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MHD FREE CONVECTION FLOW PAST AN ACCELERATED VERTICAL POROUS PLATE WITH VARIABLE TEMPERATURE THROUGH A POROUS MEDIUM

Abstract:
The effect of a uniform transverse magnetic field on the free convection flow of an electrically-conducting fluid past an uniformly accelerated infinite, vertical, porous plate through a porous medium is discussed. The plate temperature is raised linearly with time. Expressions for the velocity field and skin friction are obtained by the Laplace transform technique. The influence of the various parameters, entering into the problem, on the velocity field and skin friction is extensively discussed with the help of graphs.

Keywords:
Free convection, MHD, porous medium, Accelerated, Vertical porous plate, variable temperature

INTRODUCTION

Buoyancy forces that arise from density differences in a fluid cause free convection. These density differences are a consequence of temperature gradients with in a fluid. Free convection flow is a significant factor in several practical applications that include, for example, cooling of electronic components, in designs related to thermal insulation, material processing, and geothermal systems. Transient natural convection is of fundamental interest in many industrial and environmental situations such as air conditioning systems, human comfort in buildings, atmospheric flows, motors, thermal regulation process, cooling of electronic devices, and security of energy systems. Hydro magnetic flow is encountered in heat exchangers, pumps, flow meters, in designing communications and radar systems, and in nuclear engineering in connection with the cooling of reactor and MHD accelerators. Convective heat transfer in porous media has received considerable attention in recent years owing to its importance in various technological applications such as fibre and granular insulation, electronic system cooling, cool combustors, and porous material regenerative heat exchangers.

Gupta et al. [1] have studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [2] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis et al. [3]. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar [4]. Singh [5] studied MHD free convection flow in the stokes problem for a porous vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and

Singh [6]. Basant kumar Jha and Ravindra Prasad [7] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. Again Basant kumar Jha [8] discussed MHD free convection and mass transform flow through a porous medium. Recently Muthucumaraswamy et al. [9] studied unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion.

The hydro magnetic free convection flow past an accelerated vertical porous plate with variable temperature through a porous medium has many technical applications. Hence it is proposed to study MHD free convection flow past an accelerated vertical porous plate with variable temperature through a porous medium. The dimensionless governing equations are solved using the Laplace transform technique.

MATHEMATICAL ANALYSIS

An unsteady flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate with variable temperature through porous medium has been considered. A magnetic field of uniform strength is assumed to be applied transversely to the porous plate. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. The flow is assumed to be in x' - direction which is taken along the vertical plate in the up ward direction. The y'-axis is taken to be normal to the plate. Initially the plate and the fluid are at the same temperature T'_∞. At time $t'>0$, the plate is accelerated with a velocity $u'=u_0 t'$ in its own plane and the plate temperature is raised linearly with time t. It is assumed that the effect of viscous dissipation is negligible. Then by usual Boussinesq's approximation, the governing equations for the unsteady flow are

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=g\beta\left(T'-T'_\infty\right)+\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2 u'}{\rho}-\frac{\nu u'}{K'} \qquad (1)$$

$$\rho C_p\left(\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=\kappa\frac{\partial^2 T'}{\partial y'^2} \qquad (2)$$

With the initial and boundary conditions

$$t'\le 0,\ u'=0,\ T'=T'_\infty \quad \text{for all } y' \qquad (3)$$

$$t'>0,\ u'=u_0 t',\ T'=T'_\infty+\left(T'_w-T'_\infty\right)At' \quad \text{at } y'=0$$

$$u'=0,\ T'\to T'_\infty \text{ as } y'\to\infty .$$

where $A=\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}}$

Equation (1) is valid when the magnetic lines of force are fixed relative to the fluid.

