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# PERFORMANCE INVESTIGATION OF DC MOTOR ANGULAR VELOCITY USING OPTIMAL AND ROBUST CONTROL METHOD

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**Abstract:** In this paper, comparison of DC motor angular velocity have been analyzed using PID, H<sup>∞</sup> and μ –synthesis controllers for a step, impulse and sine wave desired ω velocity inputs. The H<sup>∞</sup> controller is designed using H<sup>∞</sup> approach while the μ –synthesis controller is designed using D-K iteration method. The step input simulation result shows that the DC motor with μ –synthesis controller have small settling time and exact steady state value while the DC motor with PID controller have small rise time and the DC motor with H<sup>∞</sup> Controllers have small percentage overshoot. The impulse input simulation result shows that the DC motor with μ –synthesis controller have small peak response while the DC motor with H<sup>∞</sup> controllers have small settling time. The sine wave input simulation result shows that the DC motor with μ –synthesis controller responds as the sine wave input signal. Finally the comparative results prove that the DC motor with μ –synthesis controller is the effective controller for this design.

**Keywords:** DC motor, PID, H<sup>∞</sup> controller, μ –synthesis controller

## INTRODUCTION

DC motor has been popular in the industry manipulate region for a long term because they have appropriate characteristics in excessive starting torque characteristics, excessive reaction overall performance and less difficult to be linear control. These machines are normally used to offer rotary (or linear) motion to a ramification of electromechanical system and servo systems.

DC motor has been extensively utilized in industry despite the fact that its upkeep expenses are higher than the induction motor. DC motor has excellent control reaction, extensive velocity manage variety and it's far broadly used in structures which need excessive manipulate requirements, which include rolling mill, double-hulled tanker, high precision digital equipment, etc.

DC motor makes use of energy and a magnetic field to provide torque, which reasons it to show. It calls for magnets of opposite polarity and an electric powered coil, which acts as an electromagnet. The repellent and attractive Electromagnetic forces of the magnets provide the torque that reasons the motor to show.

## MATHEMATICAL MODEL OF A DC MOTOR

The resistance of the armature is denoted by R (ohm) and the self-inductance of the armature with L (H). The torque (N.M) seen at the shaft of the motor is proportional to the current i (A) prompted by the implemented voltage (V),

$$\tau = K_{arm}i \quad (1)$$

where  $K_{arm}$ , the armature constant, is related to physical classification of the motor. The back electromotive force,  $V_{emf}$  (V), is a voltage proportional to the angular rate seen at the shaft,

$$V_{emf} = K_{emfc}\omega \quad (2)$$

where  $K_{emfc}$ , the EMF steady, additionally relies upon on positive physical residences of the motor.

The mechanical part of the motor equations is derived the use of Newton's laws, which states that the inertial load J (kg\*m<sup>2</sup>) times by the derivative of angular rate ω (rad/sec) equals the sum of all the torques (N.M) about the motor shaft. The end result is this equation,

$$J \frac{d\omega}{dt} = -K_{vf}\omega + K_{arm}i \quad (3)$$

where,  $K_{vf}$  is a linear approximation for viscous friction.

The electrical part of the motor equations can be described by

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{K_{arm}}{L}\omega + \frac{1}{L}V_{inp} \quad (4)$$

Given the two differential equations, you can develop a state space representation of the DC motor as a dynamic system. The current i and the angular rate are the two states of the system. The applied voltage,  $V_{app}$ , is the input to the system, and the angular speed ω is the output.

$$\frac{d}{dt} \begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_{emfc}}{L} \\ \frac{K_{rm}}{J} & -\frac{K_{vf}}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{inp} \quad (5)$$

$$y = [0 \quad 1] \begin{bmatrix} i \\ \omega \end{bmatrix} + [0]V_{inp} \quad (6)$$

The parameters of the Dc motor is shown in Table 1 below.

Table 1. DC motor parameters

No	Parameters	Symbol	Value
1	Moment of inertia of the rotor	J	$kgm^2 / s^2$
2	Damping ratio of the mechanical system	$K_{vf}$	0.4Nms
3	Electromotive force constant	$K_{am} = K_{emfc}$	0.04 Nm/A
4	Electric resistance	R	$4 \Omega$
5	Electric inductance	L	0.8H

**THE PROPOSED CONTROLLERS DESIGN**

**— PID Control**

PID control is very simple manage subsequently it is broadly used in many research and industrial applications. PID basically has a 3 manage i.e. Proportional, Integral and Derivative. A PID is a controller which calculates error between a desired values known as set point (SP) and measured value (MV). The PID ambitions to minimize the error by using manipulating the manipulate variables. For quality overall performance of PID controller, their parameters ought to be tuned depending upon the character of the system.

