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## DESIGN AND CONTROL OF FULL VEHICLE SUSPENSION SYSTEM

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**Abstract:** This paper deals with the design and control of vehicle suspension system for a full vehicle model with the aim to improve the ride comfort and to guarantee permanent contact between road and wheel. For the design of full vehicle active suspension system we will start by reduced model with 7 DOFs, this model contains all the basic elements to evaluate the ride comfort and tire load variations of the full model with 13 DOFs model. Full vehicle model with 7 DOFs contains 14 state variables which are very difficult and sometimes impossible to measure. These difficulties will be solved by using state estimator or Kalman filter. The selection of weighting factors is very important task to design an active vehicle suspension system, in such a way that during the process a defined goal function is minimized and provide the possibility to emphasize quantifiable issues of vehicle suspensions like; ride comfort and road holding for varying external conditions. Simulations are performed in SIMULINK/MATLAB for full vehicle model, both active and passive suspension system, linear and nonlinear models, while these systems are excited by the white noise disturbance and the linearization around the equilibrium point are performed in MathCAD.

**Keywords:** passive, active, suspension, optimal control, Kalman Regulator

### INTRODUCTION

In order to improve the overall suspension performance, like; ride comfort and road holding for varying external exploiting conditions different authors have discussed various control solutions.

Except contemporary suspension systems, with fixed characteristics in case of an active suspension system it may be applied with great success the theory of optimal control. This theory may be applied both for full state feedback and for limited state feedback.

These analysis has shown that despite the presence of an active force generator, according to the information of the Suspension State which may generate the control force in any shape and mark in order to ensure the better performance, however the conflict between the ride comfort and road holding still remains.

For control system design of suspension it is important to specify an input control vector  $\underline{u}$ . This force drives a system to a specified target state in such a way that during the process, a defined goal quadratic function  $J$  is minimized. During the determination of a quadratic goal performance index it is particularly important the selection of weighting matrices  $Q$  and  $R$  for different values of weighting factors (Likaj, 1998).

### NONLINEAR DYNAMIC SYSTEM OF FULL VEHICLE MODEL

For the analysis of vertical oscillation, pitch and bounce of the body mass, suspension working space and dynamic variation of tire load, is used the full car model shown in Figure 1.

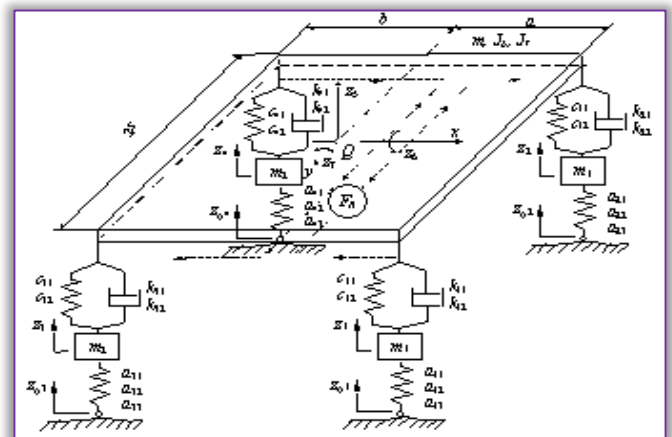


Figure 1. Full vehicle model

The general coordinates are taken in the following form (Demic 1997, Likaj 2005):

- »  $z_1, z_2, z_3$  and  $z_4$  – displacement of unsprung mass (tires),



- »  $z_5$  – vertical displacement of sprung mass (body),
- »  $z_6$  – pitch of sprung mass,
- »  $z_7$  – roll of sprung mass,
- »  $z_{01}, z_{02}, z_{03}$  and  $z_{04}$  – the amplitudes of the road excitation by the road microprofile, at each tire.
- »  $a, b$  and  $s$  – the coordinates of the center of gravity for the body mass,
- »  $J_6$  and  $J_7$  – central moments of the inertia for the body mass.

