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TEMPERATURE DISTRIBUTION STUDY OF VARIOUS INCLUSIONS FOR ESTIMATING THE EFFECTIVE THERMAL CONDUCTIVITY OF TWO PHASE MATERIALS

ABSTRACT:
In this article, the temperature distribution in the unit cell for various inclusion shapes at different conductivity ratios, contact ratios and concentration were carried out by ANSYS software with suitable boundary conditions. The software validation and mesh size has been carried out.

KEYWORDS:
Concentration; Conductivity ratio; various inclusions; Two-phase materials

INTRODUCTION

The importance of two-phase materials such as ceramics, metal foams, emulsion and suspended systems, granular materials lies in many of the applications in microelectronic chip cooling, spacecraft structures, catalytic reactors, heat recovery process, heat exchangers, heat storage systems, petroleum refineries, nuclear reactors, electronic packaging, and food processing. Many researchers have spent an enormous amount of effort on developing various analytical methods for modeling and calculating two-phase homogeneous materials with imbedded inclusions and surrounding inter phase. Moreover, this problem has importance because of its analogy with the general susceptibility of dispersed media such as dielectric constant, refractive index, magnetic permittivity, electrical conductivity, elastic modulus, and diffusion coefficient. The problem is one of the long standing issues and has been treated in many papers on the basic of unit cell approach by considering the primary parameters such as concentration of the dispersed phase (υ), conductivity ratio (α) and secondary parameters (contact resistance, heat transfer through radiation, Knudsen effect and geometrical configurations). Numerous models were developed to find out the effective thermal conductivity (ETC) of the mixtures, but one of the major limitations of the models is its suitability for specific applications. Maxwell's work (1) predicting the magnetic permittivity of a dilute suspension of spheres is the earliest reported work in the modeling of transport properties of two-phase media. But one of the limitations of the model is applicable for lower concentration of the dispersed phase. The Maxwell and phase inverted Maxwell (2) models are the minimum and maximum bounds for predicting the thermal conductivity of the two phase system. These are the most restrictive bounds proposed and every model should incorporate these bounds as a minimum and maximum. The upper and lower limits to the conductivity of two-phase materials based on parallel and series resistances were given by Wiener (3). Zehner and Schlunder (4) proposed a model considering the effect of particle contact as well as the effect of secondary parameters such as thermal radiation, pressure dependence, particle flattening, shape and size distribution for cylindrical unit cell containing spherical inclusions. An important deficiency in the model is that the deformation of the flux field is taken only as a function of concentration, not as a function of the conductivity ratio. Hsu, et al (5) obtained algebraic expressions for effective thermal conductivities of porous media by applying lumped parameter method, which is based on an electric resistance analogy. Models were developed to describe the effective thermal conductivity of randomly packed granular materials based on the unit cell method, by Crane and Vachon (6). A review of thermal conductivity of packed beds at no-flow condition was described by the Tsotsas and Martin (7). Bruggeman (8) extended Maxwell's result for lower concentration of the dispersed phase to the full range of concentration by assuming the mixture to be quasi-homogeneous. Raghavan and Martin (9) proposed a unit

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cell model that agreed exactly with field solutions of Maxwell and provided the basis for a fundamentally correct approach in the modeling of conductivity. Numerical study for effective conductivity based on a model made up of spheres in cubic lattice has been carried out by Krupiczka (10). Krischer (11) described the unit cube thermal conductivity model. A review of conduction in heterogeneous systems was studied by Meredith and Tobias (12). The purpose of this work was correcting, modifying and extending the Rayleigh (13) formula for interactions of higher order between particles. Bauer (14) developed an analytical model for the effect of randomly distributed inclusions or pores on the solution of Laplace's heat conduction equation for prediction of thermal conductivity of packed beds. The effective thermal conductivity of packed beds based on field solution approach was carried out by Dietz (15). A review of various methods for predicting the effective thermal conductivity of composite materials was proposed Progelhof et al. (16). The thermal conductivity of a saturated porous medium was calculated for a two-layer model representing as electrical resistance in an electrical circuit (Deisser and Boregli; 17). Kunii and Smith (18) proposed a unit cell model. The electrical conductivity of binary metallic mixtures was investigated by Landauer (19). Samantray et al. [20] proposed a comprehensive conductivity model by considering the primary parameters based on unit cell and field solution approaches. Later, the validity of the model was extended to predict the effective conductivity of various binary metallic mixtures with a high degree of accuracy (21). Reddy and Karthikeyan (22) developed the collocated parameter model based on the unit cell approach for predicting the effective thermal conductivity of the two-phase materials. Tai [23] deduced mathematical expressions for the equivalent thermal conductivity of two and three-dimensional orthogonally fiber-reinforced composites in a one-dimensional heat flow model. In this regard, Tai applied the fundamental definitions of thermal conductivity and the simple rule of mixtures to a unit cell of an orthogonally fiber-reinforced material. Tai, showed that whether a square slab model or a cylindrical fiber model is used makes little difference to the heat flux; while the fiber volume fraction matters. Jones and Pascal [24] developed a three-dimensional numerical finite-difference to calculate the thermal conductivity of a composite with two or more constituents to better understand how the relative quantities and distributions of the component materials, within a sample, affect the whole sample conductivity. Graham and McDowell [25] estimated the transverse thermal conductivity of continuous reinforced composites containing a random fiber distribution with imperfect interfaces using finite-element analysis. Krach and Advani [26] investigated the effect of void volume and shape on the effective conductivity of a unidirectional sample of a 3-phase composite using a numerical approach consisting of a unit cell. Their findings clearly showed that the influence of porosity on thermal conductivity could not be described solely by the void volume. They found that the shape and distribution of the voids influence the effective thermal conductivity. Al-Sulaiman et al [27] developed correlations based on a finite element analysis that predict the thermal conductivity of fibers utilizing the easy to measure thermal conductivity of the Fiber Reinforced Composite Laminates (FRCL) and the other constituents. In their model, Al-Sulaiman et al considered the FRCL cured at high pressures such that it includes no air voids. Zou et al. [28] come up with an analytical expression for transverse thermal conductivities of unidirectional fiber composites with and without thermal barrier is derived based on the electrical analogy technique and on the cylindrical filament-square packing array unit cell model (C-S model).

