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ANALYSIS OF FLOW IN WATER DISTRIBUTION NETWORK USING NUMERICAL AND EXPERIMENTAL MODELLING

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Abstract: Several studies have focused on addressing the challenges posed by increasing level of complexity and interdependence of water and other infrastructure systems to reliable design and optimal control, hence the need for such issues to be supported with insights generated beyond the traditional engineering disciplines. This work investigates the modelling techniques used in the analysis of water distribution networks. The analysis of the distribution network studied showed the similarities and disparities of the results obtained in the application of the Hardy Cross method and the Newton Raphson method. The distribution network analysed is that of a water distribution network layout of a University Campus in South West Nigeria. For simplicity purpose, only the mains of the network were analysed and emphasis was placed on the analysis of the loops the network was made up of. The flow rate of the Hardy Cross model and that of the Newton Raphson model for the pipe network was found to be 0.0536 m^3/s for both at the start of the analysis but the final flow rates in each pipes differ for both models on completion of the analysis. The results observed were compared using correlation analysis.
Keywords: Distribution, analysis, head loss, modelling, iteration

INTRODUCTION

Modern society is highly dependent on the reliable performance of critical infrastructures such as those of water, energy and transport systems. The increasing level of complexity and interdependence of such systems poses numerous challenges to reliable design and optimal control, hence the need for such issues to be supported with insights generated beyond traditional engineering disciplines (Yazdani and Jeffrey, 2020). Water distribution network analysis has gained more importance in Civil Engineering in recent years since the optimization of water distribution network has become a focus of current research (Basha and Kassab, 1996).

Ramalingam, Lingireddy and Wood (2009) highlighted the computational advantages of the wave characteristics method for transient modelling of water distribution networks. Pipe networks are composed of a number of constant diameter pipe sections containing pumps and fittings. Hydraulic performance of pipe networks is based on mass continuity and energy conservation. Generally, the hydraulic analysis of water distribution network is performed by considering the steady state situation (Tezcan, Gokkins & Sinir, 1998) and for the solution, one of the three methods containing generally the Hardy Cross, the Linear Theory and the Newton Raphson method is preferred to find the pressure and velocity at any point in the network (Tezcan et al., 1998).

— Review of Hardy Cross & Newton Raphson Methods

Hardy Cross Method

The hardy cross is an iterative method, that is, it is a method that utilizes successive corrections of obtained values during analysis. Today, most engineers use the most improved version of Hardy Cross method (Sereshki, Saffari & Elahi, 2016) for delta p, which analyses the entire loop network of pipes simultaneously (Brkic & Praks, 2019). Hardy Cross's famous equation is as follows and can be used to estimate the flow rate error in each loop:

$$Q = Q_0 + \Delta$$
$$\Delta P = RQ^n$$
$$RQ^n = R(Q_0 + \Delta)^n$$
$$RQ^n = R\left(Q_0{}^n + n\Delta Q_0{}^{n-1} + \cdots\right) \cong RQ_0{}^n + nR\Delta Q_0{}^{n-1}$$
$$\sum RQ^n = \sum RQ_0{}^n + \Delta \sum nRQ_0{}^{n-1}$$
$$\sum RQ^n = 0$$
$$\sum RQ_0{}^n = -\Delta \sum nRQ_0{}^{n-1}$$
$$\Delta = -\frac{\sum RQ_0{}^n}{\sum nRQ_0{}^{n-1}} \xrightarrow{n=2} \Delta = -\frac{\sum RQ_0{}^2}{2\sum RQ_0}$$
$$\Delta = -\frac{\sum \pm \Delta P_i}{2\sum R_i Q_i} = -\frac{\sum \pm R_i Q_i{}^2}{2\sum R_i Q_i}$$

where: Q = Actual flow rate (m^3/s); Q_o = Assumed flow rate (m^3/s); Δ = Flow rate error in loop (m^3/s); R = Flow resistance in each branch; Δp = Pressure loss for each branch (mm of water)

Newton Raphson Method

Newton Raphson technique is one of the methods that is useful for solving numerical computations. This technique is based on the definition on the definition of the derivative and the correction of it. In this technique, the initial guess of error value for the solution of the equation is estimated and then iteratively corrected (Sereshki et al., 2016). The mathematical expression can be expressed as follows;

$$f(x) = 0 \rightarrow f'(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

where: x_1: Initial guess.x_2: Answer of the next step.$f(x_1)$: Value of the function on the basis of initial guess.$f(x_2)$: Value of the function on the basis of the final answer.$f'(x_1)$: The value of the derivative of the function.