Taking into account the possible displacements of the model with 7 DOFs, the following expressions can be defined:

a) Spring deflections:

$$\begin{aligned} z_{r1} &= z_5 + a \cdot z_7 - s \cdot z_6 - z_1 \\ z_{r2} &= z_5 + a \cdot z_7 + s \cdot z_6 - z_2 \\ z_{r3} &= z_5 - b \cdot z_7 - s \cdot z_6 - z_3 \\ z_{r4} &= z_5 - b \cdot z_7 + s \cdot z_6 - z_4 \end{aligned} \quad (1)$$

b) relative velocities:

$$\begin{aligned} \dot{z}_{r1} &= \dot{z}_5 + a \cdot \dot{z}_7 - s \cdot \dot{z}_6 - \dot{z}_1 \\ \dot{z}_{r2} &= \dot{z}_5 + a \cdot \dot{z}_7 + s \cdot \dot{z}_6 - \dot{z}_2 \\ \dot{z}_{r3} &= \dot{z}_5 - b \cdot \dot{z}_7 - s \cdot \dot{z}_6 - \dot{z}_3 \\ \dot{z}_{r4} &= \dot{z}_5 - b \cdot \dot{z}_7 + s \cdot \dot{z}_6 - \dot{z}_4 \end{aligned} \quad (2)$$

c) radial tire deformations:

$$\begin{aligned} z_{r5} &= z_1 - z_{01} \\ z_{r6} &= z_2 - z_{02} \\ z_{r7} &= z_3 - z_{03} \\ z_{r8} &= z_4 - z_{04} \end{aligned} \quad (3)$$

On the basis of written deformations, non-linear forces on elastomeric elements can be written as follows:

d) Tires:

$$\begin{aligned} F_1 &= a_{11}z_{r5} + a_{12}z_{r5}^2 - a_{13}z_{r5}^3 \\ F_2 &= a_{21}z_{r6} + a_{22}z_{r6}^2 - a_{23}z_{r6}^3 \\ F_5 &= a_{31}z_{r7} + a_{32}z_{r7}^2 - a_{33}z_{r7}^3 \\ F_6 &= a_{41}z_{r8} + a_{42}z_{r8}^2 - a_{43}z_{r8}^3 \end{aligned} \quad (4)$$

e) Springs:

$$\begin{aligned} F_3 &= c_{11}z_{r1} + c_{12}z_{r1}^3 \\ F_4 &= c_{21}z_{r2} + c_{22}z_{r2}^3 \\ F_7 &= c_{31}z_{r3} + c_{32}z_{r3}^3 \\ F_8 &= c_{41}z_{r4} + c_{42}z_{r4}^3 \end{aligned} \quad (5)$$

f) Shock absorbers:

$$\begin{aligned} F_9 &= k_{11}\dot{z}_{r1} + k_{12}\dot{z}_{r1}^2 \text{sign}(\dot{z}_{r1}) \\ F_{10} &= k_{21}\dot{z}_{r2} + k_{22}\dot{z}_{r2}^2 \text{sign}(\dot{z}_{r2}) \\ F_{11} &= k_{31}\dot{z}_{r3} + k_{32}\dot{z}_{r3}^2 \text{sign}(\dot{z}_{r3}) \\ F_{12} &= k_{41}\dot{z}_{r4} + k_{42}\dot{z}_{r4}^2 \text{sign}(\dot{z}_{r4}) \end{aligned} \quad (6)$$

Based on the full vehicle model with 7DOFs which is shown in the Figure 2, the differential equation for the passive system is applied, has been obtained using the D'Alamper principle, for the small vibrations around the equilibrium position, in this form:

$$\begin{aligned} m_1 \ddot{z}_1 &= F_3 + F_9 - F_1 \\ m_1 \ddot{z}_2 &= F_4 + F_{10} - F_2 \\ m_2 \ddot{z}_3 &= F_7 + F_{11} - F_5 \\ m_2 \ddot{z}_4 &= F_8 + F_{12} - F_6 \\ m \cdot \ddot{z}_5 &= -(F_3 + F_9 + F_4 + F_{10} + F_7 + F_{11} + F_8 + F_{12}) \\ J_6 \ddot{z}_6 &= (F_3 + F_9 + F_7 + F_{11} - F_4 - F_{10} - F_8 - F_{12}) \cdot s \\ J_7 \ddot{z}_7 &= (F_7 + F_{11} + F_8 + F_{12}) \cdot b - (F_3 + F_4 + F_9 + F_{10}) \cdot a \end{aligned} \quad (7)$$