MODELING FOR VARIOUS INCLUSIONS

SQUARE CYLINDER

The effective thermal conductivity of the two dimensional medium can be estimated by considering a square cylinder with cross-section 'a x a' having a connecting bar width of 'c' as shown in the Fig. 1. The effective thermal conductivity of the two-dimensional periodic medium is assumed to be depending on the finite contact between the inclusions. The two dimensionally spread inclusions are connected by connecting plates with 'c/a' denoting the contact parameter.

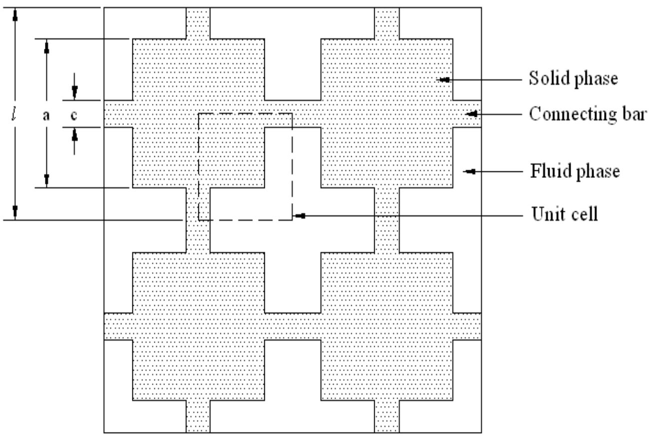

Fig. 1.Two-dimensional spatially periodic two-phase system (Touching square cylinder)

Because of the symmetry of the plates, one fourth of the square cross-section has been considered as a unit cell and is shown in Fig. 2.

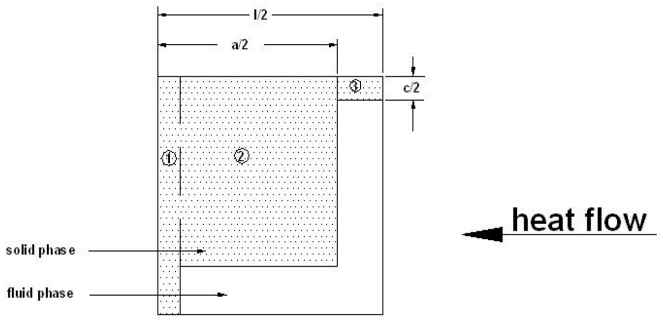

Fig.2. Unit cell of square cylinder

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The unit cell consists of three rectangular solid layers (1), (2), (3) as shown in the Fig.2. The dimensions of the first, second and third rectangular solid layer is (l/2) (c/2), (a/2) ((a-c)/2) and c/2) ((l-a)/2) respectively. The model is based on the one dimensional heat conduction in the unit cell. The concentration for square cylinder is given as

$$\upsilon = [\varepsilon^2(1-2\lambda)+2\varepsilon\lambda] \quad (1)$$

HEXAGON CYLINDER

The effective thermal conductivity of the two dimensional medium can be estimated by considering a Hexagon cylinder with cross-section 'a x a' having a connecting bar width of 'c' as shown in the Fig.3. The effective thermal conductivity of the two-dimensional periodic medium is assumed to be depending on the finite contact between the inclusions. The two dimensionally spread inclusions are connected by connecting plates with 'c/a' denoting the contact parameter

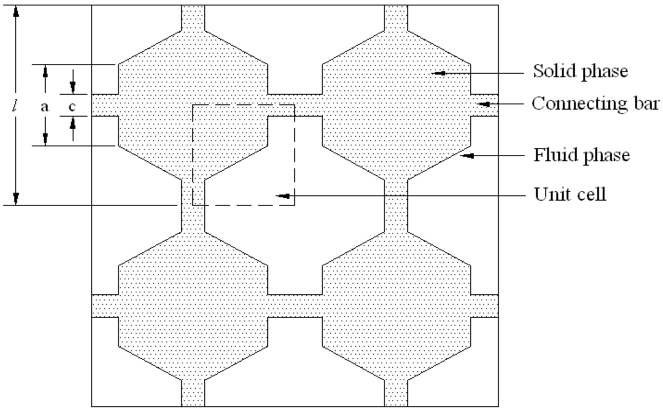


Fig.3.Two-dimensional spatially periodic two-phase system (Touching hexagon cylinder)

Because of the symmetry of the plates, one fourth of the hexagon cross-section has been considered as a unit cell and is shown in Fig.4

