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OPTIMAL DESIGN OF SEISMICALLY LOADED VESSELS

ABSTRACT:
This paper presents results of optimal design of seismically loaded thin shelled liquid containing cylindrical vessels. Three support structures are bearing plate anchored to foundations, columns and cylindrical skirt. The goal is maximisation of customer satisfaction on the structure. The goal is defined as product of fuzzy satisfaction functions for decision variables like cost and limit states like buckling and overload. Discrete design variables are used. The FE method and standards are used to verify the optimum design. The results agree satisfactorily.

KEYWORDS:
Seismic engineering applications, Steel structures, FEM calculations, Fluid structure interaction, Fuzzy design

INTRODUCTION

Background for this study is global need to utilise safely liquid containing vessels under seismic loading. A preliminary optimal design of the interconnected vessel equipments concepts is needed before detailed design.

Seismic loading excitation excites interaction between the ground, supports, shells and the inner fluid and also the neighbouring connected industrial large structures. In optimal design these have to be considered simultaneously with all interactions. First the earthquake causes overturning moments and base shear which cause bending and direct shear stresses at the vessel shells. Next seismic actions cause sloshing and tilting of the liquid level which increase the hydrostatic pressure and thus the hoop stresses.

Standards present many approaches which need to be utilised to get finalised acceptable designs. One is Nch 2369 Of.2003-API 650 2008 [1] for mechanically anchored Liquid tanks. Rules for buckling resistant designs are considered in [2] by ECCS Technical Committee 8. Structural stability and buckling of steel shells European Recommendations, 1988, NO 56. Theory and analysis of plates is considered by Szilard [3]. Steel structure design is considered in [4] Stahlbau handbuch and by Case et.al [5]. The theory of pressure vessels is considered by Harvey [6]. Malhotra, Wenk, and Wieland, [7] have proposed a simplified procedure for seismic analysis of liquid storage tanks. Malhotra [8] has studied seismic strengthening of liquid storage tanks with energy dissipating anchors.

Basic general fluid mechanics theory is discussed by White [9]. Martikka and Pöllänen have applied multi-objective optimization using customer satisfaction goal formulation with fuzzy models in [10] and in [11].

The purpose of this study is to present results of application of this methodology of fuzzy optimisation with FEM verification to designing of seismically loaded liquid storage vessel.

DESCRIPTION OF THE VESSEL MODELS.

Common support models

Liquid storage vessels are generally cylindrical. Common supports options are ground support with no skirt, elevated support with shell skirt and elevated support with columns. These are shown in Fig.1

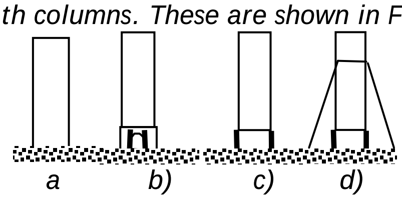


Figure 1. Support options. a) Ground support with no skirt. b) Elevated support with shell skirt. c) Support with columns. d) Cable stiffening

Basic dynamic behaviour of skirt and column supported models

Main features are described in the sketch of Fig. 2. The ground support model is considered later.

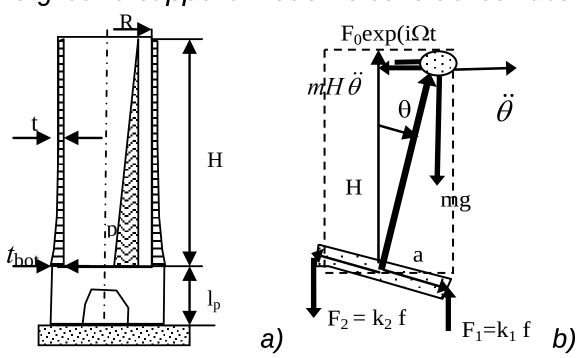


Figure 2. Shell skirt modelling. a) Sketch. b) A 2D two spring one lumped mass dynamical model.

A two spring one mass and stiff frame model, Fig.2 and Lagrange's dynamics are used to get an approximate lowest eigenfrequency. Vessel mass can be lumped to its centre of gravity. The Lagrange's function L is difference of the kinetic energy T of the mass and the potential energy V.

$$L = T - V$$
$$T = \frac{1}{2} m v^2 \Rightarrow v = H\dot{\theta} \Rightarrow T = \frac{1}{2} m H^2 \dot{\theta}^2 \qquad (1)$$
$$V \approx mgH \tfrac{1}{2}\theta^2 + kR^2\theta^2$$

The equation of motion for the one dof angular displacement is obtained with

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}}\right] - \frac{\partial L}{\partial \theta} = F_\theta \qquad (2)$$

The simplified equation of motion is

$$\ddot{\theta} + \left(\frac{g}{H} + \frac{2k}{m}\left(\frac{R}{H}\right)^2\right)\theta = F_\theta \qquad (3)$$

The solution is sum of homogeneous and particular solutions. Lowest eigenfrequency and eigenperiod T for a bearing plate anchored to ground having two springs 2k one mass m model is obtained as

$$\omega = \left(\frac{g}{H} + \frac{2k}{m}\left(\frac{R}{H}\right)^2\right)^{\frac{1}{2}} \quad , \quad T = \frac{2\pi}{\omega} \qquad (4)$$

HORIZONTAL ELASTIC SEISMIC RESPONSE SPECTRUM

For the horizontal components of the seismic action, the elastic response spectrum S_e (T) means spectral acceleration S_A. It is defined by standards EN 1998-1:20004(E) and EN 1009-1:2004(E) [2] by the following four discrete expressions

Curve 1

$$0 \le T \le T_B, \quad S_e(T) = a_g \cdot S\left[1 + \frac{T}{T_B}(\eta \cdot 2.5 - 1)\right] \qquad (5)$$

