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AXIAL VIBRATION OF A ROBOT SURGERY TENTACLE

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Abstract: *Fundamental working element of the most robot and manipulator arms is needle-sized tentacle which moves straight-line. One end of the tentacle is fixed and the other is free, and the corresponding physical model is a clamped-free beam. Operation of the tentacle is in straight-line direction and axial vibration appears. As the material of tentacle is usually with nonlinear properties, the model of the system is also nonlinear. Axial vibration is described with a nonlinear partial differential equation. In this paper an analytical method is developed for solving the equation. It is based on separation of the partial differential equations into two uncoupled strong nonlinear second order differential equations. Using boundary and initial conditions, parameters of vibration are obtained. The procedure suggested in this paper is applied for a beam with cubic nonlinearity. Frequencies of free axial vibration are determined. It is proved that they depend not only on the type of boundary conditions, but also on initial conditions. At the end of the paper the numerical solution of the axial vibration of the clamped-free beam with cubic nonlinearity is calculated.*
Keywords: *Robot surgery tentacle; Axial vibration; Fraction order nonlinearity; Clamped-free beam*

INTRODUCTION

One of the most responsible tasks for the robot arm is in surgery. Usually, the working element of the robot arm is a needle-sized tentacle. This type of tentacle is used in vascular surgery [1], in mandible reconstruction surgery [2], in orthognathic surgery [3],[4], trans nasal [5]. Robot arm may assist in renal and liver surgery [6]. Robot arm is used in laparoscopic surgery, too [7]. Unfortunately, recent investigation report about perioperative complications of robot-assisted laparoscopic surgery which use robot arms [8]. It is stated that the problem is with the accuracy of the mechanical system. To exceed the problem the control of the robot arm was improved. Results of investigation in robot control (see for example, [9]-[12]) are incorporated into the system, but did not give the expected results in motion accuracy. It gives us an idea to analyze the vibration properties of tentacle.

Tentacle is modeled as a beam which has axial vibrations. Very often the tentacle is not made of steel and the stress-strain property of tentacle material is not linear. Influence of nonlinearity on vibration of the system is already known (see for example [13]-[15]) and these results have to be included into consideration. Many papers are published which are dealing with the problem of axial vibrations of the beam with small nonlinearity [16]-[19]. In this paper the strong nonlinearity will be also investigated.

The aim of the paper is to obtain frequencies of free axial vibrations of the tentacle which is modeled as a clamped-free beam made of material with strong nonlinearity. The paper is divided into four sections. After Introduction, the model of the physical and mathematical model of the tentacle is given. In Section 3, a solving procedure is given. Method is based on variable separation which transforms the partial differential equation of vibration into two strong nonlinear second order differential equations. In Section 4, the model with cubic nonlinearity is considered. Numerical calculation is also done. Paper ends with Conclusions

MODEL OF THE SURGERY TENTACLE

In Fig.1, a surgery robot with three separate tentacles is shown. Each of tentacles is fixed at its one end to the robot arm and the other end is free. Motion of tentacles is in a straight line. Tentacle can be modeled as a clamped-free beam as one end is free, while the other is fixed (Fig.2). Cross-section properties of the beam are smaller than its length. During operation axial vibration of the beam appears.

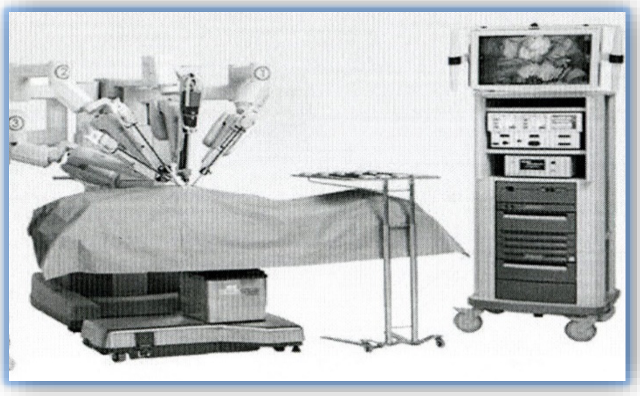

Figure 1. Robot surgery tentacle.

