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## CAPABILITY INDICES FOR MEASUREMENT PROCESS

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**Abstract:** A quality management often use the capability indices for measure its output merit. It is based on Six Sigma methodology in focus to higher quality performance. The higher sigma level, the better is process performing. On the other hand, there are some differences between needs for manufacturing process and needs for measuring process. The question of whether the measuring process gives the results of measurements in accordance with the specifications are serious questions regarding the quality assurance of measuring processes. In this paper are presented capability indices of the first, second and third generation and comparison between them, based on measurement data, focusing on their sensitivity. There is addressed the issue of the capability of a measuring process with using the capability indices and making the proposal for the use of the capability indices with confidence probability of 95 % in contrast to strait Six Sigma approach.

**Keywords:** uncertainty, confidence probability, capability index

### INTRODUCTION

Capability indices compares the desired (prescribed) the accuracy of the process to the actual process variability. The  $C_p$  index was first time mentioned by J. M. Juran and his associates in 1974 and later in 1986 his work was followed up by V. E. Kane, who introduced the  $C_{pk}$  index. These two are also called capability indices of first generation and are most commonly used in practice. To estimate the performance of the measurement process we can adopt these indices, although there are primarily designed for measuring capability of the manufacturing process. In addition to the symbol  $C_p$ , there are also other indications of this index. For example, Finley mentioned CPI (capacity potential index) or Montgomery PCR (process capability ratio) [2].

### CALCULATION PRINCIPLE

The  $C_p$  index to measure the capability of the manufacturing process will be calculated as follows:

$$C_p = \frac{USL - LSL}{6 \cdot \sigma} \quad (1)$$

where the (USL) is the upper specification limit, (LSL) is the lower specification limit and ( $\sigma$ ) is the process standard deviation.

By using the capability indices is the performance of a process monitored in long-term. This monitoring is done on the critical parameters of product

specifications, but does not apply to processes that are evaluated by characters of attribute. The difference between Long-term process capability from the preliminary is, that the values are recorded over a longer period of time and thus takes into account all the weighty parameters for process variance [4].

For proper calculation of an index is necessary to have appropriate data collected and in sufficient amount of the data. It is recommended to collect the data at regular time intervals, at least 25 sub-groups consisting of 2 to 25 values in the section, which can deliver a sufficient amount of data to be able to express all the common sources of variation affecting the process.

When we using capability indices for evaluating a measurement process the ratio of the prescribed (required) and the actually accuracy rate of the process is monitored. This can be achieved through the maximum permissible error or the expanded uncertainty, making the requirements for the measurement process. For example, if the tolerance interval  $T$  for a product that we measure is set, the measurement process should produce results with expanded uncertainty 3 to 10 times smaller than half of the tolerance interval, depending on how stringent are the requirements for the accuracy. Generally, when we have fixed these requirements, as an upper tolerance

limit USL and lower tolerance limit LSL, that defines the value of T as the difference between USL and LSL.

The requirement for expanded uncertainty U is set up directly, accordingly the needs for the measurement, or it can be determined using the formula:

$$U = \frac{T}{2 \cdot p} \quad (2)$$

where(p) is the number between 3 to 10.

The measured values with the expanded uncertainty U should be smaller than USL and greater than LSL by that value. Therefore, should be tendency to choose this p higher number, which leads to smaller U. On the other hand, this increases demands on measurement and therefore is necessary some compromises to make.

In the equation (1) was used in the denominator six sigma which represents a 99.7% confidence probability that values quality characteristic can be found in the tolerance range. Since with measurement process is normally uncertainty with confidence probability of 95% involved, it should be appropriate use the value of four sigmas, which captures the same confidence probability, in the denominator.

The formula for execution the  $C_p$  to indicate the measurement process capability will be as follows:

$$C_p = \frac{2 \cdot U}{4 \cdot \sigma} = \frac{U}{2 \cdot \sigma} \quad (3)$$

where(U) is the requirement for expanded uncertainty and ( $\sigma$ ) is the process standard deviation.

$C_p$  index structure is based on the assumption that the requirement for expanded uncertainty is properly determined and the process is centred. The systematic error is zero and the arithmetic mean is close to identical with the nominal value of the check standard.

Index can get only positive values. If the value of  $C_p$  is less than 1, the process is definitely not capable. When the index value is greater than 1, we can say that the process is capable to perform the tasks for which it is intended. But in practice we should be looking for values 1,33 and above because there always will be some fluctuations and measurement process is never in perfect state of statistically control.

The drawback is its inability to say whether the measurements are within the required tolerance range. It's just the extent of potential capability, because it does not place the actual margin of tolerance with respect to the required tolerance margin [3].

