PLASTIC DEFORMATION AND YIELD CRITERIA IN FORMING—AN OVERVIEW

Abstract: Forming is an important component in modern industry. As plastic deformities are important to processes, the present paper presents various issues of plastic deformities. Under forming, classification of processes, properties of the materials used in the process and the criteria of yield are described. For convenience, the forming processes are classified based on other parameters of the processes, namely, stress, raw material, temperature and induction of force. In addition, yield theories namely, von Mises-Henckyor Distortion energy criterion and maximum shear stress or Tressa criterion are delineated.

Keywords: Forming, Plastic deformation, Yield criterion, Metallurgy

INTRODUCTION
Forming plays a very important role in modern manufacturing industry, and itself is a major industry. It is generally a huge producer of semi-finished and finished goods and this is the reason that it is feasible to undertake large scale research and developmental projects [3].

In forming process, the product shapes are produced by plastic deformation, so it is important to know the plastic flow properties of the material for optimization of the output [2]. These depend upon the intensity and the conditions of plastic deformation during forming process. Many forming processes produce raw materials for other process which in turn produce semi-finished /finished products. Considering billets produced by steel plants are re-rolled in mills to give products like angles, channels, bars, etc. Bars may be used for manufacturing forging, wires and machined products. Similarly, wires are used further for the manufacturing of rivets, bolts and screws by the manufacturers and process them further. Plastic deformation of a material under forming process must specify the yield criteria for the analysis of stress and strain of a material loaded beyond the elastic limits and the specify the working loads up to which the material processed can withstand [4]. To describe the forming of material, it is important to know the structure of the material which may be modified by alloying elements, by heat treatment or plastic deformation.

FORMING
Forming processes are manufacturing processes which uses stresses to cause plastic deformation of material to produce required shapes without deterioration of their properties. In order to attain plastic deformation in a metal, a force must exceed the yield strength of a material [5,7]. During forming no material is removed, i.e., they are deformed and displaced. Some of the examples are sheet metal forming, extrusion, forging, rolling, thread rolling, explosive forming, electromagnetic forming, etc. It includes many manufacturing processes. Forming process can classified as followed.

Classification of forming processes
Classification of forming process is mainly based on:
(i) state of stress, 
(ii) type of raw material, 
(iii) forming temperature and (iv) methods of induction of forces into the work piece. 
According to state of stress, it is subdivided into:
(i) Direct compression type processes, here in compression force is applied in the normal direction of the metal flow. Some of the examples are forging, rolling, etc.
(ii) Tension type processes, here applied force is tensile in nature, and deformation takes place along the axis. Example stretching, bulge forming, expanding etc.
(iii) Indirect compression type processes, the metal flow is under the combined stress state. The primary force is tensile, with indirect compression force developed due to the reaction of work piece. Examples are extrusion, wire drawing, tube drawing, etc.
(iv) Shearing processes, the applied force involves shearing forces along the surface of the metal with sufficient magnitude to rupture the metal in the plane shear i.e., shear displacement, twisting etc.
(v) Bending processes, here bending moments are induced in the sheet due to the applied forces. Examples are bending with linear/rotary tool motion. These processes, which are classified based on the state of stress, are illustrated in Figure 1.

According to the type of raw material, it is divided into two types. They are:
(i) Sheet forming, the parts produced have a 3D form with approximately constant wall thickness, and
(ii) Bulk forming, the parts produced are 3D form but has very different wall thickness or cross-section [5].
Forging
Direct Compression type processes

Rolling

Stretch
Tension type processes

Shearing
Shearing processes

Wire drawing
Indirect compression type processes

Extrusion

Deep drawing
Indirect compression type processes

Bending
Bending processes

According to the forming temperature, it is subcategorized as:

(i) Hot forming, occurs at temperatures of 60% or above of the melting temperature of the metal on an absolute scale. At high temperatures, the metal decreases in its strength, hence small forces needed for deformation. Recrystallization occurs readily, causing new grains to form continuously during deformation. This continuous formation of new grains causes the ductility of the metal to remain high, allowing large amounts of deformation to be imparted without fracture. Control of final dimensions is more difficult in a hot-worked metal.