Axial deflection y of the beam depends on time t and position x. Separating an elementary part of the beam, whose length is dx and mass ρAdx, where ρ is density of tentacle material, A is cross-section, the inertial force is product of elementary mass and acceleration: $\rho A dx(\partial^2 y/\partial t^2)$. Usually, material of the tentacle is with strong nonlinear properties as its stress-strain relation is

$$\sigma = E\varepsilon^{\alpha} = E(\partial y/\partial x)^{\alpha}, \qquad (1)$$

where E is the elasticity coefficient, ε is the deformation, and α is the order of nonlinearity obtained experimentally. Coefficient α is a positive real number which need not be whole but of any fractional type. Elastic force is

$$F = \sigma A = EA(\partial y/\partial x)^{\alpha}. \quad (2)$$

Equating the elementary elastic force dF and the elementary inertial force which act on the separated part of the bean (see Fig.2) the equation for longitudinal vibrations are obtained

$$\rho A \frac{\partial^2 y}{\partial t^2} = EA \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)^{\alpha}. \quad (3)$$

The boundary conditions are

$$y(0,t) = 0, \quad F(l,t) = EA\left(\frac{\partial y}{\partial x}\right)^{\alpha}(l,t) = 0, \quad (4)$$

and initial conditions are

$$y(x,0) = Y_0(x), \quad \frac{\partial y(x,0)}{\partial t} = 0, \quad (5)$$

where l is the length of the beam.

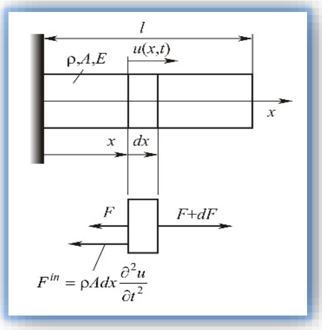

Figure 2. Model of the clamped-free beam.

Mathematical model is a second order partial differential equation with strong nonlinearity. To give a valid analysis of the beam motion, it is necessary to solve equation (3) according to the boundary (4) and initial conditions (5).

SOLVING PROCEDURE

Let us introduce a solution in the form

$$y(x,t) = X(x)T(t), \quad (6)$$

where X(x) is a deflection function and T(t) is a time function. Substituting (6) into (3) - (5), we have

$$\frac{\rho}{E} X\ddot{T} = T^{\alpha}((X')^{\alpha})', \quad (7)$$

and

$$X(0) = 0, \qquad X'(l) = 0, \quad (8)$$

$$X(x)T(0) = Y_0(x), \quad \dot{T}(0) = 0, \quad (9)$$

where $\ddot{T} = d^2T/dt^2$ and $X'=dX/dx$.

It is obvious that we can separate variables in (7) and we obtain

$$\frac{\rho}{E}\frac{\ddot{T}}{T^{\alpha}} = \frac{((X')^{\alpha})'}{X} = -k^2 = const. \quad (10)$$

i.e.,

$$\ddot{T} + c_1^2 T|T|^{\alpha-1} = 0, \alpha(X')^{\alpha-1}X'' + k^2 X = 0, \quad (11)$$

where k^2 is an unknown constant value and $c_1^2=(E/\rho)k^2$. Equations (11) are two ordinary strong nonlinear uncoupled differential equations.

Our major task is to determine frequencies of vibration based on the constants k and c_1.

Solving of the equaiton with displacement function

We rewrite the expression $(11)_2$ as

$$X''(X')^{\alpha-1} = -\frac{k^2}{\alpha}X. \quad (12)$$

For $p(X)=dX/dx=X'$ and $d^2X/dx^2 = p'p = p(dp/dX)$, equation (12) transforms into a first order equation

$$\alpha \frac{dp}{dX} p^{\alpha} = -k^2 X, \quad (13)$$

which solution is

$$X' = \left(\frac{\alpha+1}{\alpha}\right)^{1/(\alpha+1)} \left(K_1 - \frac{k^2}{2}X^2\right)^{1/(\alpha+1)}. \quad (14)$$