As was noted above, the  $C_p$  index provided that the process is centred and neglects its bias from check standard.

For this purpose, has been introduced  $C_{pk}$  index, as an indicator of the actual process capability.  $C_{pk}$  index responds to deflect the process mean from the centre of the tolerance range. From the actual nominal value of the check standard.

$C_{pk}$  index values will be obtained as minimum from the equation:

$$C_{pk} = \min \left( \frac{(X_{CS}+U)-\bar{X}}{2 \cdot \sigma}, \frac{\bar{X}-(X_{CS}-U)}{2 \cdot \sigma} \right) \quad (4)$$

where(U) is the requirement for expanded uncertainty, ( $\sigma$ ) is the process standard deviation, ( $\bar{X}$ ) is the arithmetic average of the data set and ( $X_{CS}$ ) is the nominal value of the check standard.

If we define  $X_{CS} - \bar{X} = \Delta$ , then the equation will be:

$$C_{pk} = \frac{U-|\Delta|}{2 \cdot \sigma} \quad (5)$$

In case that, the uncertainty of the check standard( $U_{CS}$ ) isn't small enough compared to requirement for expanded uncertainty, it must be taken into account in calculation.

Then:

$$C_{pk} = \min \left( \frac{(X_{CS}+U)-\bar{X}-U_{CS}}{2 \cdot \sigma}, \frac{\bar{X}-(X_{CS}-U)-U_{CS}}{2 \cdot \sigma} \right) \quad (6)$$

or

$$C_{pk} = \frac{U-|\Delta|-U_{CS}}{2 \cdot \sigma} \quad (7)$$

$C_{pk}$  index like  $C_p$  should have values greater than 1,33. Index values in excess of 1 but below 1,33 may be accepted, but must be increased attention to monitoring the process.

Using this index, we can find out how the measurement process centred. About the measurement process centralization tells us following relations [4]:

- ≡  $C_p = C_{pk}$ , the process is centered in the middle of the tolerance range,
- ≡  $C_p > C_{pk}$ , the process is not ideally centered,
- ≡  $C_{pk} = 0$ , the process is centered on the upper or lower specification limits,
- ≡  $C_{pk} < 0$ , the process is centered outside the tolerance range.

## SECOND AND THIRD GENERATION

The second generation of capability indices can be considered index  $C_{pm}$  and  $C_{pm}^*$ , which are based on the concept of Taguchi's approach to the evaluation of quality and his loss function. They quantify process capability in terms of quality indicator X variability around the target value (nominal value of check standard).

$C_{pm}$  index in measurement process will be defined as follows:

$$C_{pm} = \frac{U}{2 \cdot \sqrt{\sigma^2 + (\bar{X} - X_{CS})^2}} \quad (8)$$

The third generation of capability indices was introduced in late 90s, when people pointing on the shortcomings of previous generations. They criticized in particular the sensitivity of these indices to the use of assumptions (normality, independence of observations, process stability). [2]

The result is the  $C_{pmk}$  index, which is more sensitive to variability around the target values than the  $C_{pk}$  and  $C_{pm}$ , and is actually a combination of these indices.

$C_{pmk}$  index is defined as follows:

$$C_{pmk} = \min \left( \frac{(X_{CS}+U)-\bar{X}}{2 \cdot \sqrt{\sigma^2+(\bar{X}-X_{CS})^2}}, \frac{\bar{X}-(X_{CS}-U)}{2 \cdot \sqrt{\sigma^2+(\bar{X}-X_{CS})^2}} \right) \quad (9)$$

or we can use the simpler variant

$$C_{pmk} = \frac{U-|\Delta|}{2 \cdot \sqrt{\sigma^2+(\bar{X}-X_{CS})^2}} \quad (10)$$

### COMPARISON

In the next section we apply each of the indexes on data provide from the measuring the pH level of the filtrate in whitening process of the celluloses. The device that was used to measure level of pH was from company ABB type TB82PH. The measured values were taken once per week in a period of one year, when the sample was taken from the filtrate and compared the measured pH value with a pH of same sample measured in the laboratory. For calculation was used deviation between the value from the measuring instrument and the value from laboratory. The difference is requested as small as possible so we put the nominal value of the check standard equal zero.

Uncertainty of the instrument used in the laboratory was 0,02 according to the calibration certificate, which is a value small enough to the requirements at the expanded uncertainty, which is 0,5, so it could be neglected.

The following table shows the characteristics of the data selection used in the calculations.