Curve 2

$$T_B \le T \le T_C, S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5 \qquad (6)$$

Curve 3

$$T_C \le T \le T_D, \quad S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5\left[\frac{T_C}{T}\right] \qquad (7)$$

Curve 4

$$T_D \le T \le 4s, \quad S_e(T) = a_g \cdot S \cdot \eta \cdot 2.5\left[\frac{T_C T_D}{T^2}\right] \qquad (8)$$

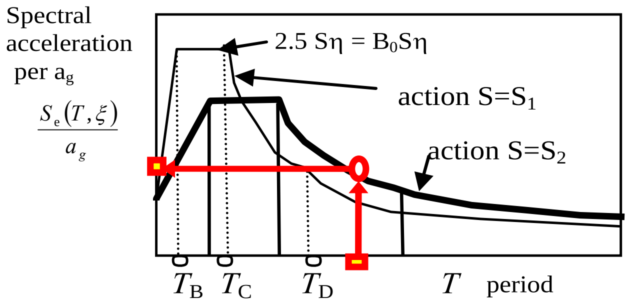


Figure 3. Spectral horizontal acceleration per ground acceleration vs.period of eigenvibration of the structure

Table 1. Some typical earthquake data values, η, damping correction factor, with reference value of 1 for 5% viscous damping.

	Seismic action j=1	Seismic action j= 2
S (j)	1	1.1
$B_0(j)$	2.5	2.3
$k_1(j)$	1	1
$k_2(j)$	2	2
$T_B(j)$	0.12	0.25
$T_C(j)$	0.35	0.9
$T_D(j)$	2.4	3
a_g (j)	2.7	1.6
η(j),	1	1

Here T= vibration period of a linear single -degree-of freedom system, a_g is design ground acceleration: Now the chosen ground type is type A (hard rock v_s > 800m/s). The soil factor S depends on the hardness of the ground.

For hard grounds (A) S =1 and for soft ground (E) S = 1.4, η is the damping correction factor with a reference value of η = 1 for 5% viscous damping

ξ is the viscous damping ratio of the structure expressed in percentages

$$\eta = \sqrt{\frac{10}{5 + \xi[\%]}} \qquad (9)$$

Typical values are

$$if\, \xi \approx 0 \Rightarrow \eta = \sqrt{\frac{10}{5+0}} = 1.4,.if\, \xi \approx 5\% \Rightarrow \eta = 1$$

SEISMIC LOAD ON A LIQUID FILLED TANK

Seismic loads and responses of liquid filled vessel are complex tasks to analyse. Thus a simple to use and also a reasonable accurate model is needed. The common method is to separate the fluid into functionally different fictive parts, convective mass on top of impulsive mass as shown in Fig.4.

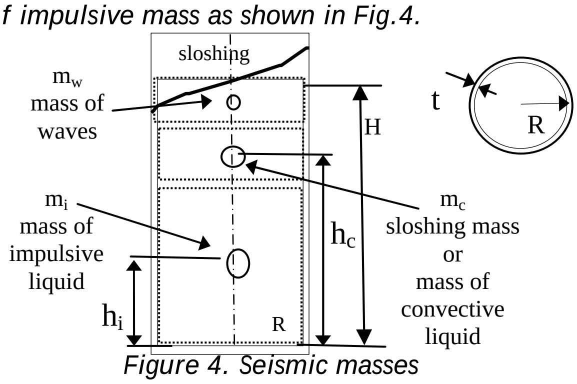


Figure 4. Seismic masses

Mass model curves are shown in Fig. 5 based on data by Malhotra et al [7].

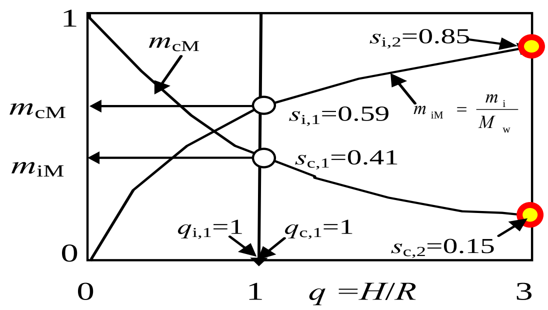


Figure 5. Mass vs. aspect ratio q curves

Model can be fitted to the experimental curves

$$m_{\mathrm{iM}} = s_{\mathrm{i1}} \cdot q^{ei} = \frac{m_{\mathrm{i}}}{M_{\mathrm{w}}}$$
$$m_{\mathrm{cM}} = s_{\mathrm{c1}} \cdot q^{ec} = \frac{m_{\mathrm{c}}}{M_{\mathrm{w}}}, q = \frac{H}{R} \quad (10)$$

Base shear V is sum of impulsive and convective components. The simplified structure has two masses and two eigenperiods, T_c for convective vibration and T_i for impulsive vibration

$$V = V_{\mathrm{i}}(T_{\mathrm{i}}, \xi_{\mathrm{i}}) + V_{\mathrm{c}}(T_{\mathrm{c}}, \xi_{\mathrm{c}})$$
$$V = (m_{\mathrm{i}} + m') \cdot (\ddot{u}_{\mathrm{i}} + \ddot{z}) + m_{\mathrm{c}} \cdot (\ddot{u}_{\mathrm{c}} + \ddot{z}) \quad (11)$$

here m' is equipment mass, $\ddot{u}$ is mass centre acceleration and $\ddot{z}$ is earthquake acceleration. This may be written as

$$V = (m_{\mathrm{i}} + m') \cdot S_{\mathrm{e}}(T_{\mathrm{i}}, \xi_{\mathrm{i}}) + m_{\mathrm{c}} \cdot S_{\mathrm{e}}(T_{\mathrm{c}}, \xi_{\mathrm{c}}) \quad (12)$$

