[
M_{i\text{max}} = f_y t^2 / 6
\]

\( t \) is the deck plate thickness, \( f_y = 235 \text{ MPa} \) is the yield stress, \( f_y^{\ast} = f_y / 1.1 \) using equation (2).

\[
r_i = \sqrt{\frac{t^2 f_y^{\ast}}{6 f_y^{\ast} a_i}}
\]

and the sought stiffener distance is

\[
x_{i0} = r_i + x_{i-1} \quad \text{if} \quad x_i \leq \frac{x_{i-1}}{1 - f_y / f_y^{\ast}}
\]

\[
x_{i0} = \frac{r_i f_y^{\ast}}{f_y} \quad \text{if} \quad x_i > \frac{x_{i-1}}{1 - f_y / f_y^{\ast}}
\]

The value of \( x_0 \) can be obtained by iteration with a MathCAD program.

Design of stiffeners

A stiffener is subject to a bending moment

\[
M_{s\text{max}} = P_M s_i^2 f_y^{\ast} / 8
\]

where \( s_i = \frac{x_{i+1} - x_{i-1}}{2} \) and the effective plate width

\[
s_{ei} = \frac{1.8}{\beta_{i0} - 0.8} s_i
\]

where

\[
\beta_{i0} = \frac{s_i f_y}{E}, \quad \text{but} \quad \beta_{i0} \geq 1
\]

\( E = 2.1 \times 10^6 \text{ MPa} \) is the elastic modulus.

The cross-sectional area of a stiffener of halved rolled I-section and the effective plate part

\[
W_{i0} = \frac{M_{i\text{max}}}{f_y^{\ast}}
\]

\( (5) \)

\( (6) \)

The cross-sectional area of a stiffener of halved rolled I-section and the effective plate part

\[
W_{i0} = \frac{M_{i\text{max}}}{f_y^{\ast}}
\]  

\( (14) \)
The section moduli are defined as

\[ W_i = I_{yi} / z_{ti} \]  

(19)

\[ T_i = 1.3C_{w1}a_{21}^2 + 1.3C_{w3}a_{21} + 2(h_i + 4h) \]  

(27)

The time of welding of the two edge radial plates to the base deck plate is

\[ T_i = 2x9835 \sqrt{1 + 0.25f_i^2} \]  

(28)

Material cost of a complete sectorial element is

\[ K_{s1} = k_wz_v, k_m = 1.0 \text{ $/kg.} \]  

(29)

where

\[ z_v = \frac{h_i + t - t_{w1}}{2} - z_{li} \]  

(17)

and

\[ z_{li} = \frac{h_i}{2} - z_{li} \]  

The moments of inertia

\[ I_w = s_{aw} + \frac{h_i + t - t_{w1}}{2} + \frac{h_{w1}}{4} \left( \frac{h_i + t}{2} - z_{li} \right)^2 + \frac{h_{w1}}{4} \left( \frac{h_i + t}{2} - z_{li} \right)^2 \]  

(20)

Kux = kux \left( \frac{\Theta_1}{\sqrt{\varphi V_1}} + 1.3C_{ux}L_{ux} \right) \]  

(21)

The painting cost of a complete sectorial element is

\[ K_m = k_pS_s, k_p = 28.8x10^{-6} \text{ $/mm}^2, \]  

(30)

\[ S_s = S + \sum_i S_{sw} + 2x53.4581x10^6 f_i \]  

(31)

The total cost of a sectorial element is

\[ K_{ux} = K_{ux} + K_{ux} + K_{ux} + K_{ux} \]  

(34)

\[ \varphi = \frac{\Delta V}{A_i \Delta h} \]  

(23)

The compression force is

\[ N_{ii} = F_m \cos 15^0 + F_y \sin 15^0 \]  

(37)

where

\[ F_y = P_{m} + \frac{F_m}{2}, L = 20000 \text{ mm,} \]  

(36)
It should be noted that the load acting on the half tank side only causes smaller forces acting on radial beams.

