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RADIATION EFFECTS ON UNSTEADY MOVING SEMI-INFINITE VERTICAL PLATE IN THE PRESENCE OF CHEMICAL REACTION

ABSTRACT:
Finite difference solution of the homogeneous first order chemical reaction on unsteady flow past an impulsively started semi-infinite vertical plate in the presence of thermal radiation have been studied. The fluid considered is a gray, absorbing-emitting radiation but non-scattering medium. The dimensionless governing equations are solved by an efficient, more accurate, unconditionally stable and fast converging implicit scheme. The effect of velocity and temperature for different parameters like chemical reaction parameter, radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time are studied. The velocity profiles are compared with available exact solution in the literature and are found be in good agreement.

KEYWORDS:
radiation, chemical reaction, vertical plate, finite-difference

INTRODUCTION

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

England and Emery (1994) have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar (1993) have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar (1996). In all above studies, the stationary vertical plate is considered. Raptis and Perdikis (1999) have studied the effects of thermal radiation and free convection flow past a moving infinite vertical plate. Again, Raptis and Perdikis (2003) studied thermal radiation effects on moving infinite vertical plate in the presence of mass diffusion. The governing equations were solved by the Laplace transform technique.

Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Many transport processes exist in nature and in industrial applications in which the simultaneous heat and mass transfer as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, e.g., polymer production, manufacturing of ceramics or glassware and food processing.

Apelblat[1] studied analytical solution for mass transfer with a chemical reaction of the first order. Chambre and Young (1958) have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al (1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Analytical or numerical work on transient natural convection along an impulsively started vertical plate under the combined buoyancy effects of heat and mass diffusion in the presence of thermal radiation and chemical reaction has not received attention of any researcher. Hence, the present study is to investigate first order chemical reaction on flow past an impulsively started semi-infinite vertical plate in the presence of thermal radiation by an implicit finite-difference scheme of Crank-Nicolson type.

MATHEMATICAL ANALYSIS

A transient, laminar, unsteady natural convection flow of a viscous incompressible fluid past an impulsively started semi-infinite vertical plate has been considered. The fluid considered is a gray, absorbing-emitting radiation but non-scattering medium. It is assumed that there is a first order chemical reaction between the diffusing species and the fluid. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. The plate starts moving impulsively in the vertical direction with constant velocity u_0 against gravitational field and the temperature of the plate and the concentration level are also raised to T'_w and C'_w. They are maintained at the same level for all time $t'>0$. Then under the above assumptions, the governing boundary layer equations of mass, momentum and concentration for free convective flow with usual Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\rho C_p\left(\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}\right)=\frac{\partial^2 T'}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}-K_l\, C' \tag{4}$$

The initial and boundary conditions are

$$\begin{aligned}
&t'\le 0: && u=0, && v=0, && T'=T'_\infty, && C'=C'_\infty &&\\
&t'>0: && u=u_0, && v=0, && T'=T'_w, && C'=C'_w && at\, y=0\\
& && u=0, && T'=T'_\infty, && C'=C'_\infty && && at\, x=0\\
& && u\to 0, && T'\to T'_\infty, && C'\to C'_\infty && && as\, y\to\infty
\end{aligned} \tag{5}$$

For the case of an optically thin gray gas the local radiant absorption is expressed by

$$\frac{\partial q_r}{\partial y}=-4a\,\sigma\left(T'^4_\infty-T'^4\right) \tag{6}$$

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty\, T'-3T'^4_\infty \tag{7}$$

By using equations (6) and (7), equation (3) reduces to

$$\rho C_p\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T}{\partial y^2}-16\,a\,\sigma T'^3_\infty\,(T'-T'_\infty) \tag{8}$$

On introducing the following non-dimensional quantities: ,

$$\begin{gathered}
X=\frac{xu_0}{\nu},\ Y=\frac{yu_0}{\nu},\ U=\frac{u}{u_0},\ V=\frac{v}{u_0},\ t=\frac{t'u_0^2}{\nu}\\
T=\frac{T'-T'_\infty}{T'_w-T'_\infty}, Gr=\frac{\nu g\beta(T'_w-T'_\infty)}{u_0^3},\\
C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ Gc=\frac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^3},\\
Pr=\frac{\nu}{\alpha},\ Sc=\frac{\nu}{D}, R=\frac{16a^*\nu^2\sigma T_\infty^3}{ku_0^2},\ K=\frac{\nu K_l}{u_0^2}
\end{gathered} \tag{9}$$