The overturning moment above the base plate at x=0

$$M = \Sigma\left[S_{\mathrm{a}}(T_{\mathrm{i}}, \xi_{\mathrm{i}}) \bullet m_{\mathrm{i}} h_{\mathrm{i}}\right] = M_{\mathrm{b}} \quad (13)$$

First this is expanded and next in calculations the equipment masses are neglected for simplicity.

$$M = (m_{\mathrm{i}} h_{\mathrm{i}} + m_{\mathrm{wall}} h_{\mathrm{wall}} + m_{\mathrm{roof}} h_{\mathrm{roof}}) S_{\mathrm{e}}(T_{\mathrm{i}}, \xi_{\mathrm{i}}) + $$
$$+ m_{\mathrm{c}} h_{\mathrm{c}} S_{\mathrm{e}}(T_{\mathrm{c}}, \xi_{\mathrm{c}}) \quad (14)$$

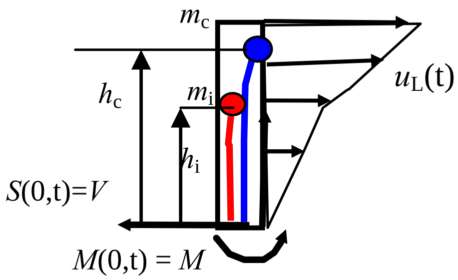


Figure 6. Base shear and overturning moment load on a vessel by seismic action

OPTIMUM DESIGN. MATERIAL DESIGN VARIABLE OPTIONS

The final success of engineering tasks is determined by the magnitude of customer's satisfaction on the delivered result. First condition of a success is optimal definition of goals and constraints. Second condition is choice of method. At the concept stage the essential design variables are few, discrete and their relationships are highly non-linear. Thus a fast enough search method is exhaustive learning search of optimum. Third condition is that all reasonable concepts are analysed and ranked in order of total satisfaction.

Options are shown in Table 2.One may also choose to use ecological merit and corrosion resistance as decision variables etc.

Table 2. Material design variables.Stress (MPa), cost (kg/m^3), Elastic modulus (MPa),material cost (eur/kg).

material code	M$(1) = " Al "	M$(2) = " St "
allowed stress	$\sigma_{all}(1) = 100$	$\sigma_{all}(2) = 150$
material cost	cm(1) = 50	cm(2) = 20
density kg/m^3)	$\rho(1) = 4000$	$\rho(2) = 8000$
Elastic modulus	E(1) = 60000	E(2) = 200000
ecological merit	eco(1) = .1	eco(2) = .7
Corr. resistance	corres(1) = .8	corres(2) = .15

UNIFIED FUZZY GOAL AND CONSTRAINT FORMULATION

Now all goals and constraints are formulated consistently by one flexible fuzzy function, as in [10], [11]. This is illustrated in Figs.7 and 8 and Table 3. These functions depend on decision variables chose as most important for the customer, like safety factors, reliability cost etc. The customers and designers can together define the most satisfactory ranges and also left or right bias.

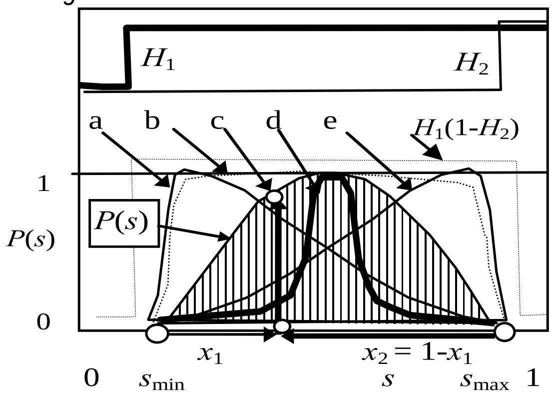


Figure 7. Definition of a typical fuzzy satisfaction function

In the design algorithm the satisfaction function is defined for each decision variable s by inputting the left and right limits and two bias parameters p. The left skewed option a is useful to get low cost designs. Flattening the shape increases indifference of choices of s. The call CALL pzz(s_{min}, s_{max}, p_1, p_2, s, P(s)) gives as output the satisfaction function P(s) which varies in the range 0…1. The decision variables s are changed to an internal dimensionless variable x_1

$$x_1 = \frac{s - s_{\mathrm{min}}}{s_{\mathrm{max}} - s_{\mathrm{min}}} \Rightarrow x_2 = 1 - x_1 \quad (15)$$

The satisfaction function depends on one variable x_1

$$P(x_1) = (p_1 + p_2)^{p1+p2} \left(\frac{x_1}{p_1}\right)^{p1} \left(\frac{1-x_1}{p_2}\right)^{p2} H_{12} \quad (16)$$

Here

$$H_{12} = H_1(s)(1 - H_2(s)) \quad (17)$$

Two step functions are used to define the inner desired range of the decision variable

$$H_1(s) = \tfrac{1}{2}\left[1 + \mathrm{sgn}(s - s_{\mathrm{min}})\right], H_2(s) = \tfrac{1}{2}\left[1 + \mathrm{sgn}(s - s_{\mathrm{max}})\right] \quad (18)$$

Outside of the desired range a small non-zero seed value is added to the satisfaction function to promote search drive for improvement.