Stress constraint for bending and compression according to Eurocode 3 [7]

\[
\frac{N_{th}}{\chi A_{J_{y}}} + k_{yy} \frac{M_{max}}{W_{y} J_{y}} \leq 1
\]

where

\[
\chi = \frac{1}{\phi + \sqrt{\phi^2 - \chi^2}}, \phi = 0.5 [1 + 0.2 (\chi - 0.2) + \chi^2]
\]

\[
\chi = \frac{10353}{r \lambda_k}, \lambda_k = \frac{E}{f_y} = 93.9
\]

\(r\) is the radius of gyration, \(A\) is the cross-sectional area,

\[k_{yy} = 0.95 \left(1 + 0.6 \chi \frac{N_{th}}{\chi A_{J_{y}}} \right)
\]

The suitable rolled I-profile is selected from an Arcelor product catalogue using the British UB profiles.

### COST OF A RADIAL BEAM

#### Material cost

\[K_M = k_w \rho V_{R} + V_{A} A_{L_R} L_R = 9825 \text{ mm}
\]

cost of welding to the inner ringbeam and to the tank shell

\[K_W = k_w \left[ \phi_2 \frac{\rho V_{R}}{2} + 1.3 C_{w} a^2 \rho V_{A} (2h_i + 4b) \right]
\]

the factor of 2 is used since the welding is mainly vertical.

#### Cost of painting

\[K_p = k_p (2h_i + 4b) L_R
\]

Total cost of a radial beam

\[K_R = K_M + K_W + K_p
\]

### ADDITIONAL COST

Material, welding and painting of a deck plate of size 200x6x9825 connecting the sectorial elements as well as welding of the sectorial elements to the radial beam

\[K_a = k_w \rho V_{A} + 1.3 C_{w} a^2 \rho V_{A} (2h_i + k_p 200 L_R
\]

Total cost of the whole roof structure

\[K = K_a + K_M + K_W + K_A
\]

### OPTIMIZATION RESULTS

Table 4 and 5 summarize the results (masses and costs) for different values of \(\omega\) for a sector and for the whole roof

#### Table 4: Masses in kg and costs for a sector containing a sectorial element and a radial beam

<table>
<thead>
<tr>
<th>(\omega)</th>
<th>(k_{th})</th>
<th>(k_{th}^S)</th>
<th>(\rho V_{A})</th>
<th>(K_{th})</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1600</td>
<td>5046</td>
<td>806</td>
<td>1352</td>
</tr>
<tr>
<td>12</td>
<td>1259</td>
<td>4112</td>
<td>729</td>
<td>1248</td>
</tr>
<tr>
<td>14</td>
<td>1072</td>
<td>3556</td>
<td>388</td>
<td>1078</td>
</tr>
<tr>
<td>16</td>
<td>927</td>
<td>3081</td>
<td>388</td>
<td>1078</td>
</tr>
</tbody>
</table>

It can be seen that \(\omega = 14\) and \(\omega = 10\) gives the minimum mass and minimum cost for the whole roof, respectively. It should be noted that the case of \(\omega = 8\) is unrealistic, since in that case the sectorial element has not a trapezoidal but a circular sector form, which needs also partial radial stiffeners beside of the circumferential ones and the cost increases.

#### Table 5: Masses in kg and costs for the whole roof

<table>
<thead>
<tr>
<th>(\omega)</th>
<th>(\rho V_{th})</th>
<th>(K_{th}^S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>24060</td>
<td>66550</td>
</tr>
<tr>
<td>12</td>
<td>23856</td>
<td>67400</td>
</tr>
<tr>
<td>14</td>
<td>23240</td>
<td>68470</td>
</tr>
<tr>
<td>16</td>
<td>24240</td>
<td>70650</td>
</tr>
</tbody>
</table>

### CONCLUSIONS

Minimum cost design of a fixed roof of a vertical steel storage tank is worked out for a numerical model structure. Load of snow and a soil layer is considered. The roof is constructed from sectorial stiffened plate elements and radial beams. The number of sectors is varied between 10 and 16. The sectorial elements are circumferential stiffened with halved rolled I-section stiffeners welded to the base plate. The non-equidistant distances of stiffeners are calculated so that the plate parts are equally stressed. The radial beams are constructed from rolled I-sections. The cost function contents the cost of material, welding and painting. The cost calculation shows that the minimum roof mass and cost corresponds to the number of sections of 14 and 10 respectively.

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#### References