Finally,

$$\int \frac{dX}{\left(1-\frac{k^2}{2K_1}X^2\right)^{1/(\alpha+1)}} = \left(K_1 \frac{\alpha+1}{\alpha}\right)^{\frac{1}{\alpha+1}}(K_2 + x), \quad (15)$$

where K_1 and K_2 are arbitrary constants. For $k^2 X^2/2K_1 = z$, we have

$$\int \frac{dz}{\sqrt{z}(1-z)^{1/(\alpha+1)}} = k\sqrt{2}\left(\frac{\alpha+1}{\alpha}\right)^{\frac{1}{\alpha+1}}(K_1)^{\frac{1-\alpha}{2(\alpha+1)}}(K_2 + x). \quad (16)$$

If the integration is in the interval (see [19],[20]), we have

$$\int_0^1 z^{-\frac{1}{2}}(1-z)^{-\frac{1}{\alpha+1}}dz =$$

$$k\sqrt{2}(K_1)^{\frac{1-\alpha}{2(\alpha+1)}}\left(\frac{\alpha+1}{\alpha}\right)^{\frac{1}{\alpha+1}}\int_0^l dx. \quad (17)$$

Introducing into (17) the definition of the complete beta function

$$B(m,n) = \int_0^1 (1-z)^{n-1} z^{m-1} dz, \quad (18)$$

we have

$$B\left(\frac{\alpha}{\alpha+1}, \frac{1}{2}\right) = kl\sqrt{2}(K_1)^{\frac{1-\alpha}{2(\alpha+1)}}\left(\frac{\alpha+1}{\alpha}\right)^{\frac{1}{\alpha+1}}. \quad (19)$$

The relation (19) gives the relation for the constant

$$k = k(K_1, \alpha). \quad (20)$$

It depends on the order of nonlinearity and on the constant K_1,, too. This result is a new one and has to be proved.

For the linear oscillator, when α=1, relation (19) transforms into

$$B\left(\frac{1}{2}, \frac{1}{2}\right) = \pi = 2kl, \quad (21)$$

i.e.,

$$k = \frac{\pi}{2l}. \quad (22)$$

Due to periodicity of the function, it is $k=(2n-1)\pi/2l$. It is already well known solution for the linear oscillator, where the value of the constant k is independent on initial and boundary conditions.

Finally, using the relation (19) and the periodic property of the function X(x), the constant k is

$$k_n = B\left(\frac{\alpha}{\alpha+1}, \frac{1}{2}\right)\frac{(2n-1)}{\sqrt{2}l}(K_1)^{\frac{\alpha-1}{2(\alpha+1)}}\left(\frac{\alpha+1}{\alpha}\right)^{\frac{-1}{\alpha+1}}, \quad (23)$$

where n=1,2,3,…

Solving of the equation with time function

Equation $(11)_1$ has a first integral

$$\frac{\dot{T}^2}{2} + \frac{c_1^2}{\alpha+1}T^{\alpha+1} = K_3 = const., \quad (24)$$

where K_3 is an arbitrary constant. Rewriting (24) we have

$$\dot{T} = \sqrt{2K_3 - \frac{2c_1^2}{\alpha+1}T^{\alpha+1}}, \quad (25)$$

and after integration we have

$$\int \frac{dT}{\sqrt{2K_3 - \frac{2c_1^2}{\alpha+1}T^{\alpha+1}}} = t + K_4, \quad (26)$$

where K_4 is the unknown constant of integration. Unfortunately, in general, we cannot find the closed form solution of (26). It is convenient to assume the approximate solution as a trigonometric function. As (24) corresponds to a conservative oscillatory system it is known that the amplitude and the period of vibration are constant. Using (25) the exact value of the period of vibration of $(11)_1$ can be calculated.