(ii) Cold forming occurs at temperatures about 30% or less of its melting temperature on an absolute temperature scale. During cold work, the metal experiences an increased number of dislocations and entanglement of these dislocations, causing strain hardening. With strain hardening, the strength of the metal increases with deformation. To recrystallize the metal, a thermal treatment, called anneal, is needed. During annealing, the strength of the metal can be reduced with a significant increase in ductility. The increase in ductility allows further deformation to occur before fracture. The final surface finish and dimensional tolerances can be well controlled in this process.

(iii) Warm forming, occurs between hot working and cold working. It occurs in the approximate temperature range of 30-60% of the melting temperature of the metal on an absolute scale. The forces required to deform metal in the warm working regime are higher than during hot working. The final finish and dimensional tolerances are better than hot working but are not nearly as good as a cold working process. Although warm work seems to have drawbacks, the primary driver for warm working is economic. If the working temperature is lowered, there can be major cost savings in the process.

According to induction of forces into the work piece, the process is categorized as:

(i) Indirect force application, the force applied to metal is indirect. Some medium is acting between the force and the material. i.e., deep drawing, and

(ii) Direct force application, force is directly applied to the metal as in the case of upsetting (Figure 2).

Figure 1: Different processes of forming based on the state of stress

Properties of the material during forming processes

During forming processes properties of the materials are considered for optimal working of the product to the required standards. In deep drawing of sheet metal, one of the requirements is that the material should be ductile. But other properties should also be considered. For example, lead is quite ductile at room temperature but lead sheet is not that suitable for drawing into cups in the way steel cups are drawn.

When a force is subjected on a material, it may result in deformation or translation/rotation of that material. The ability of material to withstand the subjected force without having any deformation is expressed in two ways, strength and hardness. We can say permanent deformation which is also known as plastic deformation, occurs where deformation cannot be recovered even if applied forces are removed. In temporary deformation also known as elastic deformation, where deformation recovers after applied forces are removed. When force is applied on the
material it undergoes elastic deformation and then followed by plastic deformation. Degree of elastic and plastic deformations will mainly depend on the temperatures surrounded, load application, kind of material, and many other conditions. Changes occur from elastic deformation state to plastic deformation state is given by the yield strength ($\sigma_0$) of the material.

Deformation can be defined as percentage change in length by unit length in three different directions. Engineering stress ($s$) which is defined as force per area on where force is subjected on the material. Engineering strain ($e$) is defined as change in length by original length. An average change in length at a particular direction is defined by engineering strain. According to definitions, $s'$ and $e'$ are as follows:

Engineering stress ($s$) = $P / A_0$. Engineering strain ($e$) = $L - L_0 / L_0$, where $P$ is the load applied over area $A_0$, $L$, the final length, $L_0$ original length. Because material dimensions change continuously under application of the load, these values are may not be the true indication of material deformation characteristics but specifies approximate values. Thus, Ludwik first proposed the concept of, true strain or natural strain ($\varepsilon$) is given by sum of all incremental strains as follows: $\varepsilon = \Sigma \varepsilon_i$, where $\varepsilon_i$ is the incremental strain. Assume material volume to be constant that is $A_0 L_0 = AL$, the expression for true strain may also be written as, $\ln \left( \frac{L}{L_0} \right) = \ln \left( 1 + \frac{\Delta L}{L_0} \right) = \ln (1 + e)$, where $e$ = engineering strain [6].

True stress ($\sigma$) is defined as load per cross-sectional area on which force is acting at a particular time.

$$\sigma = \frac{P}{A} = \frac{P}{A_0} \frac{A_0}{A} = s (e + 1)$$

Up to the elastic limit of the material, engineering stress/strain is equal to true stress/strain of the material and two values differ from each other after the beginning of plastic deformation. The engineering and true stress-strain equations above are valid only up to the uniform deformation limit, at which necking starts under tension test because the relationships are generated by assuming constant volume and homogeneous distribution of strain of the specimen along the length which is in applied tension direction. Deformation can be determined by stress-strain relationships and the relations usually obtained from different experimental tests. The tests include tension test, compression test, torsion test, plain strain compression test, etc.