On introducing the following non-dimensional quantities:

$$u=\frac{u'}{(\nu u_0)^{\frac{1}{3}}},\ t=t'\left(\frac{u_0^2}{\nu}\right)^{\frac{1}{3}},\ y=y'\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}},$$

$$\theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ P_r=\frac{\mu C_p}{\kappa},\ M=\frac{\sigma B_0^2\nu^{\frac{1}{3}}}{\rho u_0^{\frac{2}{3}}}, \qquad (4)$$

$$G_r=\frac{g\beta\left(T'_w-T'_\infty\right)}{u_0},\ \gamma=\frac{-v'}{(\nu u_0)^{\frac{1}{3}}},\ K=K'\left(\frac{u_0}{\nu^2}\right)^{\frac{2}{3}}$$

in equations (1) to (3), leads to

$$\frac{\partial u}{\partial t}-\gamma\frac{\partial u}{\partial y}=G_r\theta+\frac{\partial^2 u}{\partial y^2}-Mu-\frac{u}{K} \qquad (5)$$

$$P_r\left(\frac{\partial\theta}{\partial t}-\gamma\frac{\partial\theta}{\partial y}\right)=\frac{\partial^2\theta}{\partial y^2} \qquad (6)$$

With the initial and boundary conditions :

$$t\le 0:\quad u=0,\quad \theta=0 \quad \text{for all } y$$

$$t>0: u=t,\ \theta=t \text{ at } y=0 \qquad (7)$$

$$u=0,\quad \theta\to 0 \quad \text{as} \quad y\to\infty$$

All the physical variables are defined in the nomenclature. The solution of equations (5) and (6), subject to the boundary conditions (7) by the laplace transform technique when the prandtl number $P_r=1$, is given by

$$u=\left(1-\frac{G_r}{M}\right)\frac{t}{2}\left[e^{y\left[\sqrt{d}-\frac{\gamma}{2}\right]}erfc\left(\frac{y+2t\sqrt{d}}{2\sqrt{t}}\right)+e^{-y\left[\sqrt{d}+\frac{\gamma}{2}\right]}erfc\left(\frac{y-2t\sqrt{d}}{2\sqrt{t}}\right)\right]$$

$$-\frac{y}{4\sqrt{d}}\left[1-\frac{G_r}{M}\right]\left[e^{-y\left[\sqrt{d}+\frac{\gamma}{2}\right]}erfc\left(\frac{y-2t\sqrt{d}}{2\sqrt{t}}\right)-e^{y\left[\sqrt{d}-\frac{\gamma}{2}\right]}erfc\left(\frac{y+2t\sqrt{d}}{2\sqrt{t}}\right)\right]$$

$$+\frac{G_r t}{2M'}\left[erfc\left(\frac{y+t\gamma}{2\sqrt{t}}\right)+e^{-\gamma y}\, erfc\left(\frac{y-t\gamma}{2\sqrt{t}}\right)\right]$$

$$-\frac{G_r y}{2M'\gamma}\left[e^{-\gamma y}erfc\left(\frac{y-t\gamma}{2\sqrt{t}}\right)-erfc\left(\frac{y+t\gamma}{2\sqrt{t}}\right)\right] \qquad (8)$$

$$\theta=\frac{t}{2}\left[erfc\left(\frac{y+t\gamma}{2\sqrt{t}}\right)+e^{-\gamma y}\,erfc\left(\frac{y-t\gamma}{2\sqrt{t}}\right)\right]-\frac{y}{2\gamma}\left[e^{-\gamma y}erfc\left(\frac{y-t\gamma}{2\sqrt{t}}\right)-erfc\left(\frac{y+t\gamma}{2\sqrt{t}}\right)\right] \quad (9)$$

where $M'=M+\frac{1}{K},\ d=M'+\frac{\gamma^2}{4}$

SKIN-FRICTION

We now study skin-friction from velocity field. It is given in non-dimensional form as

$$\tau=\left.\frac{-du}{dy}\right]_{y=0} \quad (10)$$

Then from equations (8) and (10), we have

$$\tau=\left[1-\frac{G_r}{M'}\right]\left[\sqrt{\frac{t}{\pi}}e^{-td}+t\sqrt{d}\ erf\left(\sqrt{td}\right)\right]+\left[1-\frac{G_r}{M'}\right]\frac{t\gamma}{4}\left[erfc\left(\sqrt{td}\right)+erfc\left(-\sqrt{td}\right)\right]+\left[1-\frac{G_r}{M'}\right]\frac{1}{2\sqrt{d}}erf\left(\sqrt{td}\right)+\frac{G_r}{\gamma M'}erf\left(\frac{\gamma\sqrt{t}}{2}\right)+\frac{tG_r}{2M'}\left[\frac{2}{\sqrt{\pi t}}e^{-\frac{t\gamma^2}{4}}+\gamma\, erfc\left(-\frac{\gamma\sqrt{t}}{2}\right)\right] \quad (11)$$

DISCUSSION AND RESULTS

In order to get the physical insight into the problem, we have plotted velocity profiles for different parameters M (Magnetic parameter), K (permeability parameter), γ (suction parameter) and G_r (thermal grashof number) in figures (1) to (8) for the cases of heating (G_r <0) and cooling (G_r >0) of the plate. The heating and cooling take place by setting up free convection current due to temperature gradient.

