ACTA TECHNICA CORVINIENSIS – Bulletin of Engineering
Tome V (Year 2012). FASCICULE 3 [July–September]. ISSN 2067-3809

1. Juliana LITECKA, 2. Slavko PAVLENKO

MATHEMATICAL MODELLING OF GEAR HOB SURFACE WITH BASIC PROFILE

1,2 DEPARTMENT OF TECHNOLOGICAL DEVICES DESIGN, FACULTY OF MANUFACTURING TECHNOLOGIES OF TECHNICAL UNIVERSITY IN KOŠICE WITH A SEAT IN PREŠOV, ŠTÚROVA 31, PREŠOV, SLOVAKIA

ABSTRACT: Gear production is very important area of manufacturing industries because gears are the widest components in the machines and machine equipments. Mode of production and used tools are important elements of economical and quality part of production. Nowadays, there are developed the new constructional solutions of gear hobs which save time and money. For the hob which would produce precision involute gear there is possibility of finding profile which would provide this requirement. For the finding of the profile it is needed to have a good mathematical knowledge kinematic and geometrical properties investigated objects. The paper deals with mathematical description of basic hob surface with straight profile which is initial theorem for the determining of accurate profile of gear hob.
KEYWORDS: gear hob, hob profile, hob surface, mathematical description, parametric equations

INTRODUCTION

Gear hobbing is a continuous rolling method. The body enveloping is a cylindrical involute worm. Tool and workpiece rotate during the generating motion while the milling cutter executes the cutting motion as it circles around.

To manufacture spur gears, milling cutter and workpiece are shifted in relation to each other in the direction of the workpiece axis, and the generating motion is carried out at the same time. [1]

Gear hobs, shows on Figure 1. (a), are very productive cutting tools used for machining of gear wheel and other different components like spline shafts, chain wheels, ratchet gearing and parts with screw surface. Gear hobs are universal tools because with the same module we can machine gear wheels with different number of teeth, tooth inclination, corrected or uncorrected gear and worm wheels.

The characteristic element for the calculating and design of hob is basic surface of tool (Figure 1. (b)).

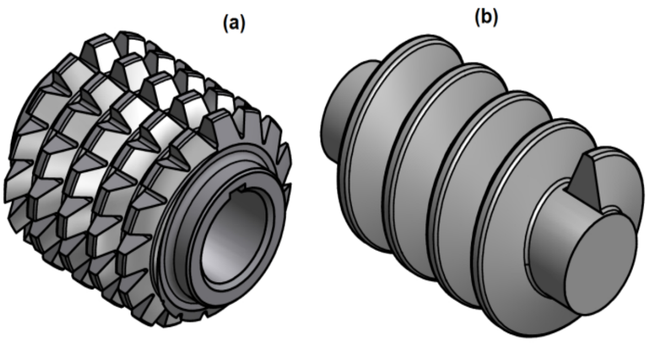

Figure1. (a) Constructional solution of solid gear hob (b) Basic surface of gear hob with straight profile

For the investigation of a basic tool surface we will use theory of three-dimensional curves and helix surfaces.

PRINCIPLES OF GEOMETRICAL THEORY

For investigate a case of hob we need to appear from three basic geometric definition: curve, surface, movement.

a. The curve

The curve is a geometrical concept, of which an exact and at the same time quite general definition presents considerable difficulties and is carried out differently in different branches of geometry.

In elementary geometry the concept of a curve is not clearly defined and is sometimes defined as "length without width" or as the "boundary of a surface". In elementary geometry the study of a curve essentially reduces to consideration of examples (a straight line, an interval, a polygon, a circle, etc.).

Since it does not have general methods at its disposal, elementary geometry has gone quite deeply into the study of properties of specific curves (conic sections, certain algebraic curves of higher orders and transcendental curves), using special methods in each case. In analytic geometry a curve in a plane is defined as a set of points whose coordinates satisfy an equation $F(x,y)=0$.

Restrictions must be imposed on the function F so that, on the one hand, the equation should have an infinite set of solutions and, on the other hand, so that this set of solutions does not fill "a piece of the plane".

An important class of curves comprises those for which the function $F(x,y)$ is a polynomial in the two variables; in this case the curve defined by the equation $F(x,y)=0$ is said to be algebraic. Algebraic curves specified by an equation of the first degree are straight lines. [2]

b. The surface

In geometry, a two-dimensional collection of points (flat surface), a three-dimensional collection of points whose cross section is a curve (curved surface), or the boundary of any three-dimensional solid.