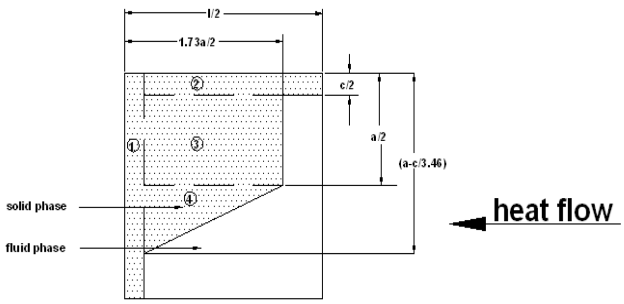


Fig.4. Unit cell of hexagon cylinder

The unit cell consists of three rectangular solid layers (1), (2), (3) and one triangular solid layer (4) as shown in Fig.4. The dimensions of the first, second and third rectangular solid layer is (l/2) (c/2), (c/2) ((l-c)/2) and ((a√3-c)/2) ((a-c)/2) respectively. The dimension of triangular solid layer is ((a√3-c)/2) ((a/2)-(c/2√3)). The concentration for hexagon cylinder is given as

$$\upsilon = (2a\lambda) + [(3\sqrt{3}-(3+\sqrt{3})\,\lambda)\,a^2]/l^2 \quad (2)$$

OCTAGON CYLINDER

The effective thermal conductivity of the two dimensional medium can be estimated by considering a Octagon cylinder with cross-section 'a x a' having a connecting bar width of 'c' as shown in the Fig.5. The effective thermal conductivity of the two-dimensional periodic medium is assumed to be depending on the finite contact between the inclusions. The two dimensionally spread inclusions are connected by connecting plates with 'c/a' denoting the contact parameter

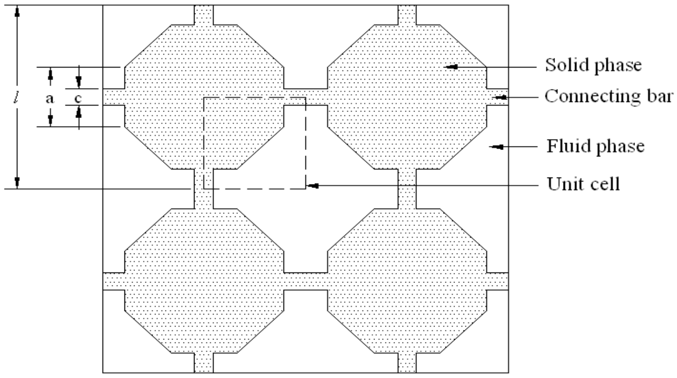


Fig.5.Two-dimensional spatially periodic two-phase system (Touching octagon cylinder)

Because of the symmetry of the plates, one fourth of the octagon cross-section has been considered as a unit cell and is shown in Fig.6.

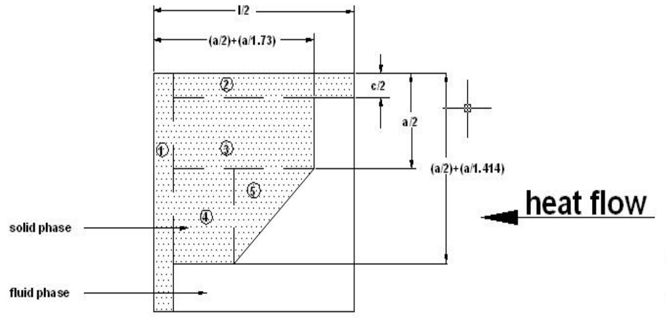


Fig.6. Unit cell of octagon cylinder

The unit cell consists of four rectangular solid layers (1), (2), (3), (4) and one triangular solid layer (5) as shown in Fig.6. The dimensions of the first, second, third and fourth rectangular solid layer is (l/2) (c/2), (c/2) ((l-c)/2), ((a+a√2-c)/2) ((a-c)/2) and (a/√2) ((a-c)/2) respectively. The dimension of triangular solid layer is (a/√2) (a/√2). The concentration for octagon cylinder is given

$$\upsilon = 2\varepsilon^2\left[\left(1+\sqrt{2}\right)(1-\lambda)\right] + 2\varepsilon\lambda \quad (3)$$

CIRCULAR CYLINDER

The effective thermal conductivity of the two dimensional medium can be estimated by considering a Circular cylinder of diameter 'a' having a connecting bar width of 'c' as shown in the Fig.7. The effective thermal conductivity of the two-dimensional periodic medium is assumed to be depending on the finite contact between the inclusions. The two dimensionally spread inclusions are connected by connecting plates with 'c/a' denoting the contact parameter.

Because of the symmetry of the plates, one fourth of the circular cross-section has been considered as a unit cell and is shown in Fig.8.

The unit cell consists of two rectangular solid layers (1), (2) and one quarter circular solid layer (3) as shown in the Fig.8.