Due to (25), it reads

$$\frac{dt}{dT} = 1/\sqrt{2K_3 - \frac{2c_1^2}{\alpha+1}T^{\alpha+1}}, \tag{27}$$

Using the periodic property of the oscillator and integrating (27), it follows

$$\int_0^P dt = 4\int_0^{K_3} \frac{dT}{\sqrt{2K_3 - \frac{2c_1^2}{\alpha+1}T^{\alpha+1}}}, \tag{28}$$

where P is the period of vibration. Introducing the new variable

$$z = \frac{2c_1^2}{\alpha+1}T^{\alpha+1}, \tag{29}$$

into (28), it is

$$P = \frac{4}{\sqrt{2}}\left(\frac{c_1^{-2}}{(\alpha+1)^{\alpha}}\right)^{1/(\alpha+1)} K_3^{(1-\alpha)/2(1+\alpha)} \int_0^1 z^{-\frac{\alpha}{\alpha+1}}(1-z)^{-\frac{1}{2}}\,dz. \tag{30}$$

Applying the definition (18), period of vibration is obtained

$$P = \frac{4}{\sqrt{2}}\left(\frac{\rho}{Ek^2(\alpha+1)^{\alpha}}\right)^{1/(\alpha+1)} K_3^{(1-\alpha)/2(1+\alpha)} B\left(\frac{1}{\alpha+1},\frac{1}{2}\right). \tag{31}$$

Based on (31) the frequency of vibration in axial direction is

$$\omega = \frac{2\pi}{P} = \frac{2\pi\sqrt{2}}{B\left(\frac{1}{\alpha+1},\frac{1}{2}\right)}\left(\frac{E}{\rho}\right)^{\frac{1}{\alpha+1}} k^{\frac{2}{\alpha+1}}(\alpha+1)^{\frac{\alpha}{\alpha+1}}\; K_3^{(\alpha-1)/2(1+\alpha)} \tag{32}$$

Substituting the constant (23) into (32) it is

$$\omega_n = \frac{2\pi}{P} = \frac{\pi\sqrt{2}}{2B\left(\frac{1}{\alpha+1},\frac{1}{2}\right)}\left(\frac{E}{\rho}\right)^{\frac{1}{\alpha+1}}(\alpha+1)^{\frac{\alpha}{\alpha+1}}\; K_3^{(\alpha-1)/2(1+\alpha)}$$

$$\left(B\left(\frac{\alpha}{\alpha+1},\frac{1}{2}\right)(K_1)^{\frac{\alpha-1}{2(\alpha+1)}}\frac{(2n-1)}{\sqrt{2}l}\left(\frac{\alpha}{\alpha+1}\right)^{\frac{1}{\alpha+1}}\right)^{\frac{2}{\alpha+1}}, \tag{33}$$

For $B(m,n) = \Gamma(m)\Gamma(n)/\Gamma(m+n)$, where Γ is the gamma function, and $\Gamma(1/2)=\sqrt{\pi}$, we have the frequencies of the free axial vibration of the beam

$$\omega_n = \frac{\pi\sqrt{2}\Gamma((3+\alpha)/2(\alpha+1))}{2\Gamma(1/(\alpha+1))\sqrt{\pi}}\left(\frac{E}{\rho}\right)^{\frac{1}{\alpha+1}}(\alpha+1)^{\frac{\alpha}{\alpha+1}}\; K_3^{(\alpha-1)/2(1+\alpha)}$$

$$\left(\frac{\Gamma(\alpha/(\alpha+1))}{\Gamma((3\alpha+1)/2(\alpha+1))}\sqrt{\pi}(K_1)^{\frac{\alpha-1}{2(\alpha+1)}}\frac{(2n-1)}{\sqrt{2}l}\left(\frac{\alpha}{\alpha+1}\right)^{\frac{1}{\alpha+1}}\right)^{\frac{2}{\alpha+1}}. \tag{34}$$

It is obvious that the frequency of vibration depends on the order of nonlinearity, but also on the constants which have to satisfy initial and boundary conditions.

For the linear oscillator, when $\alpha=1$, the frequency relation (34) is

$$\omega_n = \frac{(2n-1)\pi}{2l}\sqrt{\frac{E}{\rho}} \equiv k_n\sqrt{\frac{E}{\rho}}. \tag{35}$$

This result for the linear oscillator is already known. For the linear oscillator the frequency does not depend on the initial conditions.

Eq. $(11)_1$ has an exact solution in the form of the Ateb function [20]. Nevertheless, in this paper the approximate solution in the form of a harmonic function is assumed as

$$T_n = K_3 cos(\omega_n t + K_4), \tag{36}$$

with amplitude K_3, phase angle K_4 and frequency ω_n.