Equations (1) to (4) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{10}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=Gr\,T+Gc\,C+\frac{\partial^2 U}{\partial Y^2} \tag{11}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2}-\frac{R}{Pr}T \tag{12}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2}-KC \tag{13}$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned}
&t\le 0: && U=0, && V=0, && T=0, && C=0 &&\\
&t>0: && U=1, && V=0, && T=1, && C=1 && at\,Y=0\\
& && U=0, && T=0, && C=0, && && at\,X=0\\
& && U\to 0, && T\to 0, && C\to 0 && && as\,Y\to\infty
\end{aligned} \tag{14}$$

NUMERICAL TECHNIQUE

In order to solve these unsteady, non-linear coupled equations (10) to (13) under the conditions (14), an implicit finite difference scheme of Crank-Nicolson type has been employed. The finite difference equations corresponding to equations (10) to (13) are as follows:

$$\frac{\left[U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}+U_{i,j-1}^{n+1}-U_{i-1,j-1}^{n+1}+U_{i,j-1}^{n}-U_{i-1,j-1}^{n}\right]}{4\Delta X}+\frac{\left[V_{i,j}^{n+1}-V_{i,j-1}^{n+1}+V_{i,j}^{n}-V_{i,j-1}^{n}\right]}{2\Delta Y}=0 \tag{15}$$

$$\begin{aligned}
&\frac{\left[U_{i,j}^{n+1}-U_{i,j}^{n}\right]}{\Delta t}+U_{i,j}^{n}\frac{\left[U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}\right]}{2\Delta X}\\
&+V_{i,j}^{n}\frac{\left[U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}+U_{i,j+1}^{n}-U_{i,j-1}^{n}\right]}{4\Delta Y}\\
&=\frac{Gr}{2}\left[T_{i,j}^{n+1}+T_{i,j}^{n}\right]+\frac{Gc}{2}\left[C_{i,j}^{n+1}+C_{i,j}^{n}\right]\\
&+\frac{\left[U_{i,j-1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j+1}^{n+1}+U_{i,j-1}^{n}-2U_{i,j}^{n}+U_{i,j+1}^{n}\right]}{2(\Delta Y)^2}
\end{aligned} \tag{16}$$

$$\frac{\left[T_{i,j}^{n+1}-T_{i,j}^{n}\right]}{\Delta t}+U_{i,j}^{n}\frac{\left[T_{i,j}^{n+1}-T_{i-1,j}^{n+1}+T_{i,j}^{n}-T_{i-1,j}^{n}\right]}{2\Delta X}+V_{i,j}^{n}\frac{\left[T_{i,j+1}^{n+1}-T_{i,j-1}^{n+1}+T_{i,j+1}^{n}-T_{i,j-1}^{n}\right]}{4\Delta Y}$$
$$=\frac{1}{Pr}\frac{\left[T_{i,j-1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j+1}^{n+1}+T_{i,j-1}^{n}-2T_{i,j}^{n}+T_{i,j+1}^{n}\right]}{2(\Delta Y)^2}-\frac{(T_{i,j}^{n+1}+T_{i,j}^{n})}{2Pr} \quad (17)$$

$$\frac{\left[C_{i,j}^{n+1}-C_{i,j}^{n}\right]}{\Delta t}+U_{i,j}^{n}\frac{\left[C_{i,j}^{n+1}-C_{i-1,j}^{n+1}+C_{i,j}^{n}-C_{i-1,j}^{n}\right]}{2\Delta X}+V_{i,j}^{n}\frac{\left[C_{i,j+1}^{n+1}-C_{i,j-1}^{n+1}+C_{i,j+1}^{n}-C_{i,j-1}^{n}\right]}{4\Delta Y}$$
$$=\frac{1}{Sc}\frac{\left[C_{i,j-1}^{n+1}-2C_{i,j}^{n+1}+C_{i,j+1}^{n+1}+C_{i,j-1}^{n}-2C_{i,j}^{n}+C_{i,j+1}^{n}\right]}{2(\Delta Y)^2}-\frac{K}{2}(C_{i,j}^{n+1}+C_{i,j}^{n}) \quad (18)$$

Here the region of integration is considered as a rectangle with sides $X_{max}(=1)$ and $Y_{max}(=14)$, where Y_{max} corresponds to $Y=\infty$ which lies very well outside both the momentum and energy boundary layers. The maximum of Y was chosen as 14 after some preliminary investigations so that the last two of the boundary conditions (14) are satisfied with in the tolerance limit 10^{-5}.

After experimenting with a few set of mesh sizes, the mesh sizes have been fixed at the level $\Delta X=0.05$, $\Delta Y=0.25$ with time step $\Delta t=0.01$. In this case, the spatial mesh sizes are reduced by 50% in one direction, and later in both directions, and the results are compared. It is observed that, when the mesh size is reduced by 50% in the Y-direction, the results differ in the fifth decimal place while the mesh sizes are reduced by 50% in X-direction or in both directions, the results are comparable to three decimal places. Hence, the above mesh sizes have been considered as appropriate for calculation. The coefficients $U_{i,j}^{n}$ and $V_{i,j}^{n}$ appearing in the finite-difference equations are treated as constants in any one time step. Here i-designates the grid point along the X-direction, j along the Y-direction and k to the t-time.The values of U, V and T are known at all grid points at $t=0$ from the initial conditions.