The 3 term of PID controller performs the extraordinary control movement. P manipulate decreases the upward thrust time of a response, at the same time as there may be no improvement in offset. I control essentially used to put off the offset and steady state error however will increase the settling time, accordingly the temporary behavior of the system worsen and ultimately D manage motion used to get better brief response however stand-alone derivative control introduce a large steady state error. The transfer function of PID controller is,

$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{de(t)}{dt} \quad (8)$$

The parameters of the PID are

$$Kp = 85, \quad Ki = 225 \text{ and } Kd = 9$$

The block diagram of the Dc motor system with PID controller is shown in Figure 1 below.

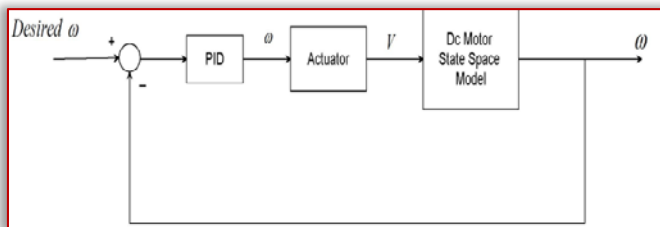


Figure 1. Block diagram of the Dc motor system with PID

**— H<sup>∞</sup> control**

H<sup>∞</sup> control are used on control theory to synthesize controllers to obtain stabilization with assured overall performance. To use H<sup>∞</sup> techniques, a control designer expresses the manipulate trouble as a mathematical optimization hassle after which reveals the controller that solves this optimization.

H<sup>∞</sup> strategies have the gain over classical control techniques in that they are simply relevant to problems regarding multivariate structures with move coupling among channels. It is vital to take into account that the ensuing controller is best premiere with appreciate to the prescribed optimal characteristic and does not necessarily represent the quality controller in phrases of the standard overall performance measures used to assess controllers consisting of settling time, power expended, and many others.

The block diagram of the DC motor with H<sup>∞</sup> controller is proven in Figure 2 below.

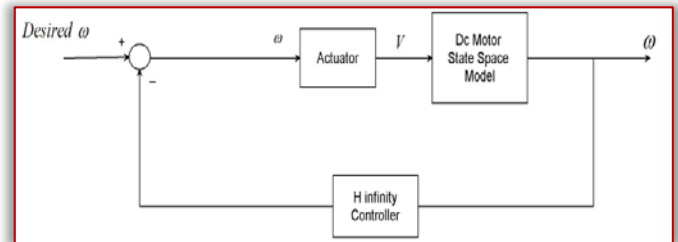


Figure 2. Block diagram of the Dc motor with H<sup>∞</sup> controller

**— μ -Synthesis Control**

The H<sup>∞</sup> optimization method may achieve strong stabilization towards unstructured system perturbations and nominal performance necessities. It is although possible that by way of making use of suitable weighting features some strong overall performance requirements can be received.

Satisfactory designs had been reported, especially while using the H<sup>∞</sup> loop-shaping design strategies. In order to acquire robust stability and robust performance, design techniques primarily based on the based singular price μ may be used. μ -Synthesis Control is used to synthesize a strong controller okay for the unsure open-loop plant model via the D-K or D-G-K set of rules.

When the plant model is uncertain, the closed-loop overall performance goal is to obtain the favored sensitivity characteristic for all plant models defined through the uncertain plant model. The block diagram of the DC motor with μ -Synthesis controller is shown in Figure 3 below.

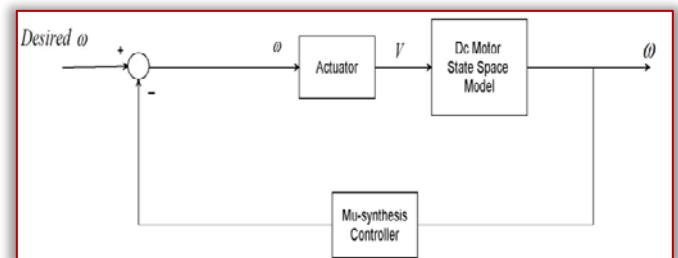


Figure 3. Block diagram of the Dc motor with μ -Synthesis controller

**RESULT AND DISCUSSION**

**— Open-loop Step Response without Controller**

The step response of the open loop DC motor without controller is shown in Figure 4 below.