In general, a surface is a continuous boundary dividing a three-dimensional space into two regions.

c. The movement

The movement is unlimited set of geometrical transformation the same type (e.g. set of rotations around axis, or set of translations along straight line) or unlimited set of geometrical affine transformations, which are analytical represented by transposed matrix:

$$T(u)=\begin{bmatrix} a_{11}(u) & a_{12}(u) & a_{13}(u) & a_{14}(u) \\ a_{21}(u) & a_{22}(u) & a_{23}(u) & a_{24}(u) \\ a_{31}(u) & a_{32}(u) & a_{33}(u) & a_{34}(u) \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

where functions a_{ij} are function of one real variable, all defined, linear and least once differentiable on interval I.

For the describing of surface in the extensive Euclid space there will be equations:

$$F\{x, y, z, 1\} = F\{x(t), y(t), z(t), 1\}.\,T(u) \ \text{ for } u \in I \qquad (2)$$

where F{x(t),y(t),z(t),1} is function of generating curve.

The equation (2) we may to write by parametric equations:

$$\begin{aligned} x &= x(t).a_{11}(u)+y(t).a_{12}(u)+z(t).a_{13}(u)+a_{14}(u) \\ y &= x(t).a_{21}(u)+y(t).a_{22}(u)+z(t).a_{23}(u)+a_{24}(u) \quad (3) \\ z &= x(t).a_{31}(u)+y(t).a_{32}(u)+z(t).a_{33}(u)+a_{34}(u) \end{aligned}$$

for each $u \in I$ and $t \in J$.

ANALYTICAL DESCRIPTION OF HOB SURFACE GEOMETRY

For the investigation of surface geometry of hob we will use the geometry of helix surface S which is created by helix movement of curve k. The curve k is a generating curve of a helix surface S. A set of all position of generating curve k for helix movement, which define helix surface S, is one system of curves which models the surface S. The helix surface S is one-parametric system of curves - all position k^u of generating curve k for helix movement which define surface S.

Each point P of helix surface S is situated on some position k^u of generating curve k (Figure 2.).

This fact we may formulate so that one-parametric variable of point P will be variable u. Position of this point P on curve k^u we describe by second parametric variable t (Figure 3.). The helix surface S is two-parametric system of points P(t,u) in the three dimensional space with coordinates (x,y,z).

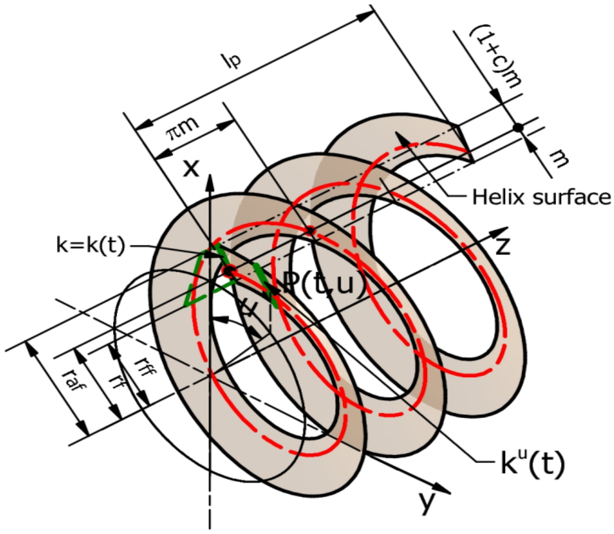

Figure 2. The generating of helix surface by curve k

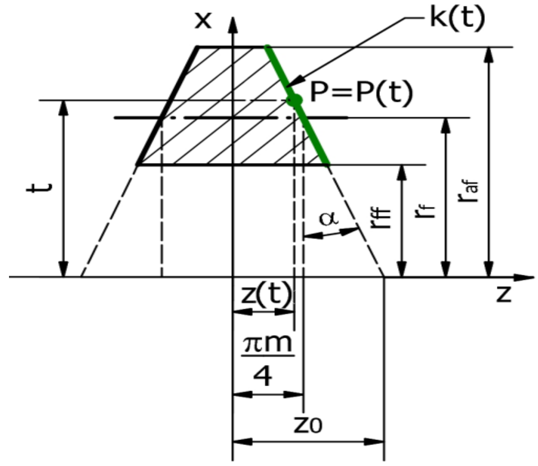

Figure 3. The generating of curve k by point P

Point P of generating curve k creates by helix movement the defining helix surface S - helicoid which is a curve on this helix surface.