The computations of U,V,T and C at time level $(n+1)$ using the values at previous time level (n) are carried out as follows: The finite difference Equation (18) at every internal nodal point on a particular i-level constitute a tridiagonal system of equations. Such a system of equations are solved by using Thomas algorithm as discussed in Carnahan et al [1]. Thus, the values of C are found at every nodal point for a particular i at $(n+1)^{th}$ time level. Similarly, the values of T are calculated from Equation (17). Using the values of C and T at $(n+1)^{th}$ time level in the equation (16), the values of U at $(n+1)^{th}$ time level are found in a similar manner. Thus, the values of C,T and U are known on a particular i-level. Finally, the values of V are calculated explicitly using the Equation (15) at every nodal point on a particular i-level at $(n+1)^{th}$ time level. This process is repeated for various i-levels. Thus the values of C,T,U and V are known, at all grid points in the rectangular region at $(n+1)^{th}$ time level.

In a similar manner computations are carried out by moving along the i-direction. After computing values corresponding to each i at a time level, the values at the next time level are determined in a similar manner. Computations are repeated until the steady-state is reached. The steady-state solution is assumed to have been reached, when the absolute difference between the values of U, as well as temperature T and concentration C at two consecutive time steps are less than 10^{-5} at all grid points.

STABILITY ANALYSIS

The stability criterion of the finite difference scheme for constant mesh sizes are examined using Von-Neumann technique as explained by Carnahan et al (1980). The general term of the Fourier expansion for U,T and C at a time arbitrarily called $t=0$, are assumed to be of the form $\exp(i\alpha X)\exp(i\beta Y)$ (here $i=\sqrt{-1}$). At a later time t, these terms will become,

$$U=F(t)\exp(i\alpha X)\exp(i\beta Y)$$
$$T=G(t)\exp(i\alpha X)\exp(i\beta Y) \quad (19)$$
$$C=H(t)\exp(i\alpha X)\exp(i\beta Y)$$

Substituting (19) in Equations (16) to (18); under the assumption that the coefficients U,T and C are constants over any one time step and denoting the values after one time step by F',G' and H'. After simplification, we get

$$\frac{(F'-F)}{\Delta t}+\frac{U}{2}\frac{(F'+F)(1-\exp(-i\alpha\Delta X))}{\Delta X}+\frac{V}{2}\frac{(F'+F)\,i\sin\beta\Delta Y}{\Delta Y}=\frac{(G'+G)Gr+(H'+H)Gc}{2}+\frac{(F'+F)(\cos\beta\Delta Y-1)}{(\Delta Y)^2} \quad (20)$$

$$\frac{(G'-G)}{\Delta t}+\frac{U}{2}\frac{(G'+G)(1-\exp(-i\alpha\Delta X))}{\Delta X}+\frac{V}{2}\frac{(G'+G)\,i\sin\beta\Delta Y}{\Delta Y}=\frac{1}{Pr}\frac{(G'+G)(\cos\beta\Delta Y-1)}{(\Delta Y)^2}-\frac{R}{2Pr}(G'+G) \quad (21)$$

$$\frac{(H'-H)}{\Delta t}+\frac{U}{2}\frac{(H'+H)(1-\exp(-i\alpha\Delta X))}{\Delta X}+\frac{V}{2}\frac{(H'+H)\,i\sin\beta\Delta Y}{\Delta Y}=\frac{1}{Sc}\frac{(H'+H)(\cos\beta\Delta Y-1)}{(\Delta Y)^2}-\frac{K}{2}(H'+H) \quad (22)$$

Equations (20) to (22) can be rewritten as,

$$(1+A)F'=(1-A)F+\frac{Gr}{2}(G'+G)\Delta t+\frac{Gc}{2}(H'+H)\Delta t \quad (23)$$

$$(1+B)G'=(1-B)G$$

$$(1+E)H'=(1-E)H$$

where,

$$A=\frac{U}{2}\frac{\Delta t}{\Delta X}(1-\exp(-i\alpha\Delta X))+\frac{V}{2}\frac{\Delta t}{\Delta Y}i\sin(\beta\Delta Y)-(\cos\beta\Delta Y-1)\frac{\Delta t}{(\Delta Y)^2}$$

$$B=\frac{U}{2}\frac{\Delta t}{\Delta X}(1-\exp(-i\alpha\Delta X))+\frac{V}{2}\frac{\Delta t}{\Delta Y}i\sin(\beta\Delta Y)-\frac{(\cos\beta\Delta Y-1)}{Pr}\frac{\Delta t}{(\Delta Y)^2}+\frac{R\Delta t}{2\,Pr}$$

$$E=\frac{U}{2}\frac{\Delta t}{\Delta X}(1-\exp(-i\alpha\Delta X))+\frac{V}{2}\frac{\Delta t}{\Delta Y}i\sin(\beta\Delta Y)-\frac{(\cos\beta\Delta Y-1)}{Sc}\frac{\Delta t}{(\Delta Y)^2}+\frac{K\Delta t}{2}$$

