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ANALYSIS OF TWO SYMMETRIC CRACKS AT A HOLE UNDER CYCLIC LOADING

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Abstract: Assessing the failure stability of engineering structures under service loadings by means of relevant crack growth concepts is one of key issues in damage tolerance analysis. Therefore, the behaviour of fatigue damages is theoretically examined through the crack growth rate. This paper describes a straightforward methodology to generate failure resistance under cyclic loading. To explore fatigue behaviour of two through–the–thickness cracks located at a hole, the stress intensity factor and life are estimated by means of developed analytical model. Stress state field is numerically analysed using the finite element method. Through relevant applications predictive capability of estimates is discussed. Relevant failure evaluations are performed in the case of complex–valued functions by employing the Euler's integration method, which is implemented in the software program developed. Whereas the type of cracks examined is taken into account through the stress intensity factor.
Keywords: fatigue resistance, two through–the–thickness cracks, finite element analysis, residual life assessment

INTRODUCTION

Notches play an important role in connections between structural components. Such stress concentrators (holes, cutouts) may often cause the formation of fatigue damages. Therefore, to avoid unexpected failures the reliable computational models have to be developed.

In the literature, extensive theoretical studies have been carried out on the residual strength of open/pin–loaded holes with crack–like damages, which provided adequate experience in exploring of the joints failure. Impellizeri and Rich [1] have analysed the fatigue behaviour of through–the–thickness crack at a hole employing the weight function method. Mahendra Babu et al. [2] have taken into account the finite element method to evaluate the stress state field of fastener holes with the same crack configuration. Furthermore, to evaluate the stability of pin–loaded hole Liu and Tan [3] have developed a numerical technique by using the boundery element method, whereas Antoni and Gaisne [4] have employed analytical concept. Boljanović and Maksimović [5] have proposed a set of relevant relationships and numerical model for the strength modeling of open/pin–loaded holes under cyclic loading.

This research work analyses the fatigue behaviour of two symmetric cracks located at a hole. The relationship between residual life and crack propagation rate is developed taking into account the effect of stress ratio. Such fracture mechanics–based computational model is verified through experimental observations, and then it is employed for analysing relevant geometrical/loading influences.

CRACK GROWTH EVALUATION

Assessing the failure stability of engineering structures under service loadings by means of relevant crack growth concepts [4, 5] is one of key issues in damage tolerance analysis. Therefore, the behaviour of fatigue damages (Figure 1) is theoretically examined through the crack growth rate [6] expressed as follows:

$$\frac{da}{dN}=C\left(e^{\alpha R}\Delta K_I\right)^m \tag{1}$$

where a is the crack length, C and m are material parameters experimentally obtained, R and ΔK_I denote the stress ratio and the stress intensity factor range, respectively.

In the fatigue analysis herein presented, the residual life is estimated, after integration of the crack growth rate from intial a_0 to final a_f crack lengths, i.e.

$$N=\int_{a_0}^{a_f}\frac{da}{C\left(e^{\alpha R}\Delta K_I\right)^m} \tag{2}$$

where N is the number of loading cycles.

Note that relevant failure evaluations are performed in the case of complex–velued functions by employing the Euler's integration method, which is implemented in the software program developed. Whereas the type of cracks examined is taken into account through the stress intensity factor (see section 3).

DRIVING FORCE ANALYSIS UNDER FATIGUE LOADING

According to fracture mechanics the fatigue response of two symmetric cracks (Figure1) should be estimated by taking into account the stress intensity factor expressed as follows:

$$\Delta K_I=\Delta S f_2\, f_{w2}\sqrt{\pi a} \tag{3}$$

where ΔS is the applied stress range.

Since the through–the–thickness cracks are located at a hole such nonlinear stress state conditions are theoretically examined here through the following correction factor [7]:

$$f_2 = 1 - 0.15\lambda + 3.46\lambda^2 - 4.47\lambda^3 + 3.52\lambda^4$$

with

$$\lambda = \frac{1}{1 + \frac{2a}{D}} \quad (4)$$

where D is the diameter of the plate.
Further, the interaction between the width and plate hole is taken into account by means of the finite–width correction factor [7] expressed as follows:

$$f_{w2} = \left(\cos\left(\frac{\pi D}{4w}\right) \cos\left(\frac{\pi (D+2a)}{4w}\right) \right)^{-0.5} \quad (5)$$

where w is half of the plate width.

