EXPERIMENTAL VERIFICATION OF OPTIMIZED ANALYTICAL CALCULATION OF HEAT TRANSFER IN FIN PIPE HEAT EXCHANGERS

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Abstract: Recently, modern CFD methods based mostly on finite elements or finite volumes are widely used for calculations of heat transfer problems. However, these numerical methods are in general very depended on a correct set-up of boundary conditions and on other parameters as well and therefore even this tool can easily give incorrect results. Therefore an appropriate and comfort verification of numerical CFD calculation has always high importance. The article deals with improved analytical calculation of heat transfer in heat exchangers equipped by fins on the air side (fluids: water-air). The validity and accuracy of used equations and relationship was experimentally verified on a car engine cooler (i.e. heat exchanger with fins – water – air). The calculation method and its correlation with experimental results are presented in this paper. The test was carried out in laboratories of Institute of Thermal Power Engineering at STU Bratislava.

Keywords: car cooler, fin heat exchanger

INTRODUCTION

Recently, modern CFD methods based mostly on finite elements or finite volumes are widely used for calculations of heat transfer problems. However, these numerical methods are in general very depended on a correct set-up of boundary conditions and on other parameters as well and therefore even this tool can easily give incorrect results. Therefore an appropriate and comfort verification of numerical CFD calculation has always high importance. One of such verification methods is to use classical analytical heat transfer methods based on thermal equations. An improved method for calculation of heat flux in finned pipes (in fact tube heat exchangers with outside fins) by classical analytical method is presented bellow. An experimental verification of this method by measuring of a car heat exchanger (engine cooler) is presented as well. The fluids water on inside of heat exchanger pipe and air on the outside fined side of the heat exchanger were considered in this verification.

A role of fins on the air side of water-air heat exchangers is generally known – due to lower convection heat transfer coefficient on the air side is needed an improvement of heat flux on this side of heat exchanger by enlarging surface. A typical representative of this type of heat exchangers is a car cooler for engines where the engine coolants are cooled by surrounding air. Analytical calculations of this type of heat exchangers by criterial equations are recently used only occasionally. And because such a “classical” analytical calculation method can be used as a useful verification method for widely used CFD calculations, we verified the accuracy of these analytical equations by experimental measurements on real car coolers. The test was carried out in laboratories of Institute of Thermal Power Engineering at STU Bratislava.

The second reason for publishing this analytical calculation method is to get it known for younger generation of technical public, because these relationships are available above all in older literatures published a few decades ago.

Figure 1: Base dimensions of heat exchanger
The basic equation of heat density in heat exchanger with two fluid flows is the well known
\[ \frac{dQ}{ds} = k (t_{liq} - t_{gas}) \] (1)

Where \( k \) is total heat transfer coefficient and \( (t_{liq} - t_{gas}) \) is a temperature difference between liquid and gaseous fluids.

Because thin-walled pipes are used in these types of heat exchangers, the total heat transfer coefficient can be expressed as
\[ k = \frac{1}{\alpha_v} + \frac{1}{(S_k / S_1) \alpha_k} \] (2)

where \( \alpha_v \) is virtual convection heat transfer coefficient applied on the whole surface on the air side of heat exchanger including fins, \( \alpha_k \) is convection heat transfer coefficient on the water side, \( S_c \) is total outside surface and \( S_k \) inside pipe surface (water side of the heat exchanger) and - of course heat flux on outside of the exchanger then is:
\[ Q = \alpha_v S_c (t_t - t_p) \] (3)

The total heat flux on the air side of heat exchanger is given by addition of heats transferred both by fins and by pipes surface, thus
\[ Q = \alpha_v S_c (t_t - t_p) + \alpha_k S_k (t_t - t_p) \] (4)

Convection heat transfer coefficients \( \alpha_r \) and \( \alpha_t \) are different in appr. 20% but due to \( S_r > S_t \) we can consider \( \alpha_r = \alpha_t \). The variables and values with sub-index “r” are connected with fin.

Supposing that \( \alpha_r \) not substantially changed with fin temperature, we can define the fin efficiency as
\[ \eta_r = \frac{\alpha_r S_r (t_t - t_p)}{\alpha_r S_r (t_t - t_p) + \alpha_k S_k (t_t - t_p)} \] (5)

Where \( t_p \) is average fin temperature and \( t_t \) is the temperature on the root of the fin. By this fin efficiency we rewrite the equation (4) as
\[ Q = \alpha_v S_c \eta_r + \frac{\alpha_k S_k}{S_c} (t_t - t_p) \] (6)

By applying (3) is possible to express virtual heat transfer coefficient \( \alpha_v \) by the convection heat transfer coefficient of real fin
\[ \alpha_v = \alpha_r \left[ 1 - \frac{S_c}{S_r} (1 - \eta_r) \right] \] (7)

The expression in brackets is efficiency of enlarged heat exchanger surface at the air side
\[ \alpha_v = \alpha_r \left[ 1 - \frac{S_c}{S_r} (1 - \eta_r) \right] \] (8)

By using equation (7) and (8) there is possible to rewrite (2) in following form
\[ k_v = \frac{1}{\alpha_v} + \frac{1}{(S_k / S_1) \alpha_k} \] (9)

Thus, the fin efficiency at boundary condition for thin flat fins with the constant cross-section can be expressed as
\[ \eta_r = \text{tgh}(m l / (m l)) \] (10)

where
\[ m = 2 \sqrt{\alpha_r / (\lambda, \delta_v)} \] (11)

\( l \) is effective fin length given in our case by half of cross distance between pipes of heat exchanger, \( \delta_v \) thickness of the fin. The expression (10) is shown on the graph bellow.