After eliminating G' and H' in Equation (23) using Equations (24) and (25), the resultant equation is given by,

$$(1+A)F'=(1-A)F+G\frac{Gr\,\Delta t}{(1+B)}+H\frac{Gc\Delta t}{(1+E)} \quad (24)$$

Equations (23) to (25) can be written in matrix form as follows:

$$\begin{bmatrix}F'\\G'\\H'\end{bmatrix}=\begin{bmatrix}\frac{1-A}{1+A} & D_1 & D_2\\ 0 & \frac{1-B}{1+B} & 0\\ 0 & 0 & \frac{1-E}{1+E}\end{bmatrix}\begin{bmatrix}F\\G\\H\end{bmatrix} \quad (25)$$

where, $D_1=\frac{Gr\Delta t}{(1+A)(1+B)}, D_2=\frac{Gc\Delta t}{(1+A)(1+E)}$

Now, for stability of the finite difference scheme, the modulus of each eigen value of the amplification matrix does not exceed unity. Since the matrix Equation (25) is triangular, the eigen values are its diagonal elements. The eigen values of the amplification matrix are $(1-A)/(1+A)$, $(1-B)/(1+B)$ and $(1-E)/(1+E)$. Assuming that, U is everywhere non-negative and V is everywhere non-positive, we get

$$A=2a\sin^2\left(\frac{\alpha\Delta X}{2}\right)+2c\sin^2\left(\frac{\beta\Delta Y}{2}\right)+i(a\sin\alpha\Delta X-b\sin\beta\Delta Y)$$

Where,

$$a=\frac{U}{2}\frac{\Delta t}{\Delta X}, b=\frac{|V|}{2}\frac{\Delta t}{\Delta Y}, c=\frac{\Delta t}{(\Delta Y)^2}$$

Since the real part of A is greater than or equal to zero, $|(1-A)/(1+A)|\le 1$ always. Similarly, $|(1-B)/(1+B)|\le 1$ and $|(1-E)/(1+E)|\le 1$. Hence the finite difference scheme is unconditionally stable. The local truncation error is $O(\Delta t^2+\Delta Y^2+\Delta X)$ and it tends to zero as $\Delta t, \Delta X$ and ΔY tend to zero. Hence the scheme is compatible. Stability and compatibility ensures convergence.

RESULTS AND DISCUSSION

Representative numerical results for the uniform heat and mass diffusion in the presence of radiation and chemical reaction will be discussed in this section. In order to ascertain the accuracy of the numerical results, the present study is compared with the available exact solution in the literature. The velocity profiles for $K=0.2$, $Sc=0.16,0.2$, $Gr=2,5$, $Gc=2,5$ and $Pr=0.71$ (corresponding to $\eta=Y/2\sqrt{t}$) are compared with the available exact solution of Das et al (1994) at $t=0.2$ in figure 1 and they are found to be in good agreement. It is observed that the present results are in good agreement with the available theoretical solution at lower time level.

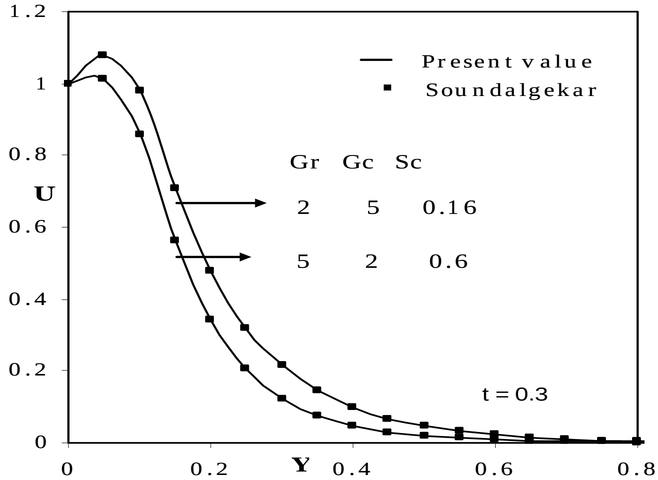

Figure 1. Comparison of velocity profiles

The effect of steady-state velocity profiles for different radiation parameters ($R=0,2,5$), $Gr=5, Gc=5, K=0.2, Pr=0.71$ and $Sc=0.6$ are shown in figure 2. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation. However, the time required for the velocity to reach steady-state depends upon the radiation parameter.