Figures (1) and (2) illustrate the influences of M(magnetic parameter) in cases of cooling and heating of the porous plate at t=0.2 and t=0.4 respectively. It is observed that the velocity decreases with increase of magnetic parameter M for both cases of cooling and heating of the plate. It is because that, the application of transverse magnetic field will result a resistitive type force(lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. It is also observed that the velocity is maximum near the plate and decreases away from the plate and finally takes asymptotic value.

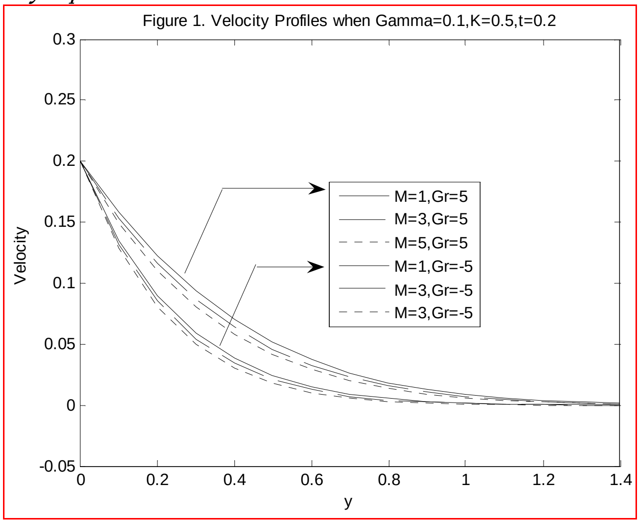
Figure 1. Velocity Profiles when Gamma=0.1,K=0.5,t=0.2


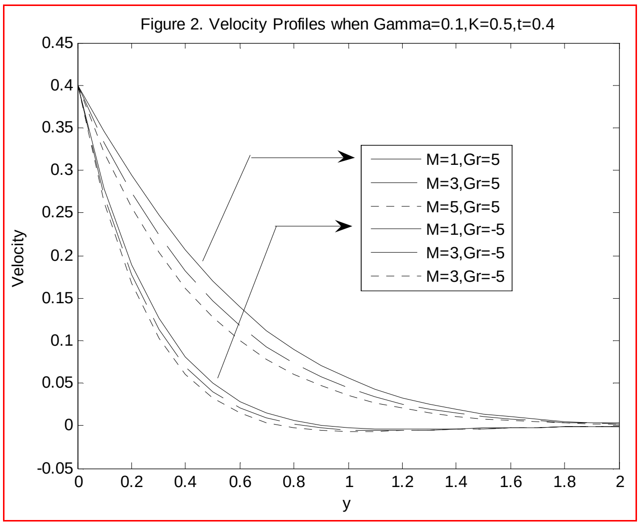
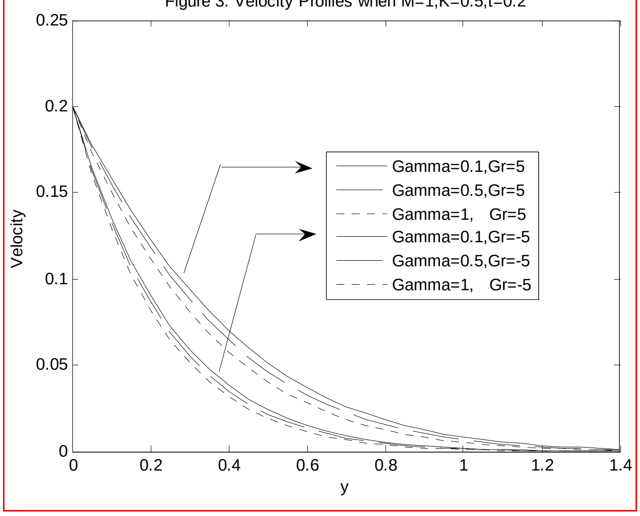
Figure 2. Velocity Profiles when Gamma=0.1,K=0.5,t=0.4