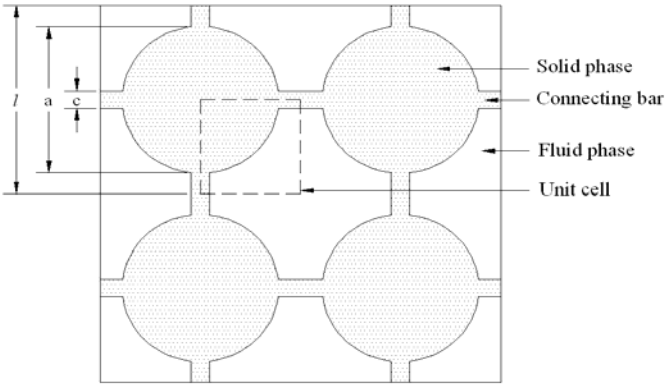


Fig.7.Two-dimensional spatially periodic two-phase system (Touching circular cylinder)

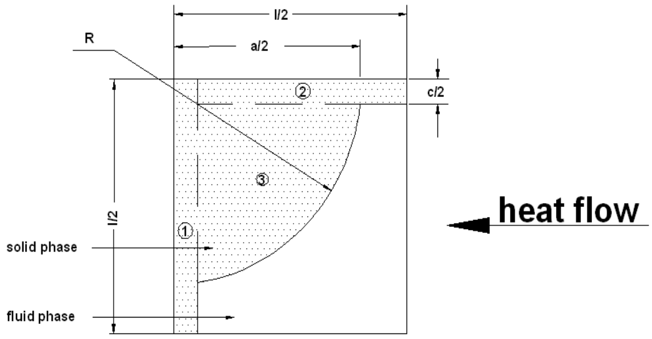


Fig.8. Unit cell of circular cylinder

The dimensions of the first and second rectangular solid layer is (l/2) (c/2), (c/2) ((l-c)/2) respectively. The radius of circular solid layer is (a/2)-(c/√2).The concentration for circular is given as

$$\upsilon = [\{\pi\, [(1/2)-(\lambda/\surd 2)]^2-\lambda^2\}\,(a/l)^2]+(2a\lambda)/l \qquad (4)$$

NUMERICAL ANALYSIS FOR VARIOUS INCLUSIONS

Numerical heat transfer analysis of the unit cell for various inclusion shapes (square, hexagon, octagon and circular cylinders) has been carried out to estimate the Effective Thermal Conductivity of the two-phase materials via the Finite Element simulation. For this heat transfer analysis ANSYS11.0, a finite element software package is used.

Boundary condition

One face of the unit cell is subjected to constant temperature and the opposite face is subjected to convective thermal environment. All other faces are kept as adiabatic in order to achieve 1D heat transfer.

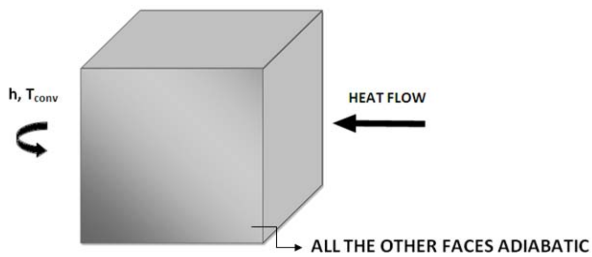


Fig 9.The Thermal boundary condition applied on the unit cell

The boundary condition imposed on the unit cell is shown in the Fig.9.

Determination of Effective Thermal conductivity

From the results of the finite element analysis, the average surface temperature on the convection wall of the unit cell is computed. Once the temperature of the convective side is known, the effective thermal conductivity across the two walls can be calculated using the following simple heat balance equation

$$hA\left(T_{wall2}-T_{conv}\right)=\frac{K_{eff}\,A\left(T_{wall1}-T_{wall2}\right)}{L} \qquad (5)$$

A - Wall area (m^2)
h - Heat transfer coefficient ($W/m^2.K$)
T_{conv}-bulk temperature of the fluid at the convection side (K)
T_{wall1} - fixed wall temperature (K)
T_{wall2}-convective wall temperature (K)

Several simulations were done for a wide spectrum of possible variation in the concentrations, conductivity ratios and contact ratios for all inclusion shapes.