Table 1: The process sample parameters

n	$\sigma$	$\sigma^2$	$\bar{X}$	$X_{CS}$	$\Delta$
52	0,1896	0,0359	-0,0713	0	0,0713

From the specified parameters we can calculate the value of each index:

$$C_p = \frac{0,5}{2 \cdot 0,1896} \cong 1,32$$

$$C_{pk} = \frac{0,5 - |0,0713|}{2 \cdot 0,1896} \cong 1,13$$

$$C_{pm} = \frac{0,5}{2 \cdot \sqrt{0,1896^2 + (-0,0713 - 0)^2}} \cong 1,23$$

$$C_{pmk} = \frac{0,5 - |0,0713|}{2 \cdot \sqrt{0,1896^2 + (-0,0713 - 0)^2}} \cong 1,06$$

The  $C_p$  value suggests that the measurement process should have enough capability, this would be true only if the process was centred perfectly.  $C_{pk}$  already capture the bias to the left of the nominal value of the check standard, but its value can still be considered capable. Surprisingly  $C_{pm}$  has a higher value than  $C_{pk}$ , this is due to its lower sensitivity to bias, especially with relatively high variance of data to the requirement. Even if the values of all indices are marginal, we can consider the measurement process as capable. Only  $C_{pmk}$  reacts strongly enough to indicate significant weaknesses of process capability.

On the histogram from the data set we can see the process is displacement to the left.

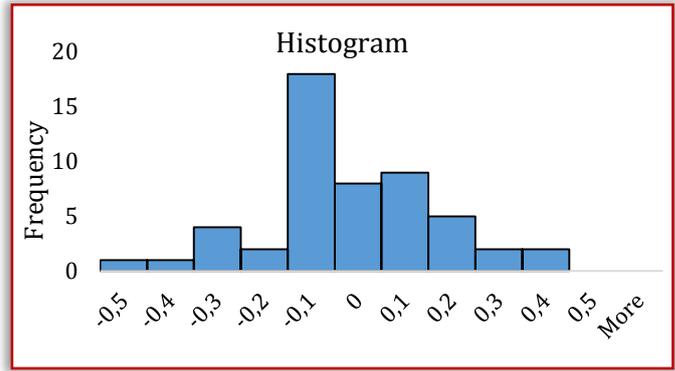


Figure 1: Histogram from process sample

Afterwards was calibration of the pH-meter performed and the probe was replaced. Then, was made 17 measurements and comparisons with laboratory results in two days in different times.

The selection characteristics are shown in the Table 2.

Table 2: The new process sample parameters

n	$\sigma$	$\sigma^2$	$\bar{X}$	$X_{CS}$	$\Delta$
17	0,1640	0,0269	0,0059	0	-0,0059

After substituting into the formula, we get:

$$C_p = \frac{0,5}{2 \cdot 0,1640} \cong 1,52$$

$$C_{pk} = \frac{0,5 - |-0,0059|}{2 \cdot 0,1640} \cong 1,51$$

$$C_{pm} = \frac{0,5}{2 \cdot \sqrt{0,1640^2 + (0,0059 - 0)^2}} \cong 1,52$$

$$C_{pmk} = \frac{0,5 - |-0,0059|}{2 \cdot \sqrt{0,1640^2 + (0,0059 - 0)^2}} \cong 1,51$$

After adjustments were made, we can see that the values of all tested indices are similar and achieve positive results. At this time the pH measurement process meets its capability requirements.

A similarity value is due to greater centralization process, which can be seen on the histogram new data sheet.

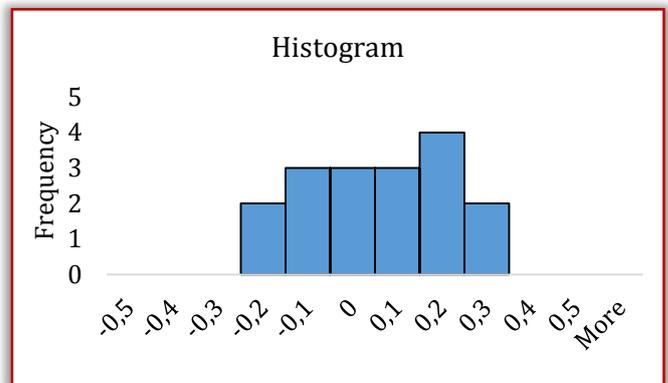


Figure 2: Histogram from new process sample

### CONCLUSIONS

The paper is presenting capability indices based on the 95% probability confidence, since the expanded uncertainty is determined with same 95% probability.

As well, the proposed indices taking into account the value of the check standard as target value and the loss function according to Taguchi.

We tested the pH-meter measuring process, where we were able lack of the capability revealed only using the  $C_{pmk}$  index. The data indicates that the studied process is in order with respect to the requirement after adjusting the measuring device.

It is also important to say that if our calculations were made using six sigma instead four sigma, the measurement process would be classified as incapable, and that would be a mistake.

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#### Note

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