Various complex problems that arise in hydraulics and water resources in general have been solved using evolution based programs and their hybrids. The applications of these techniques yield remarkable results when dealing with respect to the number of possible solutions an engineer may be faced with dealing with the design and management of hydraulic systems (Savic and Walters, 1995). Basha and Kassab (1996) in their analysis of water distribution systems by perturbation method concluded that the method may prove advantageous when a water distribution network solver is needed as part of a larger model or program as in optimization or water quality modelling. They further concluded that the system process is fast and efficient, which could prove useful in the optimization of water distribution systems wherein the network is solved for every trial set of design parameters.

However, it is very important that a realistic assessment of the network structure, efficiency or vulnerability should avoid attempting an exclusive characterization of network structure of function by using only single or even few network measurements or ultimate indicators (Yazdani and Jeffrey, 2020). The aim of this research is to improve understanding of the modelling techniques used in the analysis of water distribution networks.

MATERIALS AND METHODS

The following software packages and flow measuring devices were used for the flow analysis:

≡ Microsoft Excel Spreadsheet.
≡ Pipe Flow Expert 2016 Version 6.39 is a window based software application manufactured by Pipe Flow Software.
≡ Venturi Tube Meter.
≡ Typical Water Distribution Network.

The pipe flow expert software described above was used to analyse the flow characteristics of a typical water distribution network and the several flow results or flow characteristics of the software was compared. The distribution network analysed was the water distribution network layout of a University Campus in South West Nigeria. For simplicity purpose, only the mains of the network were analysed and emphasis placed on the analysis of the loops the network was made up of. The results of the Hardy Cross model and that of the Newton Raphson model were compared using correlation analysis.

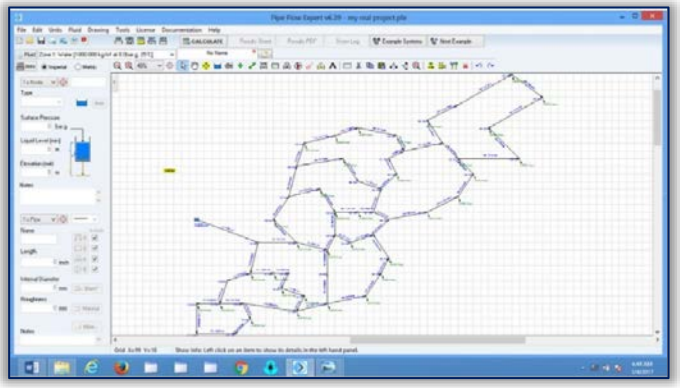

Figure 1: Graphical user interface of the pipe flow expert software for the Newton Raphson model

The Microsoft Excel spreadsheet carried out its calculations using the Hardy Cross method and the Pipe Flow Expert software employed the Newton Raphson model of analysis to obtain the final flow rates in each pipe (as represented in Figure 1) after several iterations.

The following assumptions were made before the analysis was carried out:

≡ All nodes have same elevation.
≡ The diameter for each pipe was selected based on the purpose of making the initial flows uniform before iteration.
≡ The distribution systems receive 4.6 million litres of water daily which is equivalent to 1 million gallons daily.
≡ Piping components such as fittings, valves, bends and their losses were neglected.
≡ The pressure at each node was analysed and only the final flow rate in pipe after iteration was considered.

The final results from both models, the number of iterations, change in flow uniformity based on the velocity of flow were observed on completion of the analysis and the flow rates of both models were compared using the correlation method of descriptive statistics.

— Flow Analysis using a Venturi meter

The methodology applied here was the use of the Venturi effect, that is, the reduction in fluid pressure that results when a fluid flows through a constricted section of a pipe. The flow rate was now determined by measuring the change in pressure in the Venturi meter as shown in Figures 1 and 2. The Venturi effect was applied for flowing water in the Venturi meter at varying temperature with all other factors kept constant.