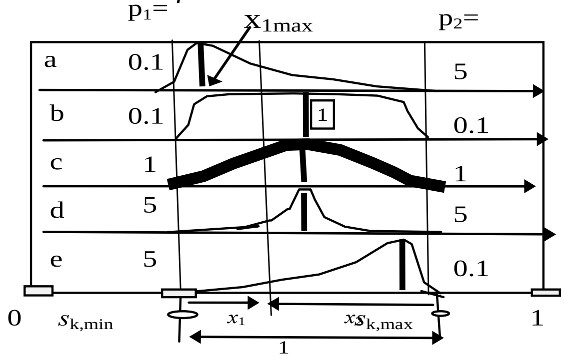


Figure 8. Satisfaction function examples

Table 3: Skewness parameter values.

	a	b	c	d	e
p_1	0.1	0.1	1	5	5
p_2	5	0.1	1	5	0.1
x_{1max}	0.02	0.5	0.5	0.5	0.98

The total design event G is junction of sub design events which are functions of decision variables

$$G(s) = G(s_1) \text{ and } G(s_2) \ldots \text{ and } G(s_n) \tag{19}$$

The design goal is to maximise the product

$$P(G(s)) \Rightarrow P(s) = P(s_1) \cdot P(s_2) \cdot \ldots\ldots \cdot P(s_n) \tag{20}$$

Here s_k is decision variable and $P(s_k)$ is satisfaction on it. The desired range for s_k is $R(s_k) = s_{kmin} < s_k < s_{kmax}$

ALGORITHM FOR OPTIMISATION

In engineering optimisation at concept stage most tasks are highly non-linear and also the design variables are few and discrete. For this reason, the exhaustive or learning enhanced search methods are deemed to be satisfactory. User can preselect the material from the list of available selections or leave it as one more design variable to the search algorithm to determine. Total satisfaction is first initialised to a low value

P_{gbest} = .0000001,
FOR ir = 1 TO N ' Radius R(ir)
FOR itt = 1 to N ' t(itt) wall thickness
FOR iH = 1 to N , H(iH) height of vessel
FOR itbot 1 to N , t_{bot}(itbot) wall thickness at bottom of the shell

Design variables for columns are preselected within feasible ranges

FOR irp = 1 to Nirp , r_p= r_p(irp) is column radius
FOR itp = 1 to Nitp, t_p = t_p(itp) is column wall thickness
FOR ilp = 1 to NIlp , l_p = l_p(ilp) is height of column

Each k = 1,2..13 decision variable s is calculated. The its range and bias pair p_1 and p_2 are given as inputs to get the satisfaction function P(s) by a call CALL pzz(s_{min}, s_{max}, p_1, p_2, s, P(s)).

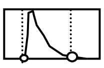
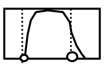
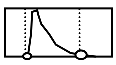
The total satisfaction is product of partial satisfactions.

P_s = 1 , the initialisation first, before the loop
FOR i = 1 TO N
Ps = Ps * Ps(i)
NEXT i
P_g = P_s
IF P_g > P_{gbest} THEN
' new optimum is better than previous
ELSE search is continued. END IF
NEXT indices

DECISION VARIABLES

An illustration of the use of decision variables is shown in Table 4.

Table 4. Typical definitions of the decision variable desired range limit s_{min}, s_{max} and biases p_1 and p_2

s_k decision variable	s_7= N Factor of safety	s_3=V Useful volume	s_5=M Cost of material
s_{min} s_{max}	1, 7	1e-5 0.002	$0.1K_{max}$ $2K_{max}$
p_1, p_2	0.1 , 5	1,1	0.1, 4
satisfaction function curves	 s_{min}, s_{max}	 s_{min}, s_{max}	 s_{min}, s_{max}

One design goal is to shift the impulsive and the convection mass eigen periods away from the large seismic acceleration period range. For seismic action S(1) the choice the main parameters are $T_B = .12$, $T_C = .35$, $T_D = 2.4$, $a_g = 2.7$.

Now ground acceleration is chosen conservatively rather high, $a_g = 3$.

The damping coefficients z are $z = z_i = 0.02$ for impulsive and $z = z_c = 0.05$ for convective motion.

Decision variable s_1= T_{imp} or impulsive mass period

$$s_1 = T_{imp}, \quad P_s(1) = P(s_1) \tag{21}$$

The aim is to constrain this into the safe range

$$\begin{aligned} &Range: s_{\min} = 0.06, \quad s_{\max} = 0.48, \\ &Biases: p_1 = 0.1, p_2 = 0.1 \end{aligned} \tag{22}$$

According to Malhotra [2]

$$\begin{aligned} &T_{\text{imp}} = C_{\text{i}} \frac{H}{\sqrt{\frac{t}{R}}} \sqrt{\frac{\rho}{E}} \\ &C_{\text{i}} = 7 \pm 1, \frac{H}{R} = 0.3....,3 \end{aligned} \tag{23}$$

The eigenvalues are

$$\omega_{\text{i}} = \frac{2\pi}{T_{\text{imp}}}, k_{\text{i}} = \omega^2 m_{\text{i}}, c_{\text{i}} = 2\xi_{\text{i}} \omega_{\text{i}} m_{\text{i}}, \tag{24}$$

Then spectral acceleration corresponding to this T=T_{imp} is calculated by CALL Se(T, z, SeT) giving as output SeTi(iv) = SeT

Decision variable s_{10}= SeTi or spectral acceleration at impulsive mass eigenperiod T_{imp}

$$s_{10} = SeTi, \quad P_s(10) = P(s_{10}) \tag{25}$$

Small value is desired and range is biased to the left

$$\begin{aligned} &Range: s_{\min} = 1, \quad s_{\max} = 20, \\ &Biases: p_1 = 0.1, p_2 = 2 \end{aligned} \tag{26}$$

Decision variable s_2 = T_{conv} or convective mass eigenperiod

Small value is desired

$$s_2 = T_{conv}, \quad P_s(2) = P(s_2) \tag{27}$$

$$\begin{aligned} &Range: s_{\min} = 0.35, \quad s_{\max} = 7, \\ &Biases: p_1 = 2, p_2 = 0.1 \end{aligned} \tag{28}$$