The tension test is commonly used, in which specimen is subjected to a continuously increasing uni-axial tensile load and measures the stress and strain simultaneously. A typical plot of stress-strain curve is shown as Figure 3.

As shown above, initial stages stress is proportional to strain up to elastic limit, after that it increases to maximum uniform deformation, and then decreases by necking i.e., non-uniform plastic deformation until fracture of the specimen happens. In the curve along the line segment ab, as defined by Hooke’s law engineering stress is directly proportional to engineering strain, so point b is defined as proportionality limit. Young modulus of elasticity of the material is given by the slope of the line ab. Further increment in stress to point c, material can still be in elastic nature, so it is known as elastic limit. We can’t define a point exactly where material plastically deformation starts. Otherwise point c may also call as yield point. Therefore yield strength ($\sigma_0$) can be assumed to be the stress value at 0.2% offset of strain, at point-e. Thereafter from the point e (yield point) the stress increments along strain and reaches the maximum value at point f, known as tensile strength ($\sigma_f$) up to here uniform plastic deformation takes place in the direction of length of the specimen and then after the point of stress reduces with non-uniform plastic deformation up to fracture of the specimen at point g, known as fracture limit, due to the starting of necking. The parameters like tensile strength and yield strength describes the material’s strength and indication of the ductility of the material which defines as the maximum deformation of the material can be withstand by the material when subjected to the applied force before fracture occurs is obtained by reduction in cross sectional area and percentage elongation in length.

![Figure 3: General stress—strain curve](image-url)

To understand the effect of strain hardening let us again consider the tension test curve as shown in Figure 4. In this figure the test piece is loaded beyond the yield point up to a point D. The test piece is then unloaded. The elastic deformation recovers via the unloading curve DP which is more or less parallel to AB. It is generally taken that there is no change in Young’s modulus during plastic deformation. The line DP depicts elastic recovery. Out of the total strain AP corresponding to the point D, the part PR is the elastic recovery. The part AP which is not recovered is the plastic strain suffered by the test specimen. Now if we reload the same test piece, it nearly follows the line PD. There is, however, some deviation due to hysteresis which is very small, and the yielding now occurs at the point D. Further loading of the test piece beyond D gives the same stress-strain curve as we would have obtained if there were no unloading. This shows that after suffering a plastic strain represented by AP, the yield strength of metal has increased from point C to point D (or $\sigma_0$ to $\sigma_f$). This is known strain hardening or work hardening. $\sigma = KE^n$, where $K$ is true stress, $e$ is true strain, $K$ is constant, $n$ is work hardening exponent is valid from the beginning of plastic to the maximum load at which the specimen begins to neck down.
During forming process, it is significant to know the force which will be needed to execute the necessary deformation. The flow stress is used to analyze what is happening at any point in forming process. In order to determine the maximum flow stress, the force needed for the maximum strain of the material must have been calculated. For different forming processes, the flow stress analysis may vary. The maximum flow stress value will be very important in forging like processes, whereas processes like extrusion, the mean flow stress value is determined in which the metal is deformed continuously. Another factor that increases load on forming equipment is the rate at which the forming process is carried out, known as strain rate [9]. At higher rates of strain the flow stress of material increases leading to higher loads on the equipment. The effect of strain rate on yield strength for an alloy is illustrated (Figure 5).

If a forming process is carried out at a temperature less than the re-crystallization temperature and at a slow rate, such a case may be taken as an isothermal process, i.e. the effect of temperature change during the process may be neglected and we may consider only the effect of strain hardening. When a forming process is carried out in hot state the re-crystallization is also present along with strain hardening and strain rate effect. The strain hardening may be nullified by recrystallization. Therefore, in hot working we may only consider the effect of temperature and strain rate on the yield strength of metal. The effect of strain rate may be written as given as \( \sigma_f = \sigma_0 + \beta \varepsilon \), where \( \sigma_0 \) are material parameters.

At hot working temperatures, flow stress also depends on strain rate (Figure 6). As strain rate increases, resistance to deformation increases, this effect is known as strain-rate sensitivity. At low strain rates the flow stress increases with increase in strain rate. At higher strain rates it still increases but at a slower rate because of the softening effect due to temperature rise in the material [9].