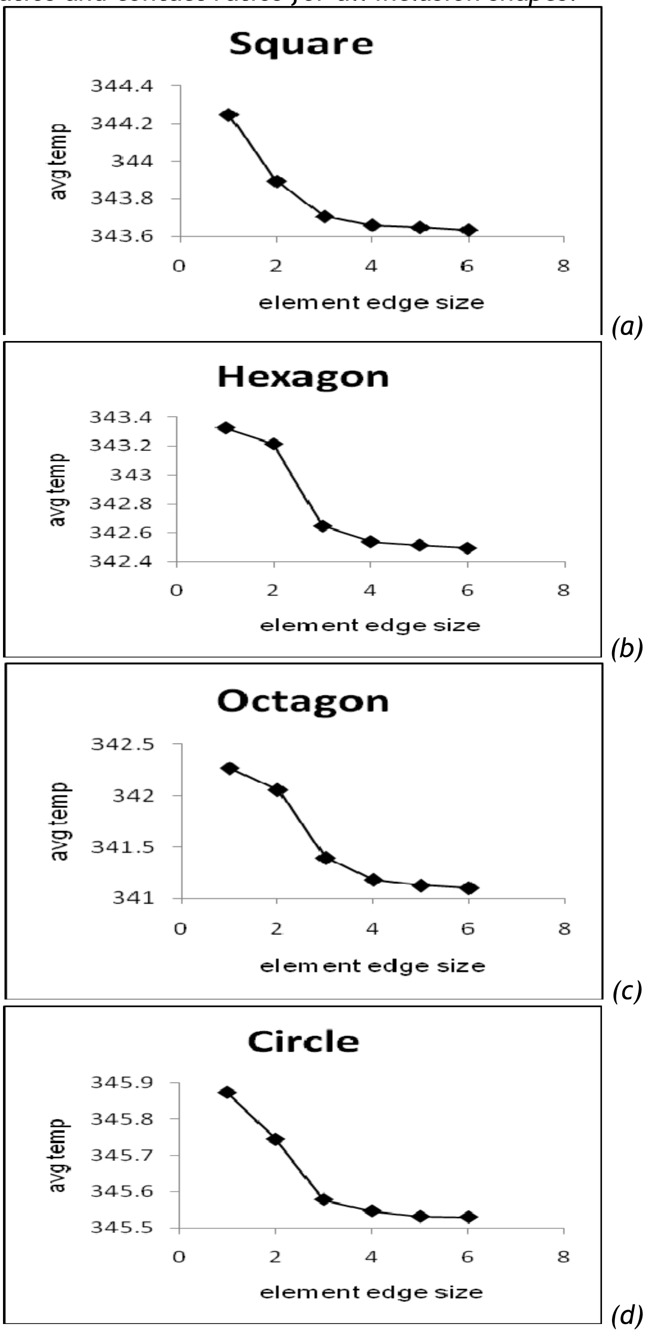


Fig10. (a)-(d) Element edge size Vs Average temperature for square, hexagon, octagon & circular cylinders

Mesh sensitivity test

This model has been first tested for mesh-independent solution. In this regard, six runs have been conducted for the case of two-phase material with conductivity ratio=800, concentration=0.5 and contact ratio=0.02. In these six runs, the finite element edge size was changed from very coarse to very fine element. In each of these six runs, the average temperature at the convective wall of the two-phase material was calculated.

Table1. Mesh Sensitivity Test

Element edge size	Square cylinder	Hexagon cylinder	Octagon cylinder	Circular cylinder
0.2	344.248	343.3291	342.2664	345.8729
0.1	343.8929	343.2161	342.0569	345.7442
0.05	343.7064	342.6477	341.3922	345.5785
0.03	343.6591	342.538	341.1802	345.5469
0.025	343.6472	342.5136	341.1242	345.5315
0.02	343.6339	342.4941	341.1	345.5225

Table.1.gives a summary of the output of these six runs indicating the element edge size and the corresponding average wall temperature obtained for various inclusions. A graph (Fig.10) is plotted between the element edge size and the corresponding average wall temperature for various inclusions. From the graph it is seen that after an element edge size of 0.03 the average wall temperature remains almost constant indicating the convergence of the solution. Looking for high accuracy and reasonable CPU simulation time, it is decided to use a medium element edge size of 0.03. This reflects clearly that the numerical solution obtained via this FE simulation is mesh size independent.

Software validation

The Finite element simulation has been bench-marked to obtain the numerical temperature distribution along the thickness of the unit cell by simulating a simple non-linear 1D analysis with thermal contact resistance. The problem chosen for validation consists of three layers of different thermal conductivity and different wall thickness. Both face of the wall is subjected to convective environment. An internal contact thermal resistance is given between each layer. The ANSYS code for the above problem is developed. The results obtained from the ANSYS software package is compared with the available analytical solution. This comparison is shown in the Fig.11.

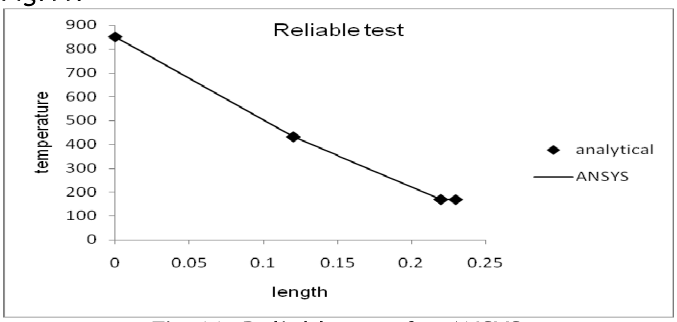


Fig.11. Reliable test for ANSYS

This figure reveals an excellent agreement between the analytical and the numerical solution obtained via ANSYS. This is considered as an excellent validation of the ANSYS software package.

TEMPERATURE DISTRIBUTION IN THE UNIT CELL FOR VARIOUS INCLUSION SHAPES

The temperature distribution in the unit cell for various inclusion shapes at different conductivity ratios and contact ratios for concentration=0.5 is shown in the Figs12-17.