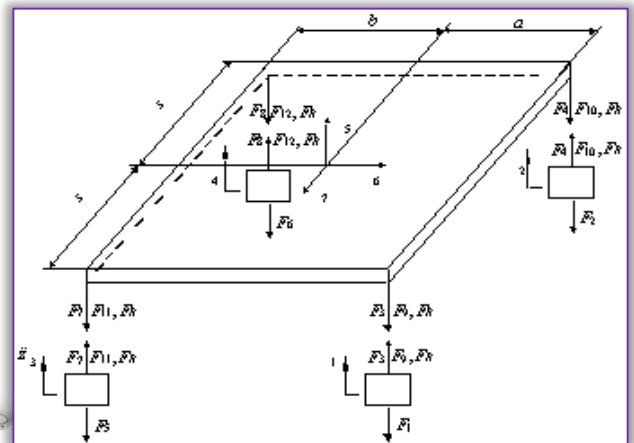


Figure 2. Acting forces in full vehicle model

## SIMULATION RESULTS

In the following table are shown the values of the full vehicle model with 7 DOFs, vehicle that have been used for the simulation of the passive and active system:

Table 1. Model parameters of full vehicle model

$m_1 = 40 \text{ kg}$	$a = 1.8 \text{ m}$
$m_2 = 35.5 \text{ kg}$	$b = 1.0 \text{ m}$
$m = 1460 \text{ kg}$	$s = 0.75 \text{ m}$
$I_6 = 2460 \text{ kg m}^2$	$I_7 = 460 \text{ kg m}^2$

While in Table 2 are given the oscillatory parameters.

Table 2. Oscillatory parameters of full vehicle model

$a_{11} = 80000 \text{ N/m}$	$c_{22} = 6000 \text{ N/m}^3$
$a_{12} = 40000 \text{ N/m}^2$	$k_{21} = 4000 \text{ Ns/m}$
$a_{13} = 20000 \text{ N/m}^3$	$k_{22} = 800 \text{ Ns}^2/\text{m}^2$
$a_{31} = 80000 \text{ N/m}$	$c_{41} = 30000 \text{ N/m}$
$a_{32} = 40000 \text{ N/m}^2$	$c_{42} = 6000 \text{ N/m}^3$
$a_{33} = 20000 \text{ N/m}^3$	$k_{31} = 4000 \text{ Ns/m}$
$c_{21} = 30000 \text{ N/m}$	$k_{32} = 800 \text{ Ns}^2/\text{m}^2$

where:

$$\begin{aligned} a_{11} &= a_{21} & a_{31} &= a_{41} & k_{11} &= k_{21} & c_{11} &= c_{21} \\ k_{31} &= k_{41} & c_{31} &= c_{41} & a_{12} &= a_{22} & a_{32} &= a_{42} \\ k_{12} &= k_{22} & c_{12} &= c_{22} & k_{32} &= k_{42} & c_{32} &= c_{42} \\ & & a_{13} &= a_{23} & a_{33} &= a_{43} \end{aligned}$$

For the full active vehicle model the following weighting factors have been used:





Table 3. Weighting factors

$q_1 = 0.1$
$q_2 = 9.0$
$q_3 = 3600$
$q_4 = 0.00000225$

The SIMULINK model for full vehicle model with 7 DOFs is shown in Figure 3.

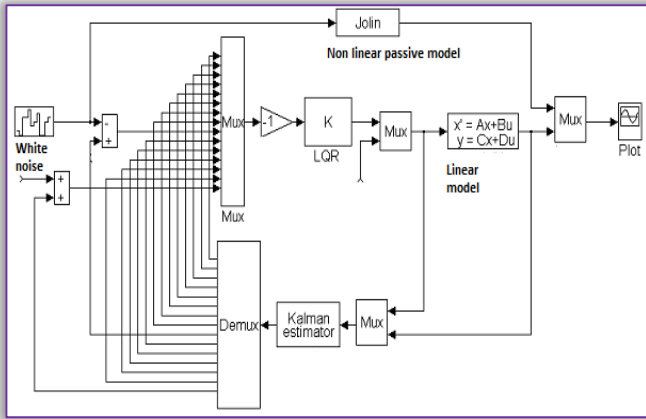
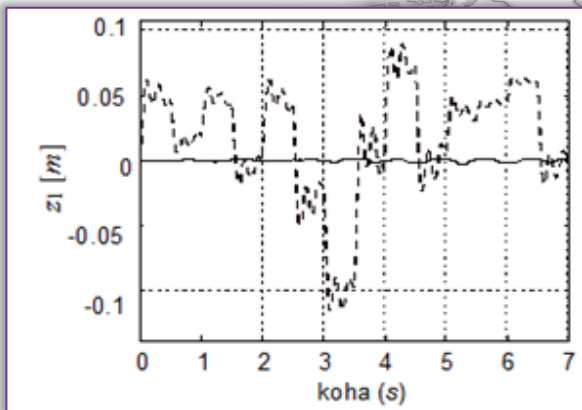
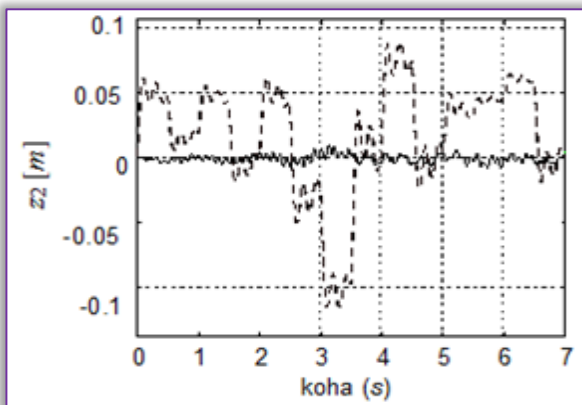