Malhotra [2] gives the simple model

$$\begin{aligned} &T_{\text{conv}} = C_{\text{c}} \sqrt{R} \\ &C_{\text{c}} = 1.5, \frac{H}{R} = 0.3....3 \end{aligned} \tag{29}$$

Eigenvalues are

$$\omega_{\text{c}} = \frac{2\pi}{T_{\text{conv}}}, k_{\text{c}} = \omega_{\text{c}}^{\;2} m_{\text{c}}, c_{\text{c}} = 2\xi_{\text{c}} \omega_{\text{c}} m_{\text{c}} \tag{30}$$

The spectral acceleration corresponding to T= T_{conv} is calculated by CALL Se(T, z, SeT) giving as output SeTc() = SeT

Decision variable s_{11}= SeTc or spectral acceleration at convective mass eigenperiod T_{conv}

$$s_{11} = SeTc, \quad P_s(11) = P(s_{11}) \tag{31}$$

Range and bias are

$$\begin{aligned} &Range: s_{\min} = 1, \quad s_{\max} = 10, \\ &Biases: p_1 = 0.1, p_2 = 4 \end{aligned} \tag{32}$$

Decision variable s_3= V or useful volume
Large volume is now desired

$$s_3 = V = \pi R\left(ir\right)^2 H\left(ih\right),, P_s\left(3\right) = P\left(s_3\right) \quad (33)$$

Range and bias are

$$\begin{aligned} &Range: s_{\min} = 1000, \quad s_{\max} = 6000, \\ &Biases: p_1 = 5, p_2 = 0.1 \end{aligned} \quad (34)$$

Decision variable s_4 = Mat(im) or mass of shell material of class im
Small mass of material class im is desired

$$\begin{aligned} &s_4 = Mat(im) = \rho(im)V_m = \rho(im)2\pi Rt(R+H) \\ &P_s(4) = P(s_4) \end{aligned} \quad (35)$$

$$\begin{aligned} &M_{\max} = 1\cdot 10^5 \\ &Range: s_{\min} = 0.1M_{\max}, \quad s_{\max} = 1M_{\max} \\ &Biases: p_1 = 0.1, p_2 = 2 \end{aligned} \quad (36)$$

Decision variable s_5 = Cost or cost of materials and construction
Low cost is now desired

$$\begin{aligned} &Cost = c_m(im)\cdot Mat(im) \\ &s_5 = Cost = K \quad , P_s(5) = P(s_5) \\ &im = 2, steel \end{aligned} \quad (37)$$

$$\begin{aligned} &K_{\max} = 1\cdot 10^6 \\ &Range: s_{\min} = 0.1K_{\max}, \quad s_{\max} = 2K_{\max} \\ &Biases: p_1 = 0.1, p_2 = 2 \end{aligned} \quad (38)$$

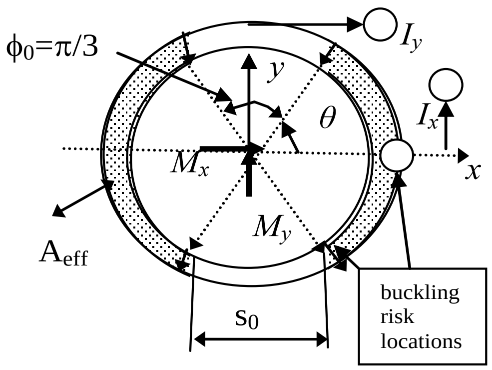


Figure 9. Skirt cross section at opening

Decision variable $s_7 = N_{skirt.cylinder}$ or skirt cylinder buckling safety factor
The buckling strength reduction factor

$$\begin{aligned} &\alpha = \alpha_0 \\ &\alpha = \alpha_0 = \frac{0.83}{\sqrt{1+0.01\frac{R}{t}}} \quad IF, \quad \frac{R}{t} < 212 \\ &\alpha = \alpha_0 = \frac{0.70}{\sqrt{0.1+0.01\frac{r}{t}}} \quad IF, \quad \frac{R}{t} > 212 \end{aligned} \quad (39)$$

The constraint becomes

$$\begin{aligned} &\sigma \le \sigma_{buckl,ideal} = 0.6\frac{Et}{R} = \sigma_{cr} \\ &\sigma_{buckl} = \alpha\sigma_{buckl,ideal} \end{aligned} \quad (40)$$

the bending stress due to overturning moment M_y= M_x = M_b is

$$\sigma_{bend} = \sigma_{Z,My} = \frac{M_y R}{I_y(\theta)} \quad (41)$$

The safety factor is of buckling endurance of the cylindrical shell parts of the skirt against both compressive loading and seismic bending loading is

$$N_{skirt.cylinder} = \frac{\sigma_{buckl}}{\sigma_z} = \frac{\alpha\sigma_{cr}}{\sigma_{z,P} + \sigma_{Z,My}} \quad (42)$$

$$s_7 = N_{skirt.cylinder} \quad , P_s(7) = P(s_7) \quad (43)$$

$$\begin{aligned} &Range: s_{\min} = 1, \quad s_{\max} = 500, \\ &Biases: p_1 = 0.1, p_2 = 1 \end{aligned} \quad (44)$$

Decision variable s8 = $N_{hoop.bot}$ or safety factor for hoop tensile stress due to sloshing at root of the main vessel
Sloshing increases fluid height by increment Z. The critical location is at bottom.

$$\begin{aligned} &\sigma_{\text{hoop}} = p\frac{R}{t_{bot}} \Rightarrow \sigma_{\text{hoop}} = \rho g(H+Z)\frac{R}{t_{bot}}, \\ &a \Rightarrow a_g = 3, g = 9.8 \\ &\sigma_{\text{hoop}} = \left(1 + \frac{R}{H}\bullet\frac{a}{g}\right)\rho gH\frac{R}{t_{bot}} = x_{ag}\sigma_{\text{hoop,bot}} \end{aligned} \quad (45)$$

The decision variable becomes

$$s_8 = N_{hoop.bot} = \frac{\sigma_{all}(steel)}{\sigma_{hoop}} \quad , P_s(8) = P(s_8) \quad (46)$$

A rather high value is desired

$$\begin{aligned} &Range: s_{\min} = 0.5, \quad s_{\max} = 8, \\ &Biases: p_1 = 2, p_2 = 0.1 \end{aligned} \quad (47)$$

Decision variable s_9 = $N_{side.top}$ or safety factor for main shell upper side buckling
The model is shown in Fig.10. This buckling risk occurs close to the top. The dynamic movement of the fluid inside the vessel is assumed to push the wall forward while causing the top sides to cave in causing compressive stresses and a buckling risk.