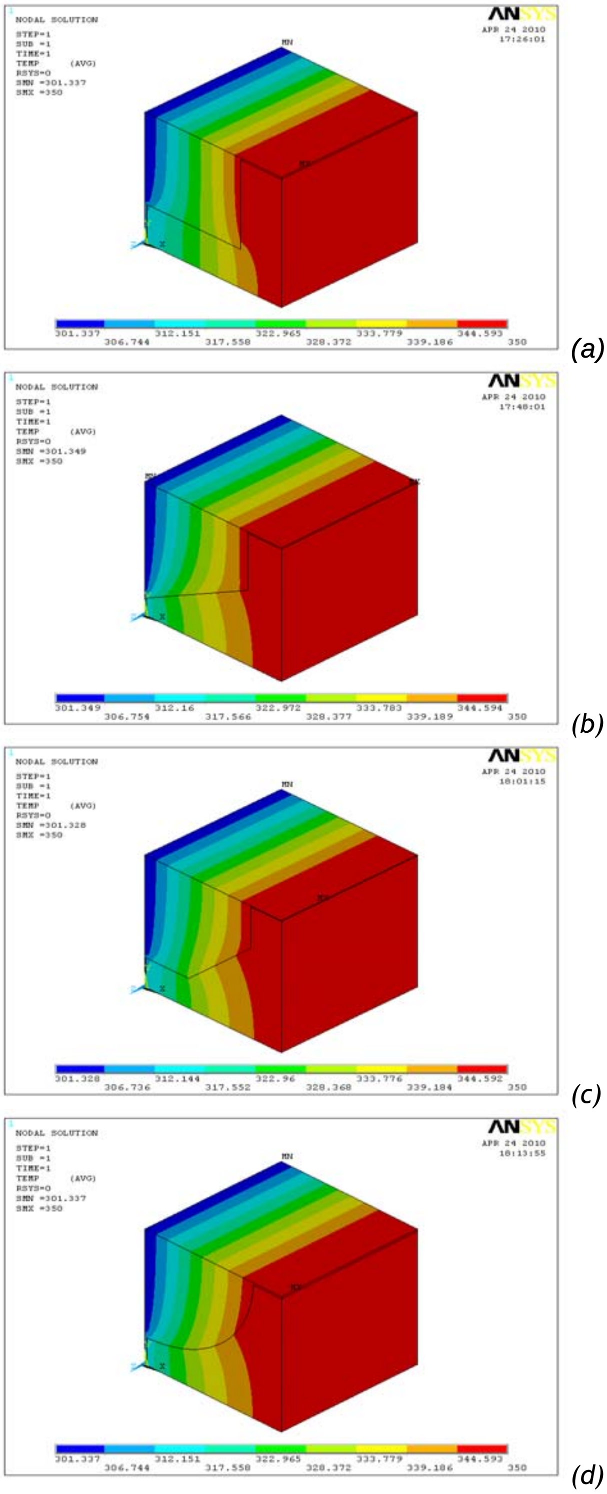
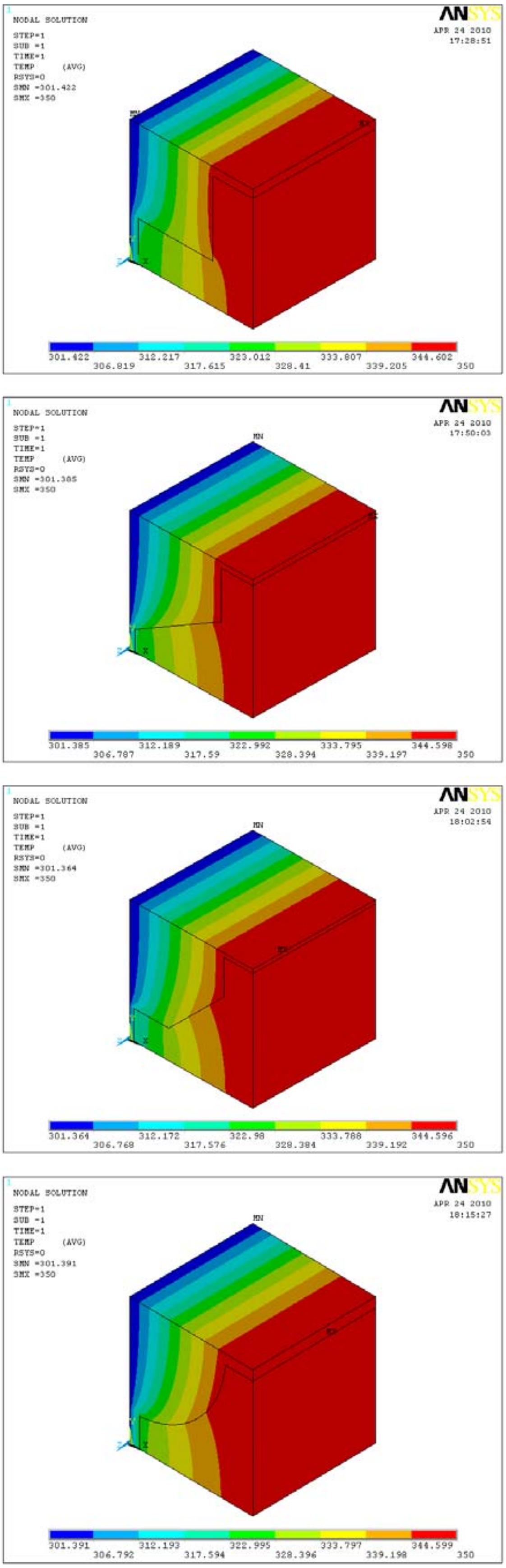


Fig12. (a)-(d) Temperature distribution in the unit cell for various inclusion shapes at a=0.1, υ=0.5 and λ=0.02.