Figure 3. Simulink model for full vehicle system with 7 DOFs (14 state variables)

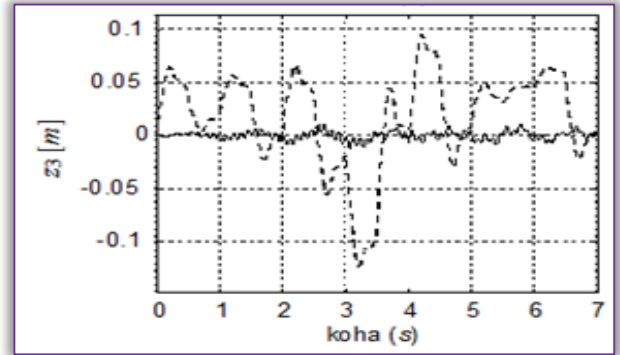
Simulation results for displacement of all DOFs are shown in the following figures.



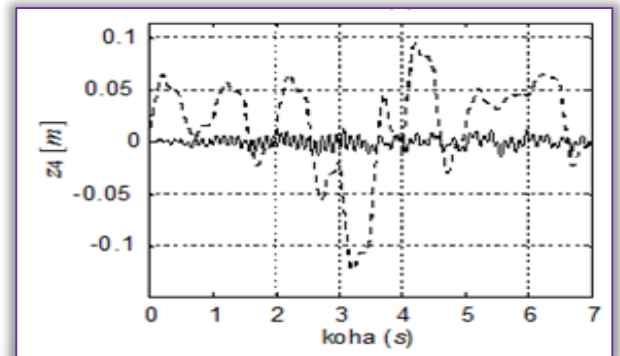
Figures 4. Displacement of z1 DOF for passive (---) and active (-) vehicle suspension system



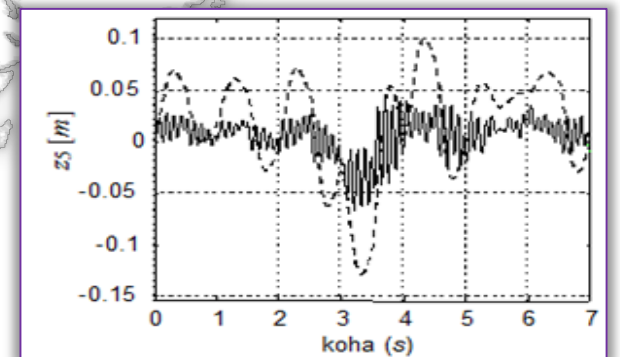
Figures 5. Displacement of z7 DOF for passive (---) and active (-) vehicle suspension system



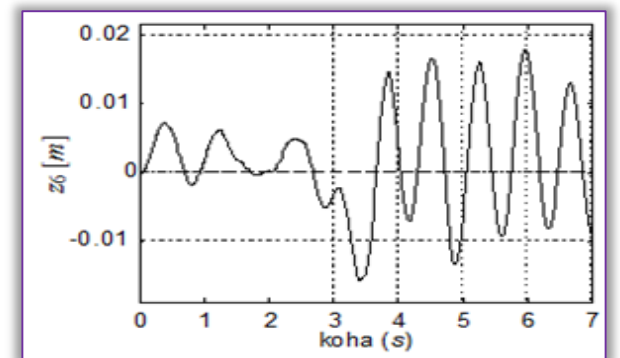
Figures 6. Displacement of z3 DOF for passive (---) and active (-) vehicle suspension system