Figure 3. Velocity Profiles when M=1,K=0.5,t=0.2


Figures (3) and (4) reveal velocity variations with γ (suction parameter) in cases of cooling and heating of the porous plate at t=0.2 and t=0.4 respectively. It is found that the velocity *decreases as the suction parameter γ increases for both cases of cooling and heating of the porous plate. It is also found that the velocity is maximum near the plate and decreases away from the plate and finally takes asymptotic value.*

Figure 4. Velocity Profiles when M=1,K=0.5,t=0.4

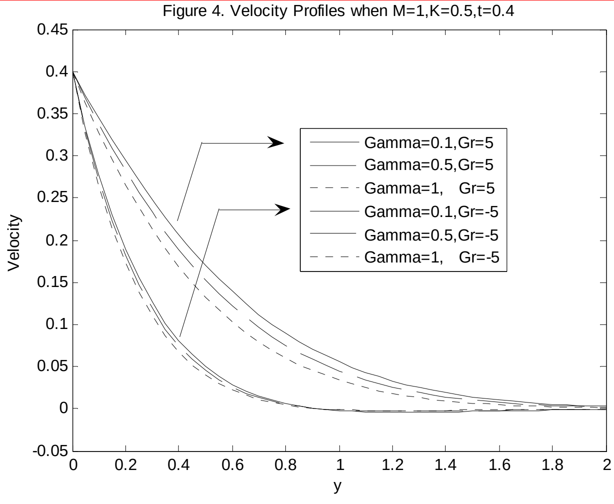


Figure 5. Velocity Profiles when Gamma=0.5,M=1,t=0.2

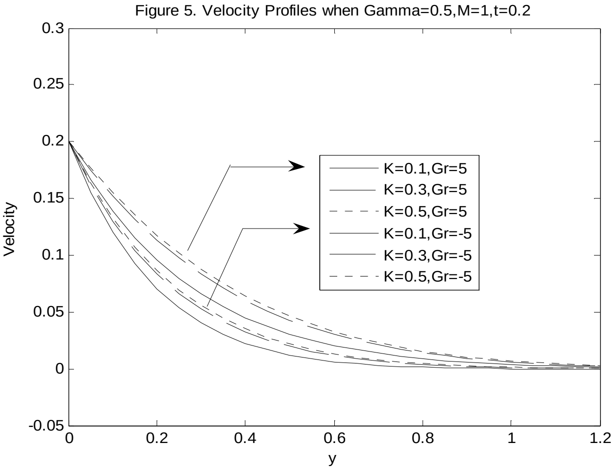
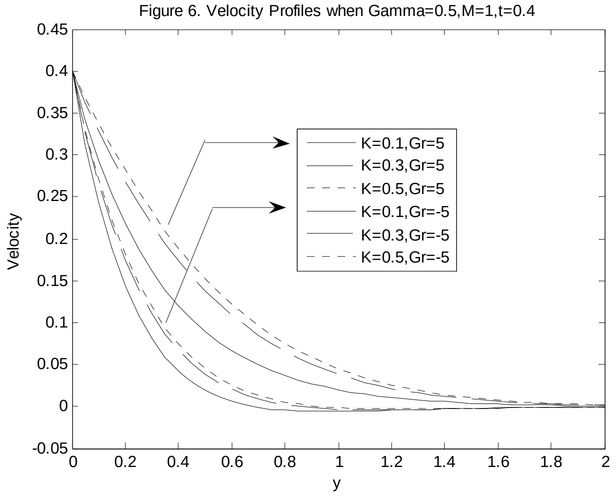


Figure 6. Velocity Profiles when Gamma=0.5,M=1,t=0.4



Figures(5) and (6) represent the velocity profiles due to variations in K (permeability parameter) in cases of cooling and heating of the porous plate at t=0.2 and t=0.4 respectively. It is observed that the velocity increases with increase of permeability parameter K for both cases of cooling and heating of the plate. This is due to the fact that the presence of a porous medium increases the resistance to flow. It is also observed that the velocity is maximum near the plate and decreases away from the plate and finally takes asymptotic value.

Figures (7) and (8) reveal velocity variations with G_r (thermal grashof number) in the cases of cooling and heating of the porous plate at t=0.2 and t=0.4 respectively. It is observed that the velocity increases with increase of thermal grashof number G_r in the case of cooling of the plate. It is due to the fact increase in the values of thermal grashof number has the tendency to increase the thermal buoyancy effect. This gives rise to an increase in the induced flow. But the reverse effect is observed in case of heating of the plate. It is also observed that the velocity is maximum near the plate and decreases away from the plate and finally takes asymptotic value.