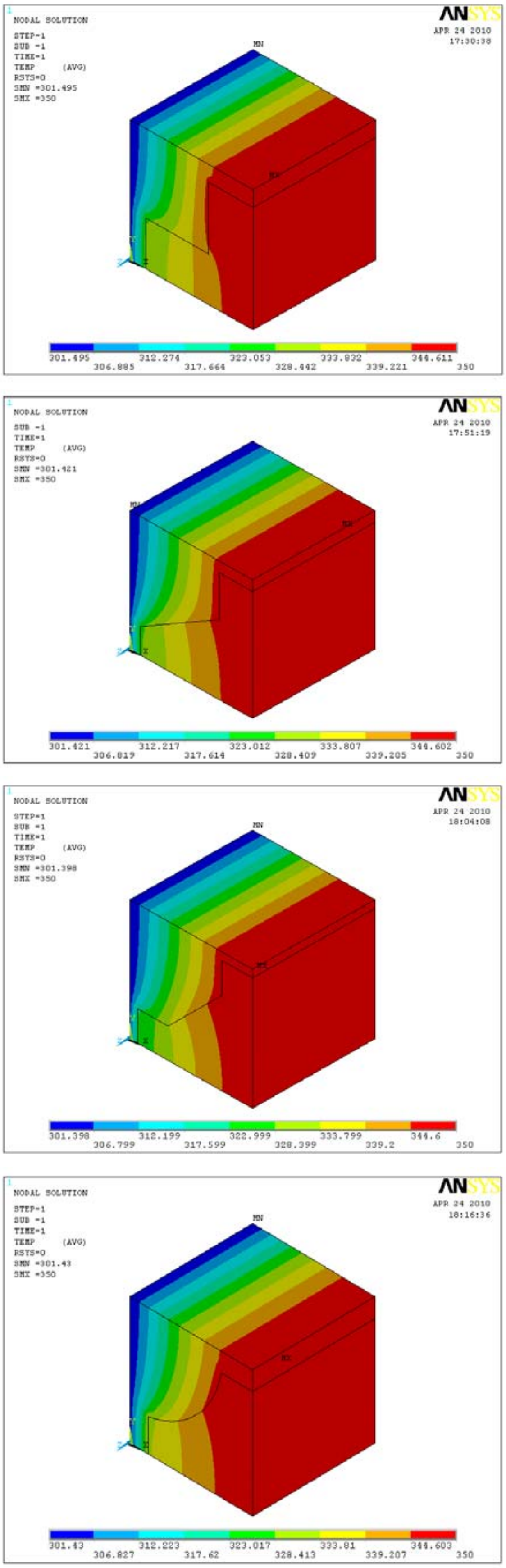
(a)

(b)

(c)

(d)

Fig13. (a)-(d) Temperature distribution in the unit cell for various inclusion shapes at a=0.1, υ=0.5 and λ=0.1



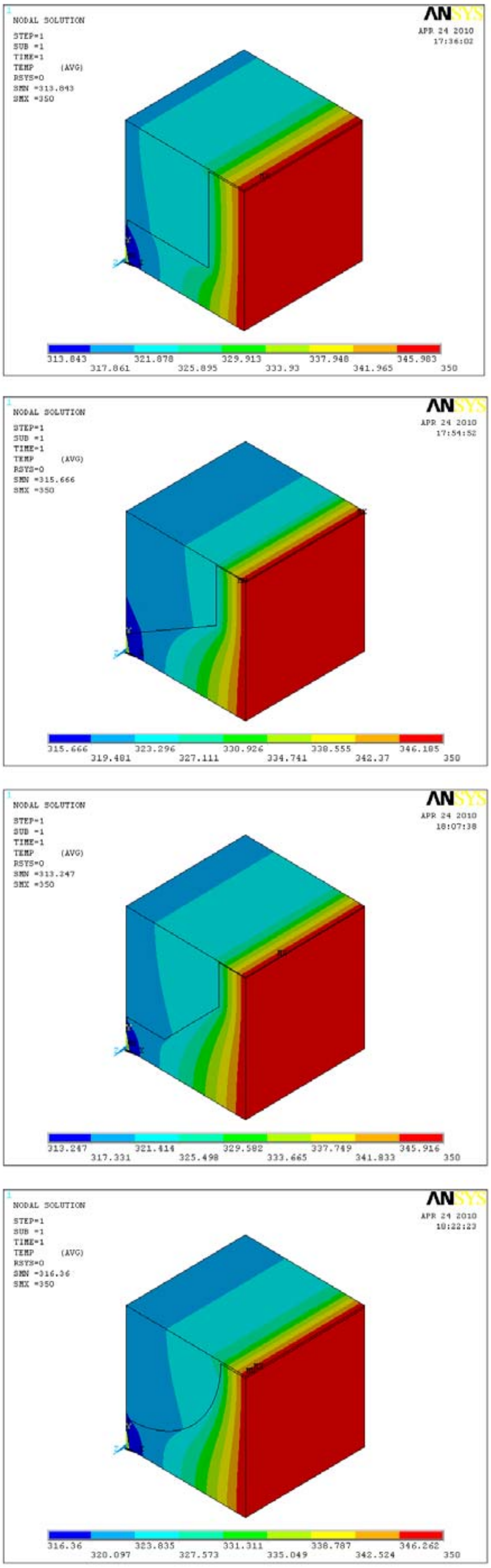
(a)

(b)

(c)