Approximate solution

Using the solutions (15) and (36) with (23) and (35), we obtain the approximate solution of (1) as a sum

$$y(x,t) = \sum_{n=1}^{\infty}\frac{\sqrt{2T_0^2K_{1n}}}{k_n(K_{1n})}sin(k_n(K_{1n})x)\,cos(\omega_n((K_{1n},K_{3n})t) \tag{37}$$

and its first time derivative is

$$\frac{\partial y(x,t)}{\partial t} = -\sum_{n=1}^{\infty}\omega_n(K_{1n},K_{3n})\frac{\sqrt{2(K_{3n})^2K_{1n}}}{k_n(K_{1n})}sin(k_n(K_{1n})x)\; sin(\omega_n(K_{1n},K_{3n}))t). \tag{38}$$

Using the condition of orthogonallity, constants K_{1n} and K_{3n} are obtained.

BEAM WITH PURE QUADRATIC NONLINEARITY

Let us assume that the nonlinearity is pure quadratic and the mathematical model of the axial vibration of the beam is

$$\rho A\frac{\partial^2 y}{\partial t^2} = EA\frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)^2. \tag{39}$$

According to the suggested procedure (39) is rewritten into two ordinary differential equations with separated variables. Specifying the nonlinearity, the equation (14) is

$$X' = 1.1447[(K_1 - \frac{k^2}{2}X^2)]^{\frac{1}{3}}. \tag{40}$$

Modifying (40) into the form

$$\int\frac{dX}{K_1^{1/3}\sqrt{1-\frac{k^2}{3K_1}X^2}} = 1.1447(K_2+x), \tag{41}$$

we have the approximate solution

$$X = \frac{\sqrt{3K_1}}{k}sin[\sqrt{k}\left(1.1447K_1^{-\frac{1}{3}}(K_2+x)\right)], \tag{42}$$

and its derivative

$$X' = \sqrt{\frac{3K_1}{k}}1.1447K_1^{-\frac{1}{3}}cos[\sqrt{k}\left((1.1447K_1^{-\frac{1}{3}}(K_2+x)\right)]. \tag{43}$$

Introducing the boundary conditions into (42) and (43), we obtain

$$K_2 = 0, cos(1.1447K_1^{-\frac{1}{3}}l\sqrt{k}) = 0, \tag{44}$$

i.e., n solutions for k are obtained

$$k_n = 0.76316(\frac{(2n-1)\pi}{l})^2K_1^{\frac{2}{3}}. \tag{45}$$

where n=1,2,… According to (45), n frequencies of vibrations follow

$$\omega_n = 1.2247(\frac{(2n-1)\pi}{2l})^{2/3}K_1^{1/9}K_3^{1/6}\left(\frac{E}{\rho}\right)^{1/3} \tag{46}$$

Frequencies depend on constants K_1 and K_3, which can be calculated according to the initial and boundary conditions.

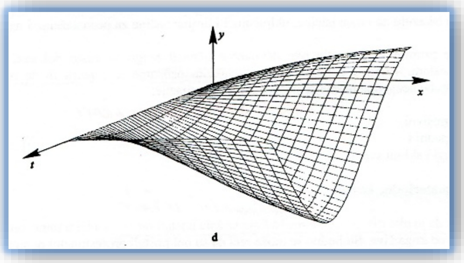

Figure 3*. Axial vibration in y-x-t space.*

In Figure 3, axial vibration of the beam in y-x-t space is plotted. The surface represents the amplitude-position-time diagram obtained analytically. As the model is assumed to be conservative, only one period of vibration is plotted. Vibrations repeat in time, and have the same form.

CONCLUSIONS

Axial vibration of the tentacle settled on the robot arm which moves straight-line is considered. System is assumed to be nonlinear. Mathematical model of the motion is given with a strong nonlinear partial differential equation. In spite of the nonlinearity the solution of the equation is a product of two functions which depend on two independent variables: a displacement and a time function. Due to nonlinear properties of the system the constant of separation depends on the order of nonlinearity and on boundary conditions. Besides, the frequency of vibration is also dependent on the order of nonlinearity and coefficient of nonlinearity but also on the initial and boundary conditions. It is a quite new result in solving the nonlinear partial differential equation. The obtained result is proved numerically.

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