Figure 7. Velocity Profiles when Gamma=0.5,K=0.5,M=1,t=0.2

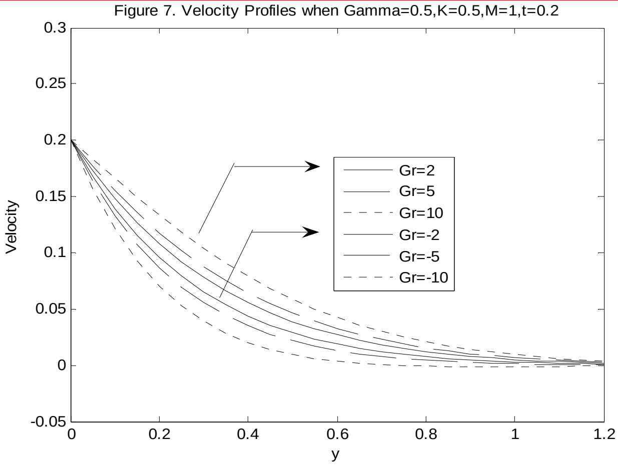


Figure 8. Velocity Profiles when Gamma=0.5,K=0.5,M=1,t=0.4

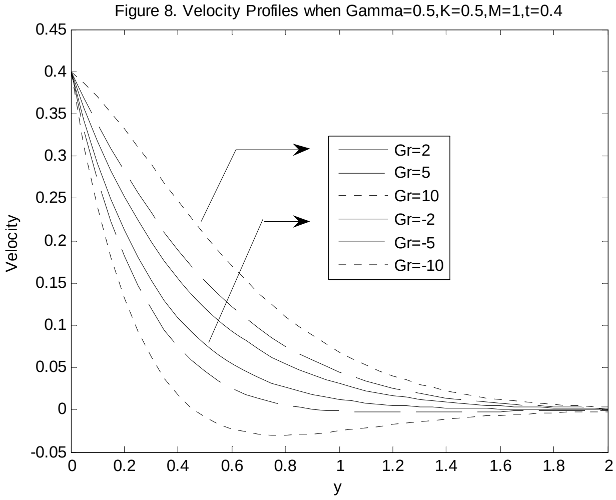


Figure 9. Skin friction when Gamma=0.5,K=0.5,

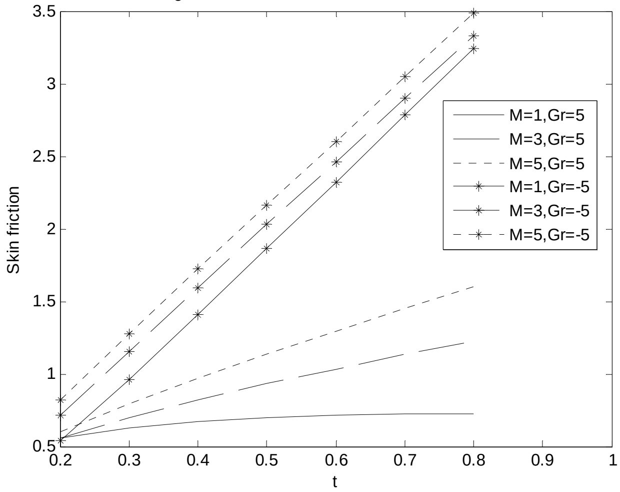


Figure 10. Skin friction when M=1,K=0.5,

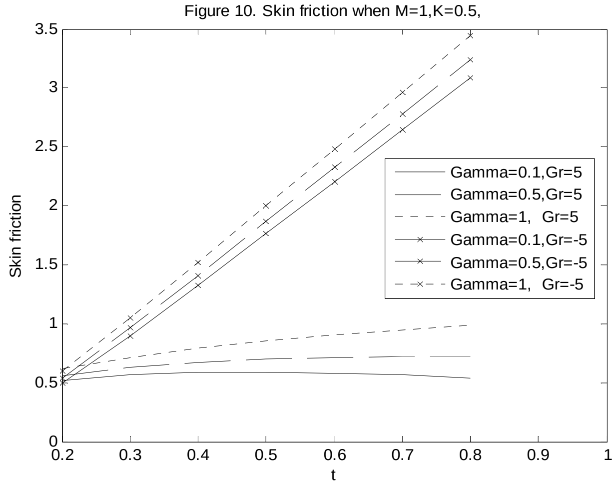
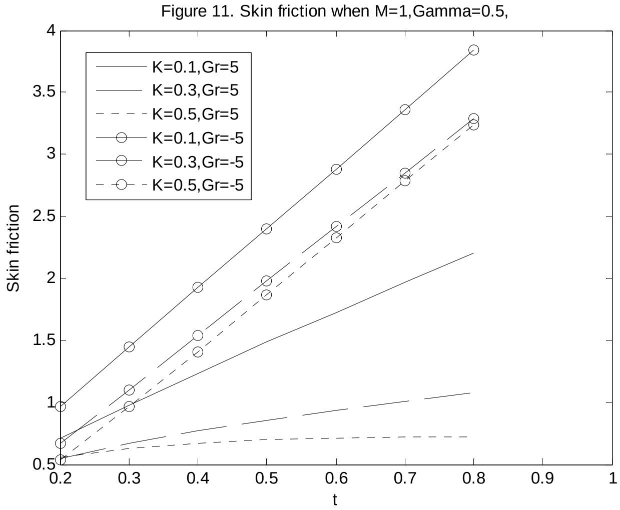
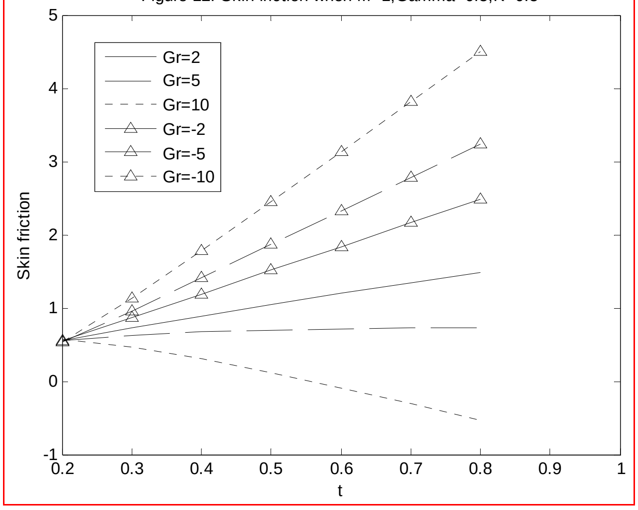


Figure 11. Skin friction when M=1,Gamma=0.5,



The skin friction is presented in figures (9) to (12). From these figures we conclude that the skin-friction increases with increase in M (Magnetic parameter) and γ (suction parameter), but decreases with an increase in K (permeability parameter) for both cooling and heating of the porous plate. It is also observed that skin-friction decreases with increase in G_r (Thermal grashof number) for cooling of the porous plate. But the reverse effect is observed in the case of heating of the porous plate.

Figure 12. Skin friction when M=1,Gamma=0.5,K=0.5



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Nomenclature

- A — *Constant*
- y' — *Coordinate axis normal to the plate*
- y — *Dimensionless coordinate axis normal to the plate*
- u — *Dimensionless velocity*
- B_0 — *External magnetic field*
- P_r — *Prandtl number*
- C_p — *Specific heat at constant pressure*
- T'_∞ — *Temperature of the fluid far away from the plate*
- T' — *Temperature of the fluid near the plate*
- T'_w — *Temperature of the plate*
- κ — *Thermal conductivity of the fluid*
- G_r — *Thermal Grashof number*
- t' — *Time*
- u' — *Velocity of the fluid in the x' -direction*
- v' — *Velocity of the fluid in the y' -direction*
- u_0 — *Velocity of the plate*
- g — *Acceleration due to gravity*
- K — *permeability parameter*

M Magnetic field parameter
t Dimensionless time

Greek symbols

μ Coefficient of viscosity
erfc Complementary error function
ρ Density of the fluid
γ Suction Parameter
τ Dimensionless skin friction
θ Dimensionless temperature
σ Electric conductivity
erf Error function
ν Kinematic viscosity
α Thermal diffusivity
β Volumetric coefficient of thermal expansion

Subscripts

w Conditions on the wall
∞ Free stream conditions

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