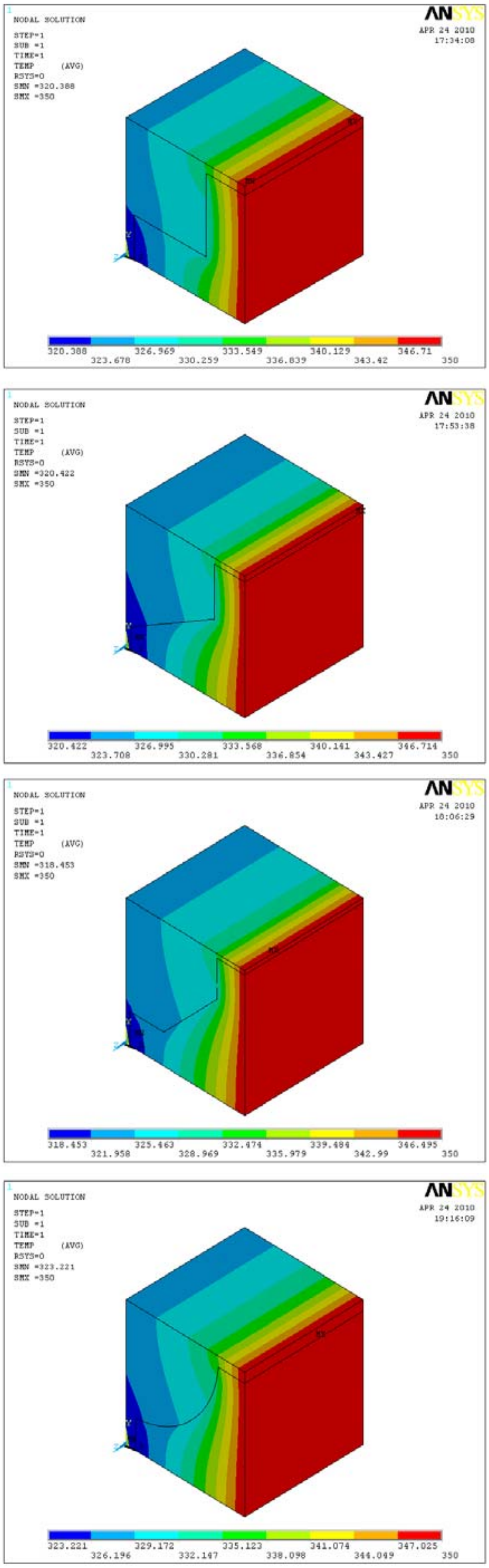
(d)

Fig14. (a)-(d) Temperature distribution in the unit cell for various inclusion shapes at a=0.1, υ=0.5 and λ=0.2



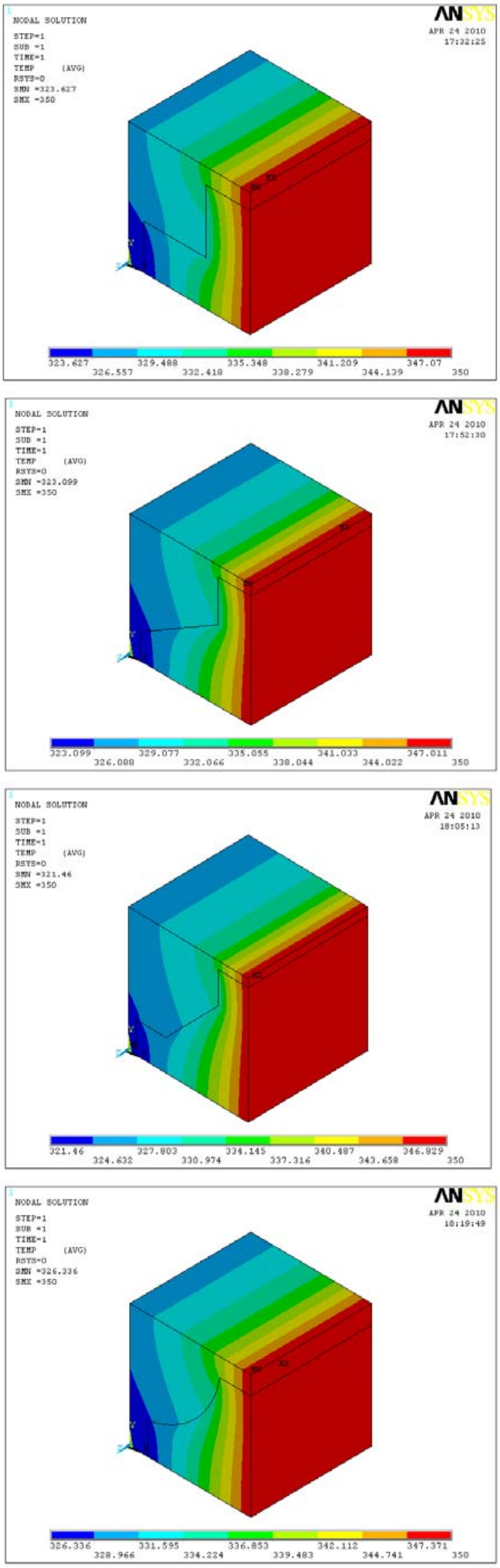
(a) (b) (c) (d)

Fig15. (a)-(d) Temperature distribution in the unit cell for various inclusion shapes at a=20, υ=0.5 and λ=0.02.



(a) (b) (c) (d)

Fig16. (a)-(d) Temperature distribution in the unit cell for various inclusion shapes at a=20, υ=0.5 and λ=0.1



(a)

(b)

(c)

(d)

Fig17. (a)-(d) Temperature distribution in the unit cell for various inclusion shapes at α=20, υ=0.5 and λ=0.2

CONCLUSION

The circular shaped inclusion has largest non-dimensional effective thermal conductivity followed by square, hexagon and octagon shaped inclusions respectively. For the same concentration and contact ratio, hexagon shaped inclusion has largest heat transfer area followed by circular, octagon and square shaped inclusions respectively. Since hexagon has the largest heat transfer area it is expected to have larger non-dimensional effective thermal conductivity than other inclusion shapes. But the geometry of hexagon and octagon shapes are not symmetric about its mutual perpendicular axis i.e., hexagon and octagon shapes exhibits anisotropic property. This is the reason for the hexagon shaped inclusion to have lower non-dimensional effective thermal conductivity than circular and square shaped inclusions.

SYMBOLS

α - Conductivity ratio(k_s/k_f)
υ - Concentration
λ - Contact ratio(c/a)
ε - Length ratio(a/l)

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