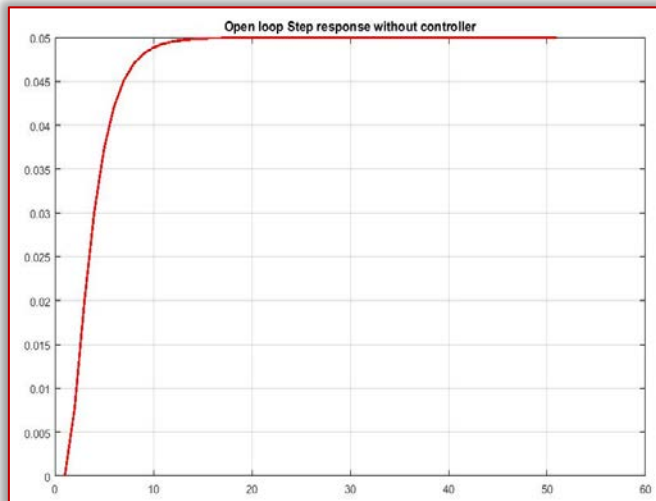


Figure 4. Step response of the open loop dc motor without controller

— Comparison of Dc Motor with PID,  $H^\infty$  and  $\mu$  – synthesis Controllers using Step Desired  $\omega$  Velocity Input Signal

The simulation result of the angular speed of the DC Motor with PID,  $H^\infty$  and  $\mu$  –synthesis Controllers using Step desired  $\omega$  velocity Input Signal is shown in Figure 5 below.

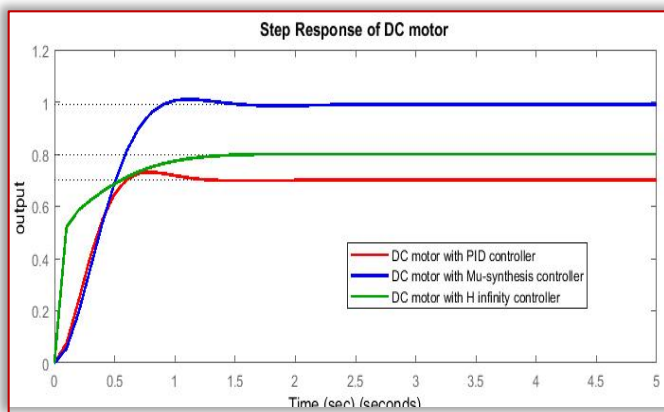


Figure 5. Step response

The simulation result for settling time, rise time, percentage overshoot and steady state value is shown in Table 2 below.

Table 2. Numerical result of the simulation output

No	Controller	Settling time sec	Rise time sec	Over Shoot %	Steady state value
1	PID	1.06	0.388	4.52	0.7
2	$H^\infty$	1.14	0.623	0.0862	0.8
3	$\mu$ – Synthesis	0.844	0.559	1.91	1

— Comparison of Dc Motor with PID,  $H^\infty$  and  $\mu$  – synthesis Controllers using Impulse Desired  $\omega$  Velocity Input Signal

The simulation result of the angular speed of the DC Motor with PID,  $H^\infty$  and  $\mu$  –synthesis Controllers using impulse desired  $\omega$  velocity Input Signal is shown in Figure 6 below.

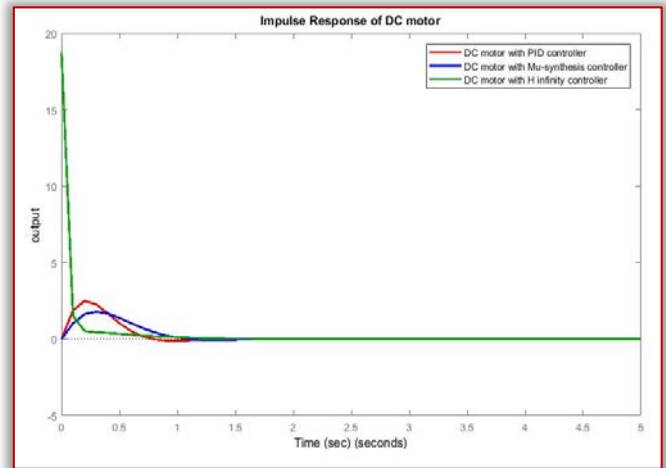


Figure 6. Impulse response

The simulation result for settling time and peak response is shown in Table 3 below.

