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## SOLVING TRAVELLING SALESMAN PROBLEM BY USE OF KOHONEN SELF-ORGANIZING MAPS

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**ABSTRACT:** This paper presents an approach for solving the travelling salesman problem (TSP) by using artificial neural network (ANN). The ANN model adopted in this paper is the Kohonen's self-organizing map (SOM) which uses competitive, unsupervised learning. The paper briefly describes the competitive learning and Kohonen's SOM model development. The possibilities of SOM were applied for solving real problem of disposal plastics waste. The Kohonen's SOM was trained using original geo coordinates for container locations in the city of Niš. The results demonstrate that the proposed approach is comparable in terms of solution quality and computational requirements to classical approaches such as Clarke-Wright saving algorithm.

**KEYWORDS:** Self-organizing maps, vehicle routing problem and simulation

### INTRODUCTION

The vehicle routing problem (VRP) can be defined as a problem of finding the optimal routes of delivery or collection from one or several depots to a number of cities or customers, while satisfying some constraints. Collection of household waste, gasoline delivery trucks, goods distribution, snow plough and mail delivery are the most used applications of the VRP [1]. The VRP plays a vital role in distribution and logistics. Huge research efforts have been devoted to studying the VRP since 1959 where Dantzig and Ramser [2] have described the problem as a generalized problem of Travelling Salesman Problem (TSP). The travelling salesman problem (TSP) is a classical optimization problem. Given a list of cities and their pair wise distances, the task is to find a shortest possible tour that visits each city exactly once. The decision-problem form of TSP is a NP complete (non-deterministic polynomial-time hard) problem [3], hence the great interest in efficient heuristics to solve it.

The problem was first formulated as a mathematical problem in 1930 and is one of the most intensively studied problems in optimization. It is used as a benchmark for many optimization methods. Even though the problem is computationally difficult, a large number of heuristics and exact methods are known, so that some instances with tens of thousands of cities can be solved.

There are many of the heuristics that utilize the paradigm of neural computation or related notions recently [4]. Most solutions have used one of the following methods: Hopfield network, Kohonen's SOM, genetic algorithm, simulated annealing and etc. The first approach to the TSP via ANNs was the work of Hopfield and Tank in 1985 [5], which was based on the minimization of an energy function. In this paper an attempt has been made to solve a real problem of disposal plastics waste in the city of Nis using the Kohonen's self-organizing map (SOM).

### SOM BASICS

Invented by Kohonen in the early 1980's, SOM, employ a dynamic mixture of competition and cooperation to enable the emergent formation of an isomorphism between a feature space and an array of neurons [6]. It simply inspects the input data for regularities and patterns and organizes itself in such a way to form an ordered description of the input data. This description may lead to a solution of the problem under consideration [7]. It maps input vectors of any dimension onto map with one, two or more dimensions (Kohonen layer). Output neurons are usually structured in a geometrical arrangement such as linear array or a two dimensional lattice on which a meaningful coordinate system for different features is created (feature map). It has only an input layer and an output layer where neurons are fully connected to all input neurons with a scalar weight. Figure 1 shows an example of Kohonen's SOM with output neurons arranged in two-dimensional lattice.

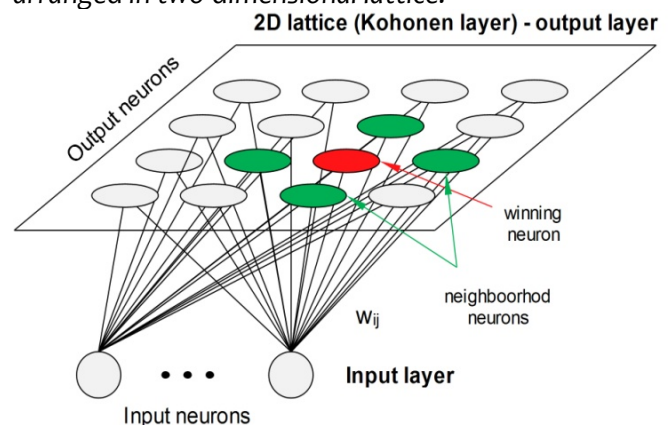


Figure 1. Schematic representation of a Kohonen SOM with 2D lattice of output neurons

SOM is one of the most popular unsupervised ANNs. When learning is unsupervised, ANN is only provided input patterns without desired outputs. The unsupervised learning process comprises two phases. In a competitive phase, a winning neuron is identified

as one which is closest to the input data (typically, the square minimum of the Euclidian distance). In the next phase, adaptive phase, the weights of winning neuron and its neighboring neurons are updated in order to approach the presented input data. The neighborhood of a neuron is usually considered to be 2D i.e. square, rectangular or hexagonal which means that each neuron has 4, 8 or 6 nearest neighbors respectively. Also one-dimensional neighborhood can be applied where each neuron has two neighbors (on the left and on the right).

The basic SOM algorithm can be described as follows [9]:

```

procedure train SOM
begin
    randomize weights for all neurons
for i = 1 to predefined iterations do
    begin
        take one random input pattern
        find the winning neuron
        find neighbors of the winner
        update synaptic weights of these neurons
    end
end
    reduce the learning rate and neighborhood function
end
    
```

**SOM APPLIED TO TSP**

To apply the SOM to the Euclidian TSP, a two-layer network, which consists of a two-dimensional input and m output neurons is used. Two dimensional input defines the coordinates of the waste disposal sites (WDS) in the two dimensional Euclidian space. The input neurons receive the coordinate values of a WDS. The input neurons are fully connected to every output neuron. To simplify the implementation, the scaled coordinates for WDS were used. The scaling to range [0-1] was performed using the equation:

$$x_{scaled} = \frac{(x - x_{min})}{(x_{max} - x_{min})} \tag{1}$$

where x is the data to be scaled, i.e. WDC's, and  $x_{min}$  and  $x_{max}$  are minimum and maximum values of the raw data. The SOM architecture consists of a one ring upon which the neurons are spatially distributed.