This was to obtain how significantly varying temperatures of an incompressible fluid such as water affect the flow rate. Flow rate was obtained for varying temperatures of 20℃, 30℃, 40℃, 50℃ and 60℃. Finally, a graph of temperature against flow rate was plotted to obtain the equation of the line and a mathematical relationship between the two variables was generated.

Figure 2: Fluid Friction Factor Board Set Up for Venturi Tube Meter

RESULTS AND DISCUSSION

— Results of Numerical Analysis

The results of the final flow rate of the numerical analysis were obtained by solving a typical water distribution network using the Hardy Cross (HC) method and the Newton-Raphson (NR) method and they are as shown below in Table 1. The final flow rate was obtained after three iterations from the initial flow rate guessed.

Table 1: Result of Numerical Analysis

Pipe	Inner Diameter (mm)	Length (m)	Flow Rate (m³/s)	
			HC Model	NR Model
1	200	700	0.054	0.054
2	140	300	0.031	0.031
3	100	700	0.017	0.018
4	70	367	0.004	0.002
5	70	400	0.004	0.002
6	100	500	0.012	0.009
7	90	100	0.01	0.009
8	95	230	0.011	0.01
9	100	300	0.005	0.012
10	100	160	0.012	0.012
11	105	330	0.014	0.013
12	140	400	0.023	0.023
13	90	400	0.008	0.009
14	90	460	0.008	0.009
15	60	360	0.002	0.002
16	65	650	0.003	8E-04
17	80	240	0.005	0.003
18	55	500	0.004	0.002
19	65	500	0.003	0.001
20	40	400	0.002	1E-04
21	30	100	0.001	0.001
22	120	260	0.019	0.018
23	90	267	0.011	0.011
24	80	267	0.009	0.009
25	55	340	0.003	0.002
26	65	320	0.005	0.003
27	70	300	0.006	0.004
28	75	600	0.007	0.005
29	100	450	0.013	0.012
30	95	253	0.013	0.011
31	40	180	0.013	0.002
32	30	260	0.002	0.001
33	75	220	0.008	0.008
34	75	284	0.008	0.008
35	75	180	0.008	0.007
36	60	167	0.005	0.004
37	100	200	0.012	0.011
38	85	330	0.01	0.009
39	50	460	0.003	0.002
40	35	300	0.002	0.001
41	30	200	0.001	0.001
42	95	233	0.013	0.011
43	75	200	0.008	0.007
44	65	360	0.007	0.005
45	65	300	0.005	0.005
46	100	150	0.012	0.012
47	95	100	0.012	0.011
48	95	100	0.012	0.011
49	50	150	0.004	0.003
50	55	300	0.005	0.004
51	110	200	0.016	0.014
52	75	353	0.008	0.007
53	55	332	0.003	0.003
54	65	460	0.005	0.049
55	27	400	0.005	0.006
56	75	150	0.073	0.007
57	80	200	0.008	0.007
58	65	300	0.005	0.004
59	65	360	0.005	0.004
60	45	340	0.002	0.002
61	70	460	0.006	0.001
62	50	360	0.004	0.003
63	80	333	0.009	0.008

Comparison of Hardy Cross and Newton Raphson Model

The Hardy Cross model uses an incremental change in flow rate to correct the initial guess work made when channelling the flow through the pipes while the Newton Raphson method attempts to solve all concerned equations simultaneously and iterations are used to solve the non-linear equations by finding the roots of the equations. This difference in methodical approach but similarity in flow principles created the interest in observing how the final flow rates calculated using these models correlate as shown in Figure 3. The correlation coefficient was calculated using the equation below

$$r = \frac{[N(\sum xy)(\sum x)(\sum y)]}{\sqrt{[N\sum x^2 - (\sum x)^2][N\sum y^2 - (\sum y)^2]}}$$

The values of X and Y represents final flow rate for the Hardy Cross method and the Newton Raphson method respectively where N represents the number of pipes which turns out to be 63. The value of the correlation coefficient ends up giving r = 0.54 which gives a positive but weak correlation between both methods. The number of iterations carried out by both models on the studied distribution network was the same; the weak correlation could be as a result of the difference in methodical approach. Although the final values of flow rates in each pipe were correct for both models based on their conformation to the principles governing the flow equations, the methodical approach is the cause if the discrepancies observed.

