Abstract: Unmanned autonomous aerial vehicles have become a real center of interest. In the last few years, their utilization has significantly increased. During the last decade many research papers have been published on the topic of modeling and control strategies of autonomous multirotors. Today, they are used for multiple tasks such as navigation and transportation. This paper presents the development of a dynamic modeling and control algorithm - backstepping controller of an autonomous hexa-rotor microcopter. The autonomous hexa-rotor microcopter is an under-actuated and dynamically unstable nonlinear system. The model that represents the dynamic behavior of the hexa-rotor microcopter is complex. Unmanned autonomous aerial vehicles applications are commonly associated with exploration, inspection or surveillance tasks.

Keywords: dynamic model, dynamic behavior, unmanned autonomous aerial vehicles, autonomous hexa-rotor microcopter, under – actuated, dynamically unstable nonlinear system, complex, control strategies, backstepping controller

INTRODUCTION
Unmanned autonomous aerial vehicles have become a real center of interest [1-14]. In the last few years, their utilization has significantly increased. Today, they are used for civil and military applications, for multiple tasks such as navigation and transportation. Unmanned autonomous aerial vehicles applications are commonly associated with exploration, inspection or surveillance tasks.

During the last decade many research papers have been published on the topic of modeling and control strategies of autonomous multirotors [15-30]. One of the unmanned autonomous aerial vehicles with a strong potential is the hexa-rotor microcopter.

The autonomous hexa-rotor microcopter have numerous advantages over quadrotors, since they can offer more:
- more payload,
- the longest flight time and
- high maneuverability

compared to quadrotor. The autonomous hexa-rotor microcopters have additional redundancy over autonomous quad-rotor microcopters.

The control design carried out for an autonomous quad-rotor microcopter can be applied to the autonomous hexa-rotor microcopter since they are modeled as a rotating rigid body dynamic system with six degrees of freedom (6 DOF), Figure 2.

The autonomous hexa-rotor microcopter is an:
- under-actuated and
- dynamically unstable nonlinear system.

The model that represents the dynamic behavior of the hexa-rotor microcopter is nonlinear and complex. This paper presents the development of a dynamic modeling and control algorithm of an autonomous hexa-rotor microcopter.

The paper is organized as follows: Section 1: Introduction. In Section 2, the dynamic modeling of a hexa-rotor microcopter is presented. In
Section 3 backstepping controller for hexa-rotor microcopter is presented. Conclusions are given in Section 4.

**DYNAMIC MODELING OF HEXA-ROTOR MICROCOPTER**

The model [31-46] of the hexa-rotor helicopter and the rotational directions of the propellers are presented in Figure 3. This cross structure is quite thin and light, however it shows robustness by linking mechanically the motors. Hexa-rotor microcopter body is rigid. The six rotors are symmetrically distributed around the center. All the propellers axes of rotation are fixed and parallel. Propellers are rigid. These considerations point out that the structure is quite rigid and the only things that can vary are the propeller speeds.

The hexa-rotor microcopter configuration has six rotors which generate the propeller forces \( F_i \) \( (i = 1,2,3,4,5,6) \) as it is shown in Figure 3. Control of quadrotor is achieved [2] by commanding different speeds to different propellers, which in turn produces differential aerodynamic forces and moments. In order to increase the altitude of the aircraft it is necessary to increase the rotor speeds altogether with the same quantity.

**Figure 3. Hexa-rotor microcopter - a non-linear dynamic system**

Each rotor consists of:
- brushless DC motor and a
- fixed-pitch propeller.
This rotorcraft is constituted by:
- three rotors which rotate clockwise \( (1,3,5) \), and
- three rotating counterclockwise \( (2,4,6) \).
Forward motion is accomplished by increasing the speed of the rotors \( (3, 4, 5) \) while simultaneously reducing the same value for forward rotors \( (1, 2, 6) \). For leftward motion the speed of rotors \( (5, 6) \) is increased while the speed of rotors \( (2, 3) \) is reduced. Backward and rightward motion can be accomplished similarly. Finally, yaw motion can be performed by speeding up or slowing down the clockwise rotors depending on the desired angle direction.