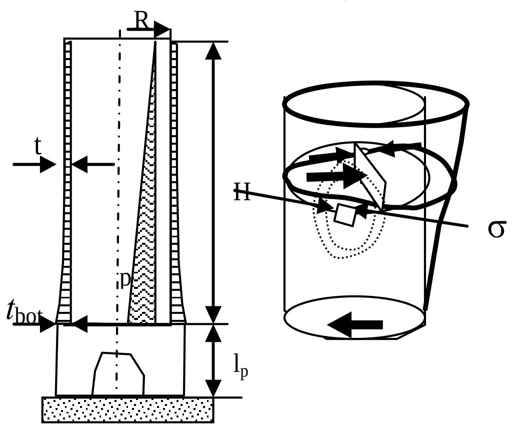


Figure 10. Shell geometry and buckling

While all other decision variable values were satisfactory this safety factor was below unity, typically only 0.03. This result predicts that some buckling probably occurs. But since this is an isolated location it is not considered as safety critical for the whole structure.

$$\begin{aligned} &A_p = D\cdot\Delta h = 2R\Delta h, \\ &\Delta h = x_{\text{H}}H = H, x_{\text{H}} = 1 \end{aligned} \quad (48)$$

Safety factor is

$$N_{side.top}=\frac{\sigma_{buckl}}{\sigma}=\frac{0.6\frac{Et}{R}}{4.8\frac{V}{4t\Delta h}\left(\frac{H}{R}\right)^2} \quad (49)$$

here V is base shear

$$V=M\cdot S_a=M2.5g$$
$$M_w=\rho V_w=\rho\pi R^2H$$
$$K=\frac{4\cdot 0.6}{4.8}\frac{x_H}{2.5\pi}=0.064, \quad (50)$$
$$p_{hydr}=\rho gH$$

Here M_w is mass of liquids H is height and p_{hydr} is hydrostatic pressure at bottom

$$s_9=N_{side.top}=K\frac{E}{p_{hydr}}\left(\frac{t}{R}\right)^2\left(\frac{R}{H}\right) \quad ,P_s(9)=P(s_9) \quad (51)$$

Now a small value is allowed due to low criticality

$$Range: s_{min}=001, \quad s_{max}=1,$$
$$Biases: p_1=0.1, p_2=1 \quad (52)$$

A stiffening ring may be used to limit the buckling amplitude. If wall thickness is about 3t then the safety factor is increased by factor of ten to a reasonable value.

Decision variable s_{12} = $N_{skirt.plate}$ is factor of safety for skirt opening sides using a plate model for sides of openings

Both direct and bending stress act on the fictive surrogate plate. The peripheral stress is small close to the edge of the opening.

For the reduced cross section the second areal moment about x axis is

$$I_x(\theta)=I_p\left[\frac{\theta}{\pi}-\frac{1}{2\pi}\sin 2\theta\right] \quad , \quad I_p=2\pi R^3t \quad (53)$$

Second area moment around y- axis is larger due to openings

$$I_y(\theta)=I_p\left[\frac{\theta}{\pi}+\frac{1}{2\pi}\sin 2\theta\right] \quad (54)$$

The compressive stress at the skirt is due to the load of the water in the vessel. The effective area is less than full area

$$\sigma_{z,P}=\frac{Mg}{A_{eff}}=\frac{\rho\pi R^2H\cdot g}{2(\pi-\phi_0)Rt}=\rho gH\frac{R}{t}\frac{1}{2\left(1-\frac{\phi_0}{\pi}\right)} \quad (55)$$

$$\sigma_{z,Mx}=\frac{M_xR}{I_x(\theta)}\le\sigma_{Z,cr}=0.53E\left(\frac{t}{b}\right)^2 \quad (56)$$
$$\sigma_{z,plate}=\sigma_{z,Mx}+\sigma_{z,P}$$

Here b is effective plate width in buckling.

Thus the decision variable is

$$s_{12}=N_{skirt.plate}=\frac{\sigma_{z,cr,plate}}{\sigma_{z,plate}}=\frac{0.53\cdot E\cdot\left(\frac{t}{b}\right)^2}{\sigma_{z,Mx}+\sigma_{z,P}}$$
$$P_s(12)=P(s_{12}) \quad (57)$$
$$\phi_0=\tfrac{1}{2}\pi, \quad ,t_{skirt}=t,$$

Now a wide range is allowed as reasonable.