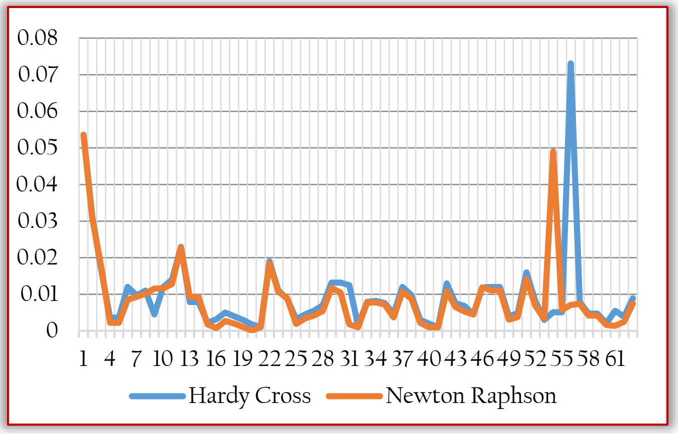

Figure 3: Image of the Correlation Plot between the Hardy Cross Model Result and the Newton Raphson Model Result

— Venturi Meter

The objective of the experiment carried out with the venturi meter was to observe the effect of temperature of water on its flow rate. This is carried out by checking for a change in the differential pressure at the manometer when water passes through the venturi tube from the divergent point to the convergent point. The temperature of the fluid rise varied between 20℃ and 60℃ and as the water flowed through the venture tube, the various differential pressure for each temperature was recorded from the differential pressure obtained, the flow rate for each temperature of water was obtained through the formula shown in the equation below:

$$Q = C_d \left[\frac{2d_p}{\rho}\right]\left[\frac{A_a}{\sqrt{\left({}^{A_a}/_{A_b}\right)^2 - 1}}\right]$$

The results of the experiment are presented in Table 2:

Table 2: Flow Rate for Varying Temperature

Temperature (°C)	Pressure Difference (mmH_2O)	Flow Rate (m^3/s)	Velocity (m/s)	Viscosity 10^{-6} (m^2/s)
20	137	0.00042	0.522	1.01
30	170	0.00046	0.572	0.81
40	206	0.00051	0.572	0.67
50	245	0.00056	0.696	0.56
60	310	0.00063	0.784	0.45

The Table 2 above shows the flow rate obtained by varying temperature for the experiment carried out using a venturi tube. It was observed that as the temperature of the fluid increased, the velocity increased accordingly, which indicates there was an increase in the flow rate of the fluid. This increase in flow rate due to temperature could be attributed to the reduction in viscosity of the fluid as temperature increased. Although, the change or increase in flow rate was not significant, the little change signifies that temperature had an effect on the velocity of flow even if it is less significant in flow models for water due to its minimal effects. From the table it can be observed that there is a 0.0000052 m^3/s average in the flow rate of water per degree rise in temperature for turbulent flow regimes in a venturi meter.

CONCLUSIONS

The study carried out here showed the similarities and discrepancies in results obtained in the application of the Hardy Cross method and the Newton Raphson method. The final flow in each of the pipe was governed by the Continuity equations and the Bernoulli equations, which was the reason the sum of head loss in each loop gives zero and the sum of flow into each junction equalled the sum flow exiting the junction.

The weak correlation in the final flow rates obtained from both methods after analysis has to do with the methodical approach towards analysing the network. It was also observed that as a result of the iterations carried out during the analysis, the final velocities changed from the initial uniform flow assumed before iteration.

The experiment carried out using the venturi meter indicates that there was an increase in velocity as a result of increase in temperature which in turn increases the overall flow rate of the water. This increase was as a result of the reduction in kinematic viscosity of the fluid due to temperature increase as this creates less resistance to shearing forces in order to enable flow.

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ISSN: 2067-3809