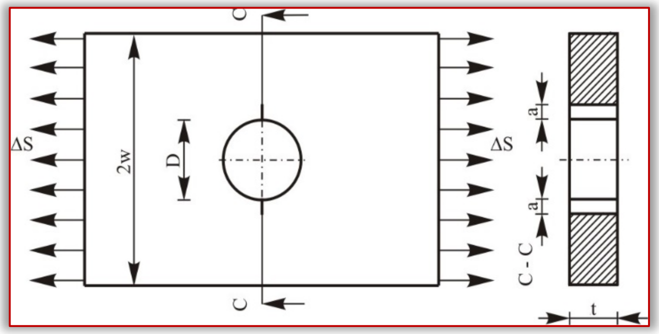

Figure 1. Geometry of the plate with two through–the–thickness cracks

The propagation of fatigue cracks at a hole is also theoretically examined by employing two–dimensional quarter–point singular elements [8] incorporated in the software package MSC/NASTRAN [9]. Relevant applications in which the stress state is evaluated through developed analytical and numerical models are discussed in the next section.

FAILURE ESTIMATION OF FATIGUE DAMAGES AT A HOLE

— RESIDUAL LIFE CALCULATION

The failure analysis presented in this section deals with residual life calculation. Two initial through–the–thickness cracks located at a plate (Figure1) are subjected to cyclic loading with constant amplitude and the following geometrical/loading parameters are assumed: a_0 = 1.625 mm, w = 60 mm, t = 6 mm, D = 17.5 mm, S_{max}= 69.44 MPa (with two different stress ratios R = 0.1 and 0.3).
The plate is made of 2024 T3 aluninium alloy and the following mechanical and fatigue parameters are adopted: E = 73.1 GPa, ν = 0.33, C = 9.45 10^{-11}, m = 3.

The behaviour of fatigue damages are herein assessed employing Eqs. (2)–(5), as it is discussed in previous sections. The evaluated fatigue life, as a function of crack length, are shown in Figure 2a and b in the case of stress ratio equal to R = 0.1 and 0.3, respectively.

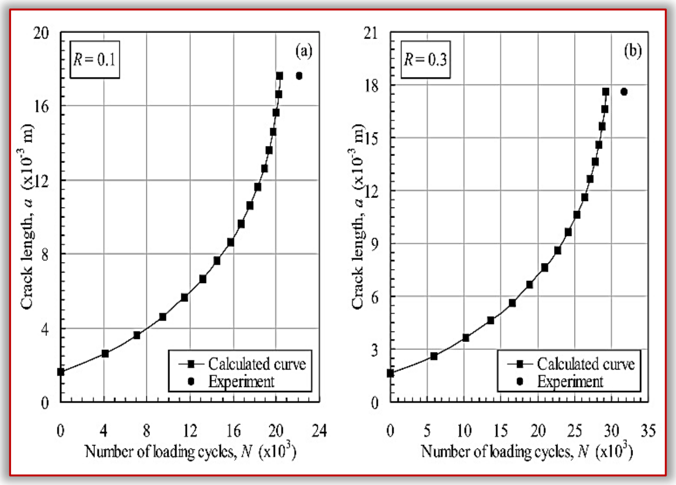

Figure 2. Life estimation for the cyclically loaded plate with two through–the–thickness cracks:
(a) R = 0.1 and (b) R = 0.3.

Furthermore, in Figure 2 the obtained final number of loading cycles and corresponding crack growth lengths are verified through experimental observations. Such comparisons show that relevant fatigue results are in a quite good agreement for the through–the–thickness crack configurations examined

— STRESS STATE ANAYSIS UNDER CYCLIC LOADING

Now the fatigue behaviour of two through–the–thickness cracks (Figure 1) is examined in terms of stress state evaluation. Such crack growth analysis is performed for the plate (a = 9.53 mm, w = 65 mm, t = 7 mm, D = 16.5 mm) subjected to maximum stress and stress ratio equal to S_{max}= 70.33 MPa and R = 0, respecitively. For the 2024 T3 aluminium alloy examined here, the same material paremeters are adopted as those mentioned in previous section.
The propagation of two through–the–thickness cracks is numerically herein analysed by using the two–dimensional singular finite elements [8], incorporated in the MSC/NASTRAN software package [9]. Actually, by assuming linear–elastic behaviour of material, for appropriate crack lengths the finite element meshes and corresponding stress distributions are evaluated. A representation of the stress distribution (a = 9.53 mm) is shown in Figure 3.
Furthermore, the failure mode is estimated through the stress intensity factor by means of Eqs. (3)–(5) and finite element method. Obtained calculations for three crack lengths (a = 3.47 mm, 6.25 mm and 9.53 mm) are listed in Table 1. It can be seen that analytical results show quite good correlation to those numerically obtained.