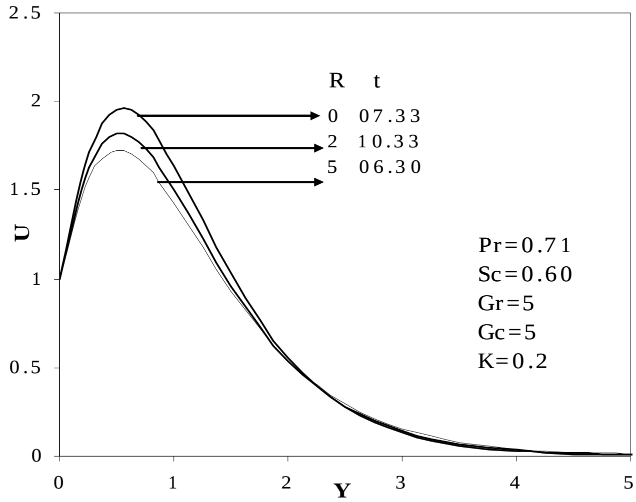

Figure 2. Velocity profiles for different values of R

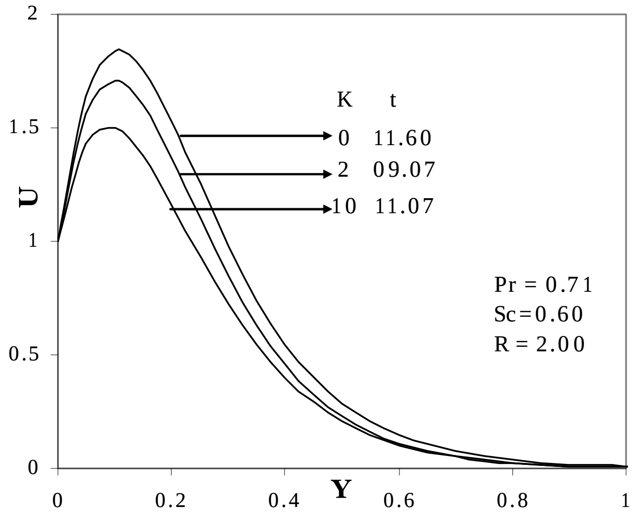

Figure 3. Velocity profiles for different values of K

The effect of velocity for different chemical reaction parameter $(K = 0,2,10)$, $Gr = 5, Gc = 5, R = 2, Pr = 0.71$ *and* $Sc = 0.6$ *are shown in figure 3. It is observed that the velocity increases with decreasing chemical reaction parameter.*

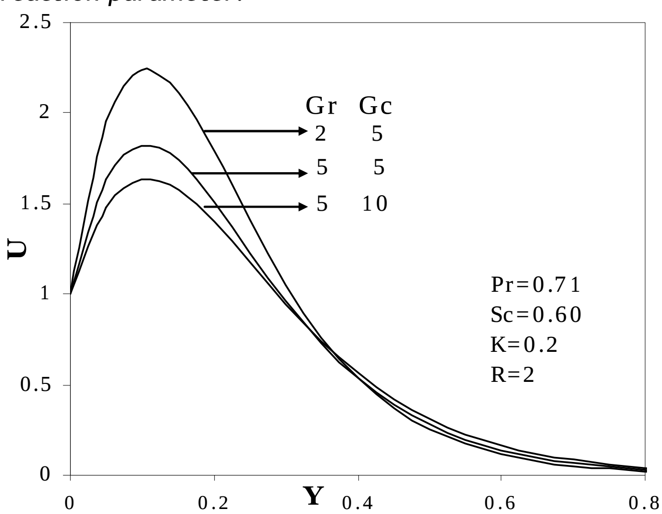

Figure 4. Velocity profiles for different values Gr, Gc

In figure 4, the velocity profiles for different thermal Grashof number$(Gr = 2,5)$, *mass Grashof number* $(Gc = 5,10)$, $Sc = 0.6$, $R = 2$, $K = 0.2$ *and* $Pr = 0.71$ *are shown graphically. This shows that the velocity increases with increasing thermal Grashof number or mass Grashof number. As thermal Grashof number or mass Grashof number increases, the buoyancy effect becomes more significant, as expected, it implies that, more fluid is entrained from the free stream due to the strong buoyancy effects as* Gr *or* Gc *increases.*