$$Range: s_{min}=1, \quad s_{max}=7,$$
$$Biases: p_1=0.1, p_2=0.1 \quad (58)$$

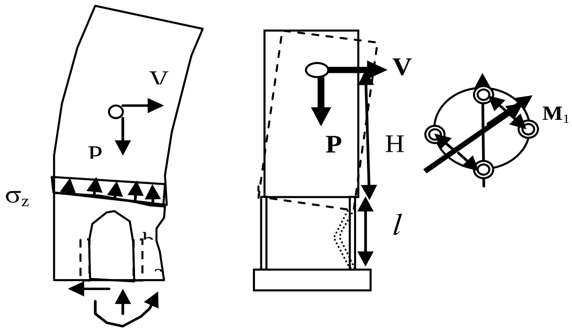


Figure 11. a) Skirt side buckling; b) Column support buckling

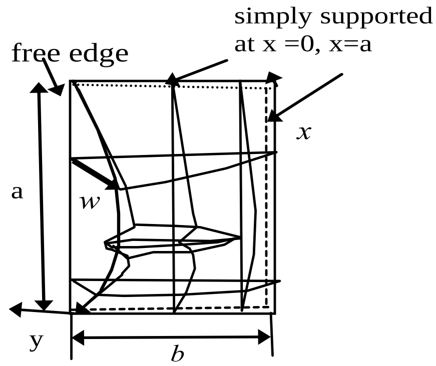


Figure 12. Plate buckling model

Decision variable s_{13} = N_{Euler} is safety factor for Euler buckling of columns

According to Case et al [5] it has been found from tests on mild-steel pin ended struts that failure of an initially curved member takes place when the yield stress is first attained in one of the extreme fibres. First the column cross section area is calculated

$$A_{col}=2\pi r_p t_p \quad (59)$$

The Euler buckling strength is calculated as stress

$$\sigma_{Euler}=E\frac{\pi^2}{2}\left(\frac{r_p}{l_p}\right)^2 \quad (60)$$

The total load due to water on the struts is P

$$P=M_wg \quad V=M_wg\cdot x_v, \quad x_v=2.5 \quad (61)$$

Here V is seismic shear stress causing bending at height H lever. The strength reduction factor is

$$\eta=0.003\left(\frac{l_p}{r_g}\right), \quad r_g=\tfrac{1}{2}r_p \quad (62)$$

The radius of gyration of thin shelled columns is r_g = $r_p/\sqrt{2}$. The total column stress is due to normal and bending stress action

$$\sigma_{tot}=\frac{F_{tot}}{A_{col}}=\frac{1}{A_{col}}\left[\frac{1}{4}P+\frac{V(H+l_p)}{x_R R}\right], x_R=2.8 \quad (63)$$

The buckling instability strength of a strut

$$A_{strut}=\tfrac{1}{2}\sigma_y+(1+\eta)\sigma_{Euler} \quad ,$$
$$B_{strut}=\left[\tfrac{1}{4}A_{strut}^{\,2}-\sigma_y\sigma_{Euler}\right]^{\frac{1}{2}} \quad (64)$$
$$\sigma_{strut}=\tfrac{1}{2}A_{strut}-B_{strut}$$

The safety factor is

$$s_{13} = N_{Euler.column} = \frac{\sigma_{strut}}{\sigma_{tot}} \quad , P_s(13) = P(s_{13}) \qquad (65)$$

Where

$$\begin{aligned} &Range: s_{\min} = 0.5, \quad s_{\max} = 5 \\ &Biases: p_1 = 2, p_2 = 0.1 \end{aligned} \qquad (66)$$

RESULTS

Using this optimisation method the design goals are formulated just as the customer wishes using fuzzy ideas. In a case study the effect of emphasising simultaneously desire for very low cost and desire for very high useful volume and maintaining satisfaction of other goals is studied. The result is a trade off between the contradictory and non-contradictory requirements. Results are shown in Table 4. As expected, the cost and volume satisfactions were both low. Other goals were however satisfactory.

Table 4. Emphasis on very low cost and on high useful volume gave satisfaction $P_G = 3.8 \cdot 10^{-7}$. Constraint s_6 is not needed and passed by setting $P(s_6)=1$

properties for optimal model	numerical values	p_1 p_2
$P(s_1)$, $s_1 = T_{imp}$, impulsive mass period	0.95, 0.0137	0.1 0.1
$P(s_2)$, $s_2=T_{conv}$, convective period	0.164, 2.6 3.1*,	2, 0.1
$P(s_3)$, s_3=V, Volume of inner fluid	0.005, 382	5, 0.1
$P(s_4)$, s_4=Mat, mass of material	0.58, 4750	0.1 2
$P(s_5)$, s_5 = cost of shell material	0.078, 95000	0.1 5
$P(s_6)$, not used	1,0.	-
$Ps(s_7)$, $s_7 = N_{skirt.cylinder}$, skirt.cyl. buckling	0.997, 4e5	0.1 0.1
$P(s_8)$, $s_8 = N_{hoop.bot}$ tension at l bottom	0.567, 5.37	2 0.1
$P(s_9)$, $s_9 = N_{side.top}$, buckling of shell	0.79, 0.032)	0.1 ,0.1
$P(s_{10})$, s_{10}= acceleration at T_{imp} period	0.343, 10.6	0.1 2
$P(s_{11})$, s_{11}=acceleration at T_{conv} period	0.993, 1.31	0.1 4
$P(s_{12})$, $s_{12} = N_{skirt.plate}$ skirt plate buckling	0.935, 1.5e4	0.1 0.1
$P(s_{13})$, $s_{13} = N_{Euler.column}$ column buckling	0.08, 1.56	2 , 0.1

* Convective mode period with FEM model T=3.1s

The main shell geometry:Radius R = 4.5 wall in upper section t =0.002, height H = 6, wall at bottom t_{bot} =0.035,

The column geometry: radius R_p=0.08, wall thickness t_p = 0.003, height l_p = 2.

Some results are discussed to show the essential features in this design case study.

The critical decision variables are those with least satisfaction. Some decision variables have high level of satisfaction over the range of design variables. This means that they are not sensitive to changes and thus need no closer attention. The impulsive acceleration is over ten and the convective is somewhat over one. Thus it is much more critical.

Decision variable $s_9 = N_{side.top}$ or safety factor for buckling of main shell was small. The consequences to the overall structure can be determined by FEM. But by adding a stiffening ring the buckling factor of safety can be increased.