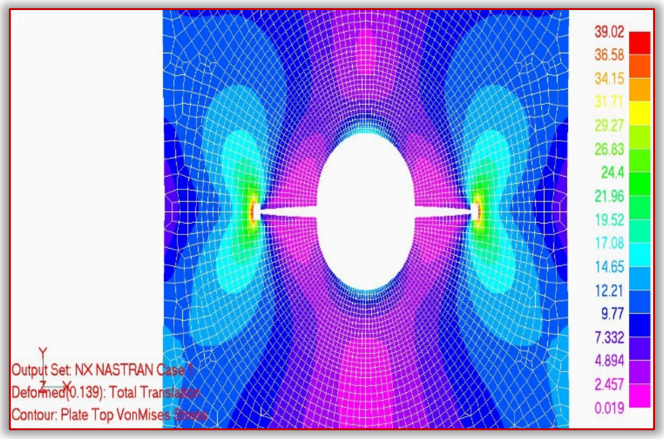

Figure 3. Numerical analysis of two througth–the–thuckness cracks at a hole

Table 1. Calculated stress intensity factors using analytical model and finite elements

	K_{max} (MPam$^{0.5}$)	
a (mm)	Analytical	FEM
3.47	15.94	17.89
6.25	18.56	20.17
9.53	21.84	23.86

— EFFECTS OF STRESS RATIO AND HOLE DIAMETER ON THE FATIGUE RESPONSE

The fatigue strength is here investigated taking into account the effects of stress ratio and diameter of the plate (Figure 1). The residual life is evaluated for damaged plates (a_0 = 2 mm, w = 70 mm, t = 7 mm, S_{max}= 61.22 MPa), made of 2024 T3 aluminium alloy, in the case of three different stress ratios and diameters: (a) R = 0.1, 0.25, 0.5 together with D = 18 mm and (b) D = 19 mm, 21.85 mm, 24.7 mm, R = 0.2, respectively.

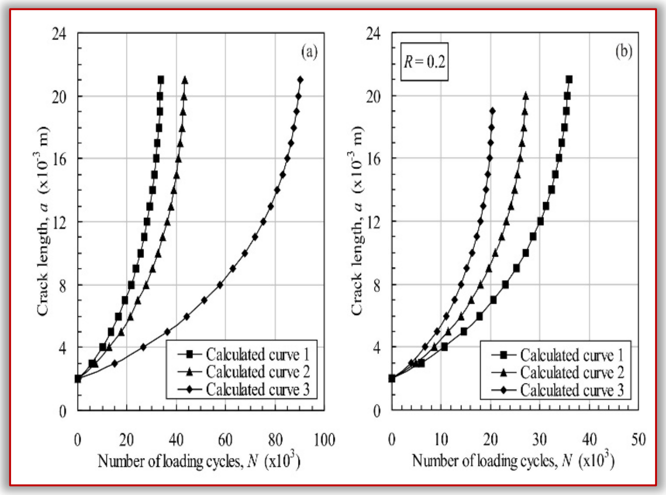

Figure 4. Life estimation for two through–the–thickness cracks at a hole: (a) 1 – R = 0.1, 2 –R = 0.25, 3 – R = 0.5 and (b) 1 – D = 19 mm, 2 – D = 21.85 mm, 3 – D = 24.7 mm.

The failure analysis presented in this section is examined in terms of the stress intensity factor and number of loading cycles (Eqs. (2)–(5)). Relevant life evaluations, obtained for different stress ratios and diametrers of the plate, are shown in Figure 4a and b, respectively. By examining such figures it can be inferred that stress ratio and hole diameter may significantly affect the durability of components with the through–the–thickness cracks at a hole.

CONCLUSIONS

The fatigue degradation is considerable problem in industry which can often cause unexpected catastrophic accidents. Thus, relevant relationships are herein proposed to assess the failure strength of through–the–thickness cracks at a hole taking into account geometrical and loading effects.

Nonlinear stress state field is numerically evaluated by means of quarter–point singular finite elements. Conservative trend of estimates with respect to experimental observations show that developed fracture mechanics based model can be employed for analysing two symmetric trough cracks at a hole.

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