Recovery and Re-crystallization, is very important in forming process, because the grain structure in material may undergo different stages during process. The flow of deformation of grain structure is shown in Figure 7.

The material gets strain hardened, i.e. its yield strength, UTM and hardness increase while ductility decreases. The strain hardening occurs because the dislocation density increases due to cold deformation. With increase in temperature the movement of dislocations gets easier and they readjust due to stresses locked in the lattice. Some dislocations having opposite sign may annihilate each other. This is called recovery process in which the residual stresses are reduced, however, the enhanced properties due to cold working are only little affected.

Now if the compressed metal piece is heated to certain higher temperature (0.4 to 0.6T_{melt}), new grains will start emerging at the boundaries of old grains and at sites of other defects. If this temperature is maintained for some time the new grains will grow to cover the entire structure. This is called primary re-crystallization. Re-crystallization removes the strain hardening effect and hence reduces strength but increases ductility. However, the process does not stop there. Some grains start growing at the expense of other grains till the complete structure is covered by bigger grains. This is called secondary re-crystallization and grain growth. The mechanical properties like yield strength depend upon the grain size. The relationship between grain size and flow stress is given by the Hall-Petch formula, \( \sigma_f = \sigma_0 + \lambda d^{-1/2} \), where \( \sigma_0 \) is the flow stress of material, \( d \) is the average grain size and \( \sigma_0 \) and \( \lambda \) are material parameters.

The parameters \( \sigma_0 \) and \( \lambda \) are not absolute constants but are functions of strain, strain rate and temperature. During hot working the processes of...
strain hardening and subsequent stress relieving/recovery and re-
crystallization may occur depending upon the temperature and strain. 
Bonnanand, Bramley and Mynores [1] have employed the following 
formulation for the effect of these for yield strength, which includes 
the effect of strain, strain rate, temperature, grain size and re-
crystallization for calculations related to backward extrusion process 
\[ \frac{2}{\sqrt{3}(1-\eta)} Ke^n(\varepsilon')^m \exp(-\beta T), \] 
where K is material constant in stress units, \( \varepsilon \) = equivalent strain, 
\( n = \) strain hardening exponent, \( m = \) strain-rate-effect exponent, 
\( T = \) temperature (Kelvin) and \( \beta = \) material parameter.

To identify the constant \( k \), by defining the stress state in pure 
strain \( \sigma_1 = -\sigma_3 = T, \sigma_2 = 0 \). 
\[ \frac{1}{6} (\sigma_1^2 + \sigma_2^2 + 4\sigma_3^2) = k^2; \sigma_1 = k, \] 
where, \( k \) is the yield stress under pure shear condition, 
whereas \( \sigma_0 \) represents the yield stress under unidirectional tension. 
These two stresses can be related as \( k = \frac{1}{\sqrt{3}} \sigma_0 = 0.577 \sigma_0 \).

For design purposes it is convenient to include a chosen safety factor \( N \) 
in the calculations so that the stress state will be safely inside the failure-
stress ellipse. \( N = \frac{\sigma'}{\sigma_0} \) where \( \sigma' \) is effective stress, \( S_y \) is tensile yield strength

**Maximum-shear-stress or Tresca criterion:**

It was first proposed by coulomb and later described by Tresca. It states 
that yielding occurs when the maximum shear stress in a system exceeds 
the shear stress in a tensile test. The principal stresses \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) 
are arranged in a descending order. Maximum shear stress, 
\[ T_{\text{max}} = \sigma_1 - \sigma_3 \]

This criterion may be easier than the von Mises-Hencky criterion, however 
it is necessary to know the minimum and maximum principal stresses 
from the system before. Under uni-axial tension test \( \sigma_1 = \sigma_0, \sigma_2 = \sigma_3 = 0 \), 
\[ T_{\text{max}} = \frac{\sigma_1^2 - \sigma_2^2}{2} = \frac{\sigma_0}{2}, \] 
therefore \( \sigma_1 - \sigma_3 = \sigma_0 \).