The transient temperature profiles for different values of the thermal radiation parameter $(R = 0,2,5)$, $Gr = Gc = 5$ *and* $K = 0.2$ *are shown in figure 5. It is observed that the temperature increases with decreasing* R. *This shows that the buoyancy effect on the temperature distribution is very significant in air*$(Pr = 0.71)$. *It is known that the radiation parameter and Prandtl number plays an important role in flow phenomena because, it is a measure of the relative magnitude of viscous boundary layer thickness to the thermal boundary layer thickness.*

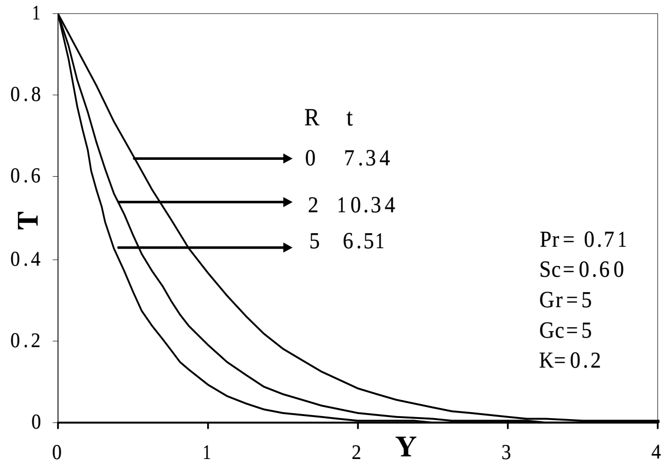

Figure 5. Temperature profiles for different values of R

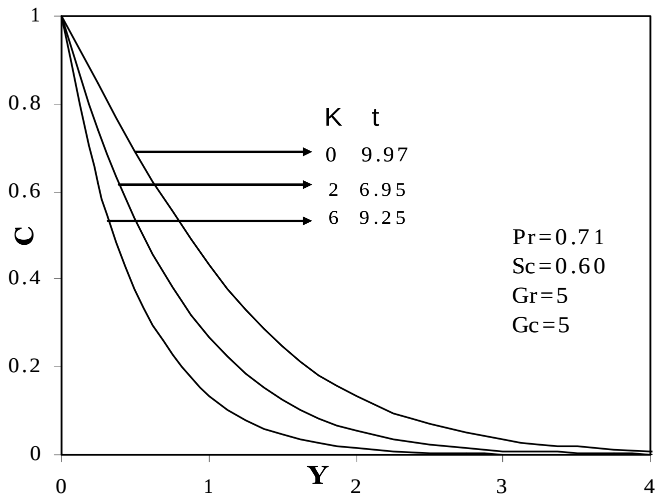

Figure 6. Concentration profiles for different values of K

The steady-state concentration profiles for different chemical reaction parameter $(K = 0,2,6), Gr = Gc = 5$ *and* $Sc = 0.6$ *are shown in figure 6. The effect of chemical reaction parameter play an important role in concentration field. There is a fall in concentration due to increasing the values of the chemical reaction parameter.*

Knowing the velocity and temperature field, it is customary to study the skin-friction and the Nusselt number. The local as well as average values of skin-friction and Nusselt number in dimensionless form are as follows:

$$\tau_X = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{28}$$

$$\bar{\tau} = -\int_0^1 \left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX \tag{29}$$

$$Nu_X = -X\left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right] \tag{30}$$

$$\overline{Nu} = -\int_0^1 \left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right] dX \tag{31}$$

$$Sh_X = -X\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{32}$$

$$\overline{Sh} = -\int_0^1 \left(\frac{\partial C}{\partial Y}\right)_{Y=0} dX \tag{33}$$

The derivatives involved in the Equations (28) to (33) are evaluated using five-point approximation formula and then the integrals are evaluated using Newton-Cotes closed integration formula.

The Local values of the skin-friction, Nusselt number and Sherwood number for fixed parameters $Gr = Gc = 5, Pr = 0.71, R = 2, K = 0.2$ *and* $Sc = 0.6$ *are plotted in figures 7, 8 and 9 respectively. Local skin-friction values are evaluated from equation (28) and plotted in figure 7 as a function of the axial coordinate. The local wall shear stress increases with increasing chemical reaction parameter or chemical reaction parameter. It is clear that there is a fall in local skin-friction with decreasing radiation parameter. Such effect is predominant with respect to radiation parameter than for chemical reaction parameter. This is because, the velocity is affected more by* R *than by* K. *The value of the skin-friction becomes negative, which implies, that after some time there occurs a reverse type of flow near the moving plate. Physically this is also true as the motion of the fluid is due to plate moving in the vertical direction against the gravitational field. The rate of heat transfer increases with decreasing values of the radiation parameter.*