To describe the motion of a 6 DOF rigid body it is usual to define two reference frames (Figure 2):
- the earth inertial frame (E-frame), and
- the body-fixed frame (B-frame).
The equations of motion are formulated using the Newton-Euler laws with the following reasons:
- the inertia matrix is time-invariant;
- advantage of body symmetry can be taken to simplify the equations;
- measurements taken on-board are easily converted to body-fixed frame;
- control forces are almost always given in body-fixed frame. The E-frame \( (Oxyz) \) is chosen as the linear position (in meters) and the angular position (in radians) of the quadrotor.

The B-frame is attached to the body. The origin of the B-frame is chosen to coincide with the center of the hexa-rotor microcopter cross structure. This reference is right-hand, too. 

The linear position of the helicopter \( (X, Y, Z) \) is determined by the coordinates of the vector between the origin of the B frame and the origin of the E-frame according to equation.

The angular position of the hexa-rotor microcopter \( (\Phi, \theta, \psi) \) is defined by the orientation of the B-frame with respect to the E-frame. This is given by three consecutive rotations about the main axes which take the E-frame into the B-frame. In this paper, the “roll-pitch-yaw” set of Euler angles \( (\Phi, \theta, \psi) \) were used.

The vector that describes quad-rotor position and orientation with respect to the E-frame can be written in the form:
\[
s = [x, y, z, \Phi, \theta, \psi]^T
\]
(1)

The rotation matrix between the E- and B-frames has the following form:
\[
R = \begin{bmatrix}
    c\psi c\theta & c\psi s\theta c\phi - s\psi s\phi & c\psi s\theta s\phi + c\phi s\psi \\
    -s\psi c\theta & s\psi s\theta c\phi + c\psi s\phi & -s\psi s\theta s\phi + c\phi c\psi \\
    -s\theta & c\theta s\phi & c\theta c\phi
\end{bmatrix}
\]
(2)

Now, the model of hexa-rotor dynamics can be described by a system of equations:
\[
\begin{align*}
    \dot{x} &= (\sin\psi \sin\phi + \cos\phi \cos\psi \cos\theta) \frac{U_x}{m} + A_x \\
    \dot{y} &= (-\cos\psi \sin\phi - \sin\phi \cos\psi \cos\theta) \frac{U_x}{m} + A_y \\
    \dot{z} &= -g + \cos\theta \cos\phi \frac{U_x}{m} + A_z \\
    \dot{\phi} &= \frac{I_z - I_x}{I_x} \theta \omega + \frac{U_y}{I_x} \psi + A_\phi \\
    \dot{\theta} &= \frac{I_z - I_x}{I_x} \phi \omega + \frac{U_z}{I_x} \psi + A_\theta \\
    \dot{\psi} &= \frac{I_z - I_x}{I_x} \phi \omega + \frac{U_z}{I_x} \psi + A_\psi
\end{align*}
\]
(3)

**BACKSTEPPING CONTROLLER FOR HEXA-ROTOR MICROCOPTER**

In this paper, controller design for the hexa-rotor microcopter is proposed by using backstepping technique. Backstepping is a recursive design methodology that makes use of Lyapunov stability theory to force the system to follow a desired trajectory. The hexa-rotor microcopter is controlled by angular speeds of six motors. Each motor produces a thrust and a torque, whose combination generates the main trust, the yaw torque, the pitch torque, and he roll torque acting on the hexa-rotor microcopter. First, the dynamical model is rewritten in state-space form:
\[
\dot{X} = f(X, U)
\]
(4)
The position of the hexa-rotor microcopter in the earth reference frame as space vector of the system:

\[
X = \{x_1, x_2, \ldots, x_9\}^T \in \mathbb{R}^9
\]

as space vector of the system:

\[
X_1 = \phi \quad x_1 = x_1 \quad x_9 = Y
\]

\[
X_2 = \dot{x}_1 \quad x_2 = \dot{x}_1 \quad x_{10} = \dot{Y}
\]

\[
X_3 = \theta \quad x_3 = X \quad x_{11} = Z
\]

\[
X_4 = \dot{x}_3 \quad x_4 = \dot{x}_3 \quad x_{12} = \dot{X}
\]