Set of all helicoids which are created by each point of generating curve k is second system of curves which models helix surface S. All these helicoids have unit axis z, all they are clockwise or anticlockwise and they have the same size of convolution:

$$p = \pi.m \qquad (4)$$

where m is module of hob.

On Figure 2 there is illustrated model of helix surface which is created by curve k by helix movement in the coordinate system (x,y,z) where axis z is axis of helixoid. Two systems of curves, it means system position curve k and system of helicoid created points of curve k, create model of helix surface.

For the mathematical descriptiom of helix surface we will use parametric equations of curve k, where k:{x=x(t), y=y(t), z=z(t), $t \in \langle t_0, t_1 \rangle$}, which is straight line of basic hob profile. Their parametric equations we may describe on based of Figure 3.:

$$x(t) = t \quad y(t) = 0 \quad z(t) = z_o - t.tg(\alpha) \qquad t \in \langle r_{ff}, r_{af} \rangle \tag{5}$$

where

$$z_o = \frac{\pi.m}{4} + r_f.tg(\alpha) \tag{6}$$

r_f is a radius of hob pitch circle

r_{af} is a radius of hob addendum circle

r_{ff} is a radius of hob dedendum circle.

In the case of hob we consider, helix surface which is created by curve k, clockwise and convolution is p. The transposed matrix of movement of curve k will be represented:

$$T(u) = \begin{bmatrix} \cos(u) & -\sin(u) & 0 & 0 \\ \sin(u) & \cos(u) & 0 & 0 \\ 0 & 0 & 1 & \frac{\pi.m.u}{360} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

After the writing the equations (5),(6),(7) to equation (2) we get final parametric equations of hob surface in the coordinates (x,y,z).

$$x = t.\cos(u) \quad y = t.\sin(u) \quad z = \frac{\pi.m}{4} + r_f tg(\alpha) + \frac{\pi.m.u}{360} \qquad t \in \langle r_{ff}, r_{af} \rangle \quad u \in \langle 2i\pi, 2(i+1)\pi \rangle \tag{8}$$

where i is number of convolutions.

CONCLUSIONS

In the case of investigation of surface geometry of gear hob we was based on transformation of movement straight line curve which rotates around axis z and at the same time translates along the same axis z. The describing of the movement was realized by 4x4 transposed matrix and the results were represented by parametric equations of the surface. By these parametric equations we may investigate the hob movement in depends of gear movement for gear production in the next part of research of influence the hob profile to gear production. By analysis of results of the research we may to design new profile of gear hob which will be machine gear with higher accuracy.

REFERENCES

[1.] Tschätsch, H.:Applied Machinig Technology, Springer Dordrecht Heildelberg London New York, pp. 398, 2009.

[2.] Hazewinkel, M.: Encyclopaedia of Mathematics, Supplement III, Springer, pp. 568, 2007.

[3.] Maščenik, J., Batešková, E.: Design and computing of gearing with Autodesk Inventor. In: MOSIS '09. - Ostrava, P. 227-230., 2009

[4.] Pavlenko, S., Haľko, J., Maščenik, J., Nováková, M.: Časti strojov II, 1. edition - Prešov: FVT TU, 2008.

[5.] Pavlenko, S.: K profilovaniu odvaľovacích fréz, FVT TU, Prešov, pp. 106, 2006, ISBN 80-8073-493-3

[6.] Pavlenko, S., Haľko, J., Paško, J.: Impact of spur-gear-hob diameter on tooth profile accuracy, In: Scientific Bulletin. Vol. 20, serie C (2006), p. 321-324, 2006,

[7.] Litvin, L. F., Fuentes A.: Gear geometry and applied theory, Cambridge University Press, 2004

[8.] Lawrence, D. J. : A catalog of special plane curves, Dover Publications. pp. 168,171–173., 1972, ISBN 0-486-60288-5.

[9.] Jüttler, B. , Piene, R., Dokken, T.: Geometric modeling and algebraic geometry, Springer, pp. 231, 2008, ISBN 978-3-540-72184-0

[10.] Radzevich, S. P. : Kinematic geometry of surface machining, CRC Press, pp. 508, 2007

ACTA TECHNICA CORVINIENSIS – BULLETIN of ENGINEERING

ISSN: 2067-3809 [CD-Rom, online]