The satisfaction on $s_7 = N_{skirt.cylinder}$ or the skirt cylinder buckling safety factor was not high enough. Thus some strengthening is needed.

The satisfaction on $s_{12} = N_{skirt.plate}$ or skirt surrogate plate buckling at the sides of the opening was too small. Thus some strengthening is also needed.

The satisfaction on $s_8 = N_{hoop.bot}$ or safety factor on the hoop tensile stress at shell bottom was over five but according the set satisfaction function it was not high enough. Justification for desiring high safety factor was that the bottom shell is a safety critical area of a large vessel. However, this shows that some rational fine-tuning of desire levels is needed

Comparison of the skirt and column support choices. The simplified dynamical model of Fig.1 has one lumped mass and two effective springs

A. Skirt supported model. Eigenfrequency period is small T_{skirt} = 0.0026 s and damping z = 0.02 give the spectral acceleration is Se_{Tskirt} = 3.17

B. Column supported model. Eigenfrequency period is now long T_{column} = 0.036 s and damping z=0.02 gives for the spectral acceleration $Se_{Tcolumn}$ = 5.26.

Thus there is not very great difference in Se values and selection between them may be made using other criteria.

Simple dynamical model showed that the skirt supported structure is somewhat more satisfactory than the column supported model. The main reason is that the stiffness of the skirt support is high giving short eigenperiod and thus it generates a relatively small spectral acceleration. But the stiffness of column support is low causing long eigenperiod and spectral acceleration which is higher than for the cylindrical skirt. For both support types the safety factor against buckling is only about unity. This shows that more stiffening is needed.

FEM MODEL RESULTS

The main geometry of the FEM model is shown in Fig.13. Radius R = 4.5and height G = 6m are the same as obtained by optimum design. But now the advantage of FEM was used to choose different wall thickness which us structurally and also optimal to manufacture

Layer 1, z= 0…1m, wall is t=0.025.

Layer 2 ,z = 1…3m, wall is t=0.010.

Layer 3, z= 3…6m, wall is t=0.004.

Accurate convective mode period was obtained by standard (1) as T_2 = 3.1. The approximate model gave less 2.6. This accurate standard modelling gave the impulsive pressure on the projection area between heights z =0 to z=4.5 m and convective equivalent pressure extends from z= 6-4.116 to 6.

This means that they overlap. This pressure distribution is transferred to FEM model.

The deformation result is shown in Fig. 14.

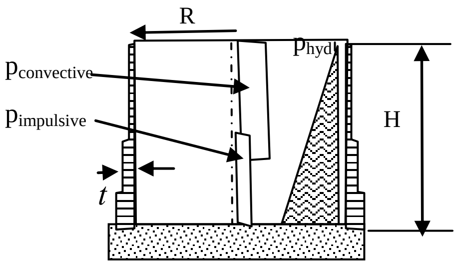


Figure 13. FEM model dimensions

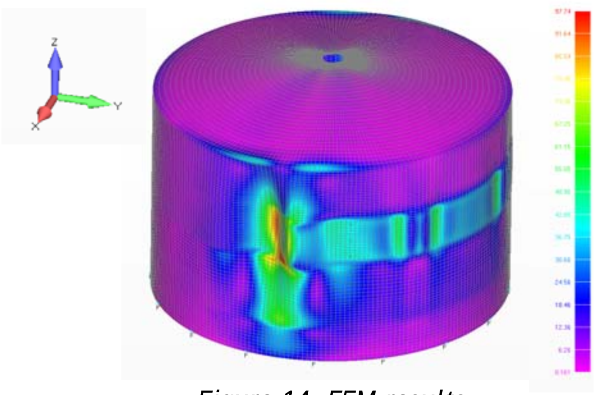


Figure 14. FEM results

By comparing the FEMin Fig. 14 it seems that the result resembles the prediction of shell theory for thebuckling risk of main shell upper side buckling in Fig. 15.

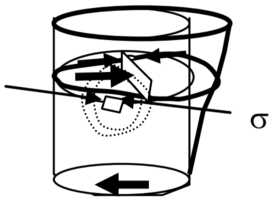


Figure 15. Buckling risk sketch for the shell theory prediction for main shell upper side buckling.

CONCLUSION

A preliminary optimal design of seismically loaded liquid containing vessels is essential get the main dimensions within correct ranges before detailed design by FEM. This methodology makes possible to consider the simultaneous interaction of various choices like loads, dimensions, materials and limit states on the result. All important design events like cost and limit states are expressed as decision variables and the fuzzy customer satisfaction function distribution on them. Then the total satisfaction is calculated as product of functions.

The optimisation method is composed of analytical probing of assumed risk locations with physical variable models.

The optimisation goal is to obtain optimal main dimensions and shapes, the critical locations using basic mechanics and simplified standard calculation. It showed that the impulsive seismic acceleration was more critical for optimality than the convective acceleration.

The suggested optimum result was checked by FEM modelling. Both models predict a buckling risk at upper sides of the vessel caused by to fluid motion against the wall.

The FEM results show reliably and graphically the behaviour of the structure under loads.

Both methods supplement each other by adding their strong points and compensating weaknesses by synergy.

The FEM methods is done is steps. First the main dimension of the vessel and seismic environments data is assembled. Most of this data is given by the customer. Next the relevant standards are used to get loading data for the FEM models.

Third the FEM model shows the deformations, stresses and eigen frequencies and modes for some parts. The fourth step is to make iterative optimising changes to the structure and rerun the model until result is satisfactory.

The future vision is to combine the three main design methods. First is the analytical concept innovation and optimisation to get main parameters. The second is fine-tuning with FEM. The third step is to use as guidelines in both steps the requirements of standards and global megatrends in ecology and technology.

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