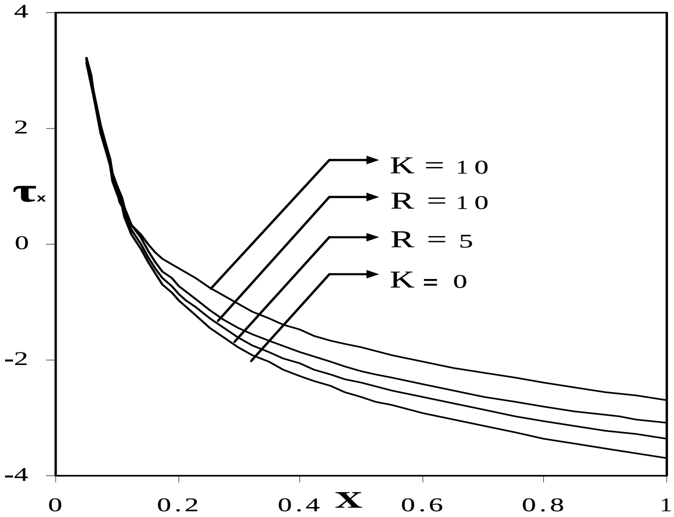

Figure 7. Local skin-friction

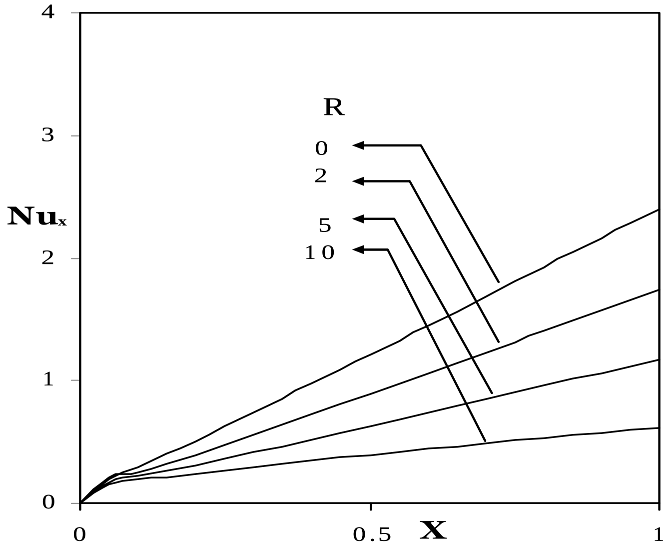

Figure 8. Local Nusselt number

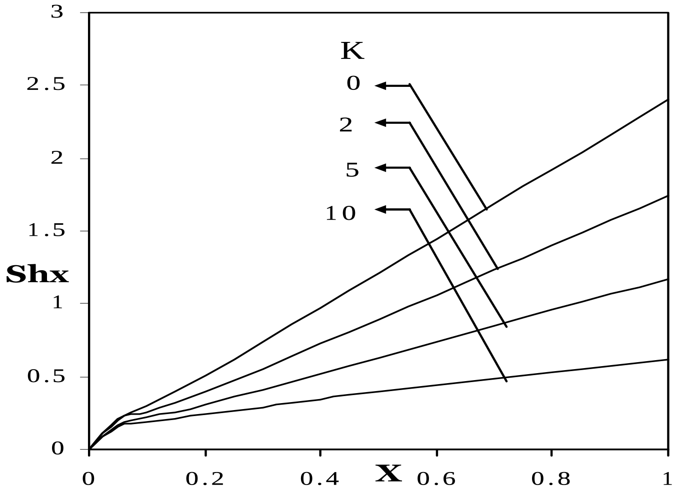

Figure 9. Local Sherwood number

The Local Sherwood number for different values of the chemical reaction parameter are shown in figure 9. The trend shows that the rate of concentration decreases the presence of chemical reaction than its absence.

The average values of the skin-friction, Nusselt number and Sherwood number are shown in figures 10, 11 and 12 respectively. The effects of the radiation parameter on the average values of the skin-friction are shown in figure 10. The average skin-friction decreases with decreasing radiation parameter. The average Nusselt number increases with increasing radiation parameter. The average Sherwood number increases with increasing values of the chemical reaction parameter.