Figures 7. Displacement of z4 DOF for passive (---) and active (-) vehicle suspension system

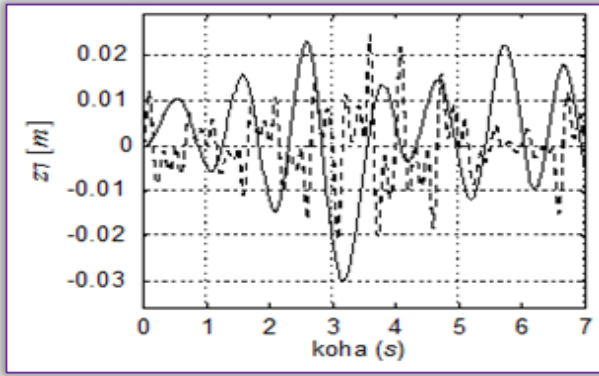


Figures 8. Displacement of z5 DOF for passive (---) and active (-) vehicle suspension system



Figures 9. Displacement of z6 DOF for passive (---) and active (-) vehicle suspension system





Figures 10. Displacement of z7 DOF for passive (---) and active (-) vehicle suspension system

## CONCLUSIONS

For the purpose of designing the performance of an active suspension system the model 7 DOFs model has been used, which is a universal model and can be easily modified to be applied to a wide range of vehicle models. Optimal output control (full feedback control) is performed by minimizing of the quadratic goal function. Through the quadratic performance index can be assigned a Kc matrix of the output feedback, which will obtain Jmin (Likaj, 1998) for the given weighting factors. Such acquisition of values for the weighting factors is based on the maximum allowed variance of the output variables. From the results obtained in the simulations can be concluded that the displacements of the full vehicle model for active suspension system are significantly smaller than the passive nonlinear suspension system.

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## References

- [1] Likaj,R.(2005). Fuzzy Logic Control of Nonlinear Vehicle Suspension System, PHD thesis, Prishtina
- [2] Likaj,R.(1998). Active Suspension Design Using Linear optimal Control, Master thesis, Prishtina
- [3] Demic, M. (1997), Optimizacija oscilatornih sistema motornih vozila, Skver, monography, Kragujevac 1997.
- [4] Ikenaga, S., Lewis, F. L., Campos, J., & Davis, L. (2000). Active suspension control of ground vehicle based on a full-vehicle model. In American Control Conference, 2000. Proceedings of the 2000 (Vol. 6, pp. 4019-4024). IEEE.
- [5] Choi, S. B., Lee, H. S., & Park, Y. P. (2002). H8 Control Performance of a Full-Vehicle Suspension Featuring Magnetorheological Dampers. Vehicle System Dynamics, 38(5), 341-360.

- [6] Sam, Y. M., Osman, J. H., & Ghani, M. R. A. (2004). A class of proportional-integral sliding mode control with application to active suspension system. Systems & control letters, 51(3), 217-223.
- [7] Dumitru, A., Preda, I., & Mogan, G. (2016). Aspects Concerning Modeling and Simulation of a Car Suspension with Multi-Body Dynamics and Finite Element Analysis Software Packages.
- [8] Sharma, S. K., Pare, V., Chouksey, M., & Rawal, B. R. (2016). Numerical studies using full car model for combined primary and cabin suspension. Procedia Technology, 23, 171-178.
- [9] Yazici, H. (2016). Design of a Parameter-Dependent Optimal Vibration Control of a Non-Linear Vehicle Suspension System. Mathematical and Computational Applications, 21(2), 13.
- [10] Likaj R.; Shala A.; Bruqi M.; Qelaj M. (2009). Optimal Design of Quarter Car Vehicle Suspension System, Trends in the Development of Machinery and Associated Technology - ISSN 1840-4944; pp. 417-420; Livorno, Italy.
- [11] Likaj R.; Shala A.; Bruqi M.; Bajrami Xh.. (2016). Optimal Design and analysis of Quarter Vehicle Suspension System by using Matlab, 27 DAAAM, ISIMA, Vienna, Austria
- [12] Mohammadzadeh A. and Haidar S. (2009), Analysis and Design of Vehicle Suspension system using MATLAB and SIMULINK. Grand Valley State University Michigan.



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