Table 3. Numerical result of the simulation output

No	Controller	Settling time sec	Peak response at a time sec
1	PID	1.26	2.48
2	$H^\infty$	0.442	18.8
3	$\mu$ –Synthesis	1.55	1.78

— Comparison of Dc Motor with PID,  $H^\infty$  and  $\mu$  – synthesis Controllers using sine wave Desired  $\omega$  Velocity Input Signal

The simulation result of the angular speed of the DC Motor with PID,  $H^\infty$  and  $\mu$  –synthesis Controllers using sine wave desired  $\omega$  velocity Input Signal is shown in Figure 7 below.

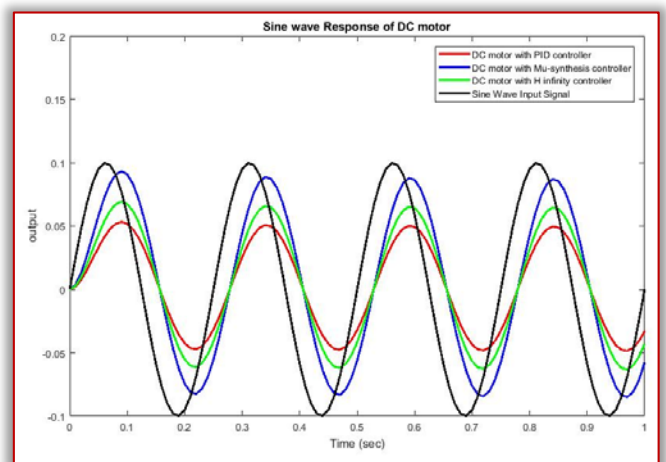


Figure 7. Sine wave response

The simulation result for peak amplitude is shown in Table 4 below.

Table 4. Numerical result of the simulation output

No	Controller	Peak amplitude
1	Sine Wave	0.1
2	PID	0.05
3	$H^\infty$	0.07
4	$\mu$ –Synthesis	0.095

Table 4 shows that the DC Motor with  $\mu$  –synthesis Controller responds as the sine wave input signal than the DC Motor with PID and  $H^\infty$  Controllers.

#### CONCLUSION

In this paper, the design and mathematical model of a Dc motor have been done and the analysis and simulation have been done using Matlab/Script. PID,  $H^\infty$  and  $\mu$  –synthesis controllers have been designed for the Dc motor in order to compare the performance of the system. The comparison is done by using step, impulse and sine wave input signals. The step input simulation proved that the DC motor with  $\mu$  –synthesis controller have small settling time and exact steady state value while the DC motor with PID controller have small rise time and the DC motor with  $H^\infty$  controllers have small percentage overshoot. The impulse input simulation proved that the DC motor with  $\mu$  –synthesis controller have small peak response while the Dc motor with  $H^\infty$  controllers have small settling time. The sine wave input simulation proved that the DC motor with  $\mu$  –synthesis controller responds as the sine wave input signal. Finally the comparative simulation results prove the effectiveness of the presented DC motor with  $\mu$  –synthesis controller.

#### Reference

- [1] R Chandrasekaran et al. “Modelling and Analysis of DC Motor Drive” International Journal of Pure and Applied Mathematics, Vol. 118, No. 20, 2018.
- [2] Panagiotis K. et al. “Fuzzy Q-Learning Agent for Online Tuning of PID Controller for DC Motor Speed Control” Algorithms for PID Controller (Special Issue), Vol. 11, Issue. 10, 2018.
- [3] Aung Ye et al. “DC Motor Control System with PID Controller” International Journal of Science and Engineering Applications, Vol. 7, Issue. 8, pp. 250-253, 2018.
- [4] Ali A. Hassen et al. “Comparative Study for DC Motor Speed Control using PID Controller” International Journal of Engineering and Technology, Vol. 9, Issue. 6, pp. 4181-4192, 2017.
- [5] Thang Nguyen et al. “The Control Structure for DC Motor Based on the Flatness Control” International Journal of Power Electronics and Drive Systems”, Vol. 8, Issue. 4, 2017.
- [6] Peicheng Shi et al. “Design of Dual DC Motor Control System Based on DSP” The Second Annual International Conference on Information System and Artificial Intelligence, 2017.
- [7] Tayfun A. et al “Modelling and Optimal Control of DC Motor” International Journal of Engineering Trends and Technology, Vol. 32, Issue. 3, pp. 146-150, 2016.



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