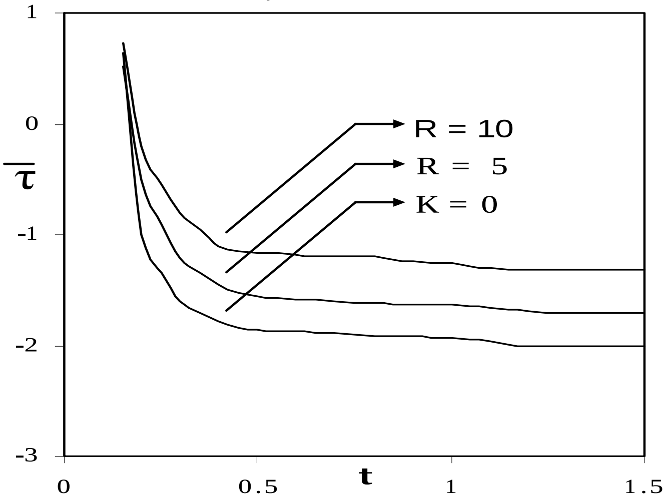

Figure 10. Average skin-friction

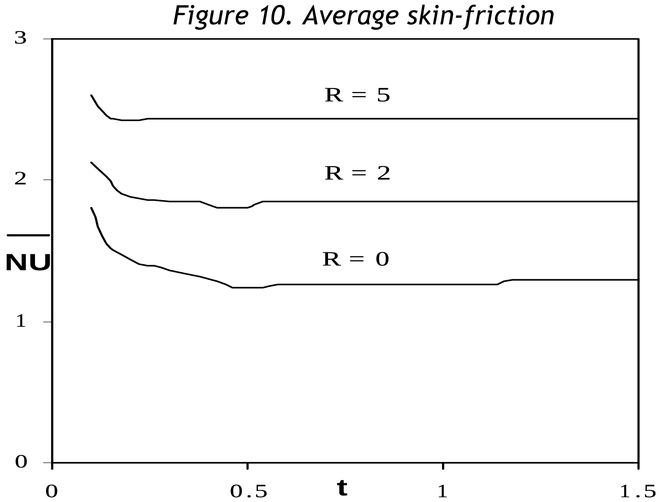

Figure 11. Average Nusselt number

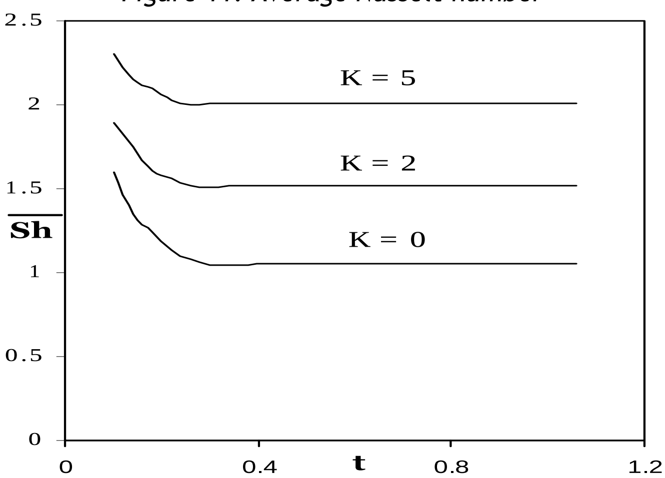

Figure 12. Average Sherwood number

CONCLUSION

Finite difference study has been carried out for unsteady thermal radiation effects on flow past an impulsively started semi-infinite vertical plate in the presence of chemical reaction of first order. The dimensionless governing equations are solved by an implicit scheme of Crank-Nicolson type. A comparison between the present numerical results and available theoretical solution in the presence of chemical reaction are also made. The agreement between the two is found to be very good. The effect of velocity, temperature and temperature for different parameter are studied. The local as well as average skin-friction and Nusselt number are shown graphically. It is observed that the contribution of mass diffusion to the buoyancy force increases the maximum velocity significantly. It is also observed that the velocity decreases in the presence of thermal radiation or chemical reaction. The study shows that the number of time steps to reach steady-state depends strongly on the radiation parameter and chemical reaction parameter.

NOMENCLATURE

a^* absorption coefficient
C' species concentration in the fluid
C dimensionless concentration
D mass diffusion coefficient
g acceleration due to gravity
Gr thermal Grashof number
Gc mass Grashof number
K thermal conductivity of the fluid
K_l chemical reaction parameter
Nu_X dimensionless local Nusselt number
$\overline{Nu}$ dimensionless average Nusselt number
Pr Prandtl number
R radiation parameter
Sc Schmit number
Sh_X dimensionless local Sherwood number
$\overline{Sh}$ dimensionless average Sherwood number
T' temperature
T dimensionless temperature
t' time
t dimensionless time
u_0 velocity of the plate
u,v velocity components of the fluid in X,Y-directions respectively
U,V dimensionless velocity components in X,Y-directions respectively
x spatial coordinate along the plate
X dimensionless spatial coordinate along the plate
y spatial coordinate normal to the plate
Y dimensionless spatial coordinate normal to the plate
Greek symbols
α thermal diffusivity
β coefficient of volume expansion
β^* volumetric coefficient of expansion with concentration
μ coefficient of viscosity

υ kinematic viscosity
τ_x dimensionless local skin-friction
$\overline{\tau}$ dimensionless average skin-friction
Subscripts
w conditions at the wall
∞ conditions in the free stream

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