Next, the \( x \) -coordinates are transformed into the new \( z \) – coordinates:

\[
z_1 = x_{1\text{ ref}} - x_1
\]

\[
z_2 = x_2 - \dot{x}_{1\text{ ref}} - \alpha_z z_1
\]

\[
z_3 = x_3 - x_{3\text{ ref}} - \alpha_z z_2
\]

\[
z_4 = x_4 - \dot{x}_{3\text{ ref}} - \alpha_z z_3
\]

\[
z_5 = x_5 - x_{5\text{ ref}} - \alpha_z z_4
\]

\[
z_6 = x_6 - \dot{x}_{5\text{ ref}} - \alpha_z z_5
\]

\[
z_7 = x_7 - x_{7\text{ ref}} - \alpha_z z_6
\]

\[
z_8 = x_8 - \dot{x}_{7\text{ ref}} - \alpha_z z_7
\]

\[
z_9 = x_9 - x_{9\text{ ref}} - \alpha_z z_8
\]

\[
z_{10} = x_{10} - \dot{x}_{9\text{ ref}} - \alpha_z z_9
\]

\[
z_{11} = x_{11} - x_{11\text{ ref}} - \alpha_z z_{10}
\]

\[
z_{12} = x_{12} - \dot{x}_{11\text{ ref}} - \alpha_z z_{11}
\]

By introducing the partial Lyapunov functions [2], to all \( x \) – coordinates results in the following backstepping controller:

\[
U_i = \frac{m}{I_i} (z_i - \alpha_z (z_{i+1} + \alpha_z z_i) - \alpha_{\phi, z_i})
\]

\[
U_i = \frac{m}{I_i} (z_i - \alpha_z (z_{i+1} + \alpha_z z_i) - \alpha_{\psi, z_i})
\]

\[
U_i = \frac{m}{I_i} (z_i - \alpha_z (z_{i+1} + \alpha_z z_i) - \alpha_{\theta, z_i})
\]

\[
U_i = \frac{m}{I_i} (z_i - \alpha_z (z_{i+1} + \alpha_z z_i) - \alpha_{\phi, z_i})
\]

\[
U_i = \frac{m}{I_i} (z_i - \alpha_z (z_{i+1} + \alpha_z z_i) - \alpha_{\psi, z_i})
\]

\[
U_i = \frac{m}{I_i} (z_i - \alpha_z (z_{i+1} + \alpha_z z_i) - \alpha_{\theta, z_i})
\]

The position of the hexa-rotor microcopter in the earth reference frame is illustrated in Figure 4.

Figure 4. Position of the hexa-rotor microcopter in the earth reference frame.

Conclusions

This paper presents the development of a dynamic modeling and control algorithm of an autonomous hexa-rotor microcopter. During the last decade many research papers have been published on the topic of modeling and control strategies of autonomous multirotors.

The autonomous hexa-rotor microcopter is an under-actuated and dynamically unstable nonlinear system. The model that represents the dynamic behavior of the hexa-rotor microcopter is complex. Unmanned autonomous aerial vehicles have become a real center of interest. In the last few years, their utilization has significantly increased. The autonomous hexa-rotor microcopters have additional redundancy over autonomous quad-rotor microcopters.

The control design carried out for an autonomous quad-rotor microcopter - backstepping controller, can be applied to the autonomous hexa-rotor microcopter since they are modeled as a rotating rigid body dynamic system with six degrees of freedom.

References