To determine the heat transfer rate (W) of this exchanger, a set of another dimensionless parameters is used, particularly efficiency of heat exchanger
\[ \epsilon = \frac{Q}{Q_{max}} = \frac{W_r (t_{1,1} - t_{1,2})}{W_{max}(t_{1,1} - t_{1,2})} = \frac{W_p (t_{2,1} - t_{2,2})}{W_{max}(t_{2,1} - t_{2,2})} \] (12)

where \( W_{max} \) is smaller heat capacity from the used fluids and \( W_{max} \) the bigger ones.

Then we need also
\[ NTU = \frac{k_v S_c}{W_{min}} \] (13)

and ratio
\[ \frac{W_{max}}{W_{min}} \] (14)

The general relationship between them is
\[ \epsilon = f (NTU, \frac{W_{max}}{W_{min}}) \] (15)

which can be expressed even graphically.

From the heat exchanger efficiency given by the graph bellow is possible to determine the output temperatures of both fluids.

Figure 2: Efficiency of straight fine

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Figure 3: Efficiency of cross flow heat exchanger (unmixed flows)

For determination of NTU (number of transfer units) the calculation of total heat transfer coefficient \( k_v \) from (9) is needed. \( \alpha_k \) in (9) is obtained from criterial equations for convection flows in pipes. Nusselt number which is needed for determination of \( \alpha_k \) was obtained from the well known equation in [1]
The relationship between the constant $K$ and Prandtl number $Pr$ is in the table below.

**Table 1. The constant $K$ and Prandtl number $Pr$**

<table>
<thead>
<tr>
<th>$Re \cdot 10^3$</th>
<th>2.2</th>
<th>2.3</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
<th>7.0</th>
<th>8.0</th>
<th>9.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>2.2</td>
<td>3.6</td>
<td>4.9</td>
<td>7.5</td>
<td>10.0</td>
<td>12.0</td>
<td>16.5</td>
<td>20.0</td>
<td>24.0</td>
<td>27.0</td>
<td>30.0</td>
<td>32.0</td>
</tr>
</tbody>
</table>

The determining temperature was the mean temperature of water in exchanger and the determining dimension was the hydraulic diameter of the pipe.

Determination of convection heat transfer coefficient $\alpha$: There were used experimental results measured at a heat exchanger with similar geometry [2], where the determining dimension is the hydraulic diameter of the air channel.

From the graph ($St \cdot Pr^{2/3}$) (Stanton, Prandtl) is then obtained the convection heat transfer coefficient $\alpha$ and at the end also the total heat transfer coefficient $k$. Then we can finally finalize the calculation of cooling output of the heat exchanger.

**CALCULATION METHOD IN STEPS — SUMMARY**

- a) Obtaining geometry data of heat exchanger (Lengths, surfaces, volumes, diameters ...)
- b) Mean water velocity in pipes at 353 Kelvin. The air velocities are given: in front of exchanger as 7 m/s and 11 m/s at 303 Kelvin
- c) Calculation of Reynolds numbers $Re$ on water side and air side — using $\alpha_r$ (picture 4) and $\alpha_k$ (eq. 4)
- d) From (10) and (11) is calculated the fin efficiency $\eta_f$ (pic. 2) and from (8) efficiency $\eta_v$
- e) From (9) is calculated the total heat transfer coefficient $k$.
- f) From (13) and (14) and from flows are calculated parameters NTU and $W_{min}/W_{max}$
- g) From the graph (pic. 3) is determined the heat exchanger efficiency $\epsilon$
- h) From (12) are (by estimate) determined the unknown temperatures of air and water so that their mean values were in coincidence with given values
- i) At the end, from the equation $Q = W(t_1 - t_2) = W_p(t_{p2} - t_{p1})$ can determined the cooling output (capacity) of the heat exchanger.

**EXPERIMENTAL VERIFICATION AND CONCLUSION**

Theoretical calculation model was verified by heat exchanger applied as a car cooler for petrol engines with volume 1300 - 1500 cm³. The front side of the heat exchanger was 0.138 m², water flow 1.6 \cdot 10^3 m³/s. The fins were calculated as flat ones. The material was steel, heat conductivity \( \Xi = 50 \text{ W/(m K)} \). Arranging of experimental workplace with measurements on heat exchanger is visible on Picture 5. Hot water was obtained and stored in 650 litres water tank with electrical heating unit. Hot water temperature was set on 80°C with operating velocities 7 and 11 m/s regulated by pump (P1).

**Figure 6: Placement of temperature sensors (left - water side, right - air side of heat exchanger)**

The accuracy — or coincidence between the presented calculation method and experimental results varied between — 9.28% and +16.3%.

This accuracy is very acceptable for purposes like approximate verification of CFD calculations of heat exchangers or even for frame non-computer calculations of these types of heat exchangers. When higher calculation accuracy is needed then a profound experimental verification comes in place — or even a well validated and verified CFD calculation.
REFERENCES