Under pure shear stress conditions \( \sigma_1 = -\sigma_3 = \sigma_0, k = \frac{\sigma_1 - \sigma_3}{2} = \frac{1}{2} \sigma_0 \). The safety factor for the maximum shear-stress theory 
given by \( N = \frac{S_{yz}}{T_{\text{max}}} = \frac{S_y}{(\sigma_1 - \sigma_3)} \), where \( S_{yz} \) is shear yield 
strength (Figure 8).

The yield locus of the maximum shear stress criterion falls inside of the 
distortion-energy yield ellipse. The two yielding criteria anticipate the 
same yield stress for uni-axial stress conditions. The greatest deviation 
between the two yield theories occurs for pure shear stress \( \sigma_1 = -\sigma_3 \) 
where yield stress from distortion-energy criterion \( \frac{1}{\sqrt{3}} \sigma_0 \) is 
15.5% greater than the yield stress from maximum shear stress criterion 
(\( \frac{1}{2} \sigma_0 \)).

**Figure 7:** Recovery and recrystallization in forming process; A-Initial 
structure, B-After compression, C-Start of crystallization, D-After 
re-crystallization, E-Grain growth

**YIELD CRITERIA**

Yield criteria are also called theories of yielding. A number of yield criteria 
have been developed for ductile and brittle materials. Yield criteria is the 
relationship between the stress state and the strength of the material, 
when the criteria met, then plastic deformation occurs. In uni-axial 
tensile tests, yield occurs that is at yield stress \( (\sigma_0) \) where macroscopic 
plastic flow starts [2]. There are two accepted theories which are used in 
anticipating the yielding distribution in ductile material are:

(i) von Mises-Hencky or Distortion-energy criterion, and 
(ii) Maximum-shear-stress or Tresca criterion.

**von Mises-Hencky Distortion-energy criterion:**

It states that when the second invariant of stress deviator \( J_2 \) over comes 
the critical value then yield occurs. It is calculated as 
\[ J_2 = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]; \sigma_1, \sigma_2 \] 
and \( \sigma_3 \) are principal stresses. Criterion is independent of hydrostatic stress 
due to the presence of differences of normal stresses. Distortion energy is 
defined as total strain energy divided by unit volume, which changes its 
shape to obstruct the change in volume.

To determine, let’s consider the yield in unidirectional tension 
test(\( \sigma_1 = \sigma_0, \sigma_2 = \sigma_3 = 0 \)). Thus \( \frac{1}{6} (\sigma_0 - \sigma_0)^2 = k^2; \sigma_0 = \sqrt{3}k \). 
\[ \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \]

**Figure 8:** Yield loci for the two yield criteria in plane stress

Once the necking started, non-uniform plastic deformation is initiated 
in the material, then unidirectional state of stress turns into multi-
directional/tri-axial state of stress. A mild notch effect is present in the 
necked region. Because of the tri-axial state of stress, the uni-axial yield
The average true stress is greater than the state of stress which would cause flow in tension be more. Under tri-axial stress state, the true stress state from calculated stress in axial direction can be measured by mathematical analysis by Bridgman. Following assumptions are considered during his analysis, they are: (a) Necked region cross-section remains circular; (b) Necking cross-section is approximated by arc of the circle; (c) Distortion-energy yield theory is applicable; (d) Over the cross-section area, strains are constant.

For the flow curve, from the point E that is the onset of necking Bridgman’s correction is applicable. Corrected yield stress in tri-axial stress state is

\[
\sigma = \frac{(\sigma_{x})_{avg}}{1 + 2(R)} \left[\ln(1 + \frac{a}{2R})\right]
\]

where, \(\sigma_{x}\) – average measured stress in the axial direction, \(a\) – smallest radius in the neck region, \(R\) – radius of the curvature of neck (Figure 9).

**Figure 9: Necked region in the specimen**

**CONCLUSION**

Thus the forming is an important component in modern industry, as it uses stresses to cause plastic deformation of material without deteriorating their properties. And the present paper presents various issues of forming and plastic deformities. The properties of materials are to be considered as important issue for the production at the required standards. A theory of yielding is also important issue to be understood in the industry, as it is related to the stress state and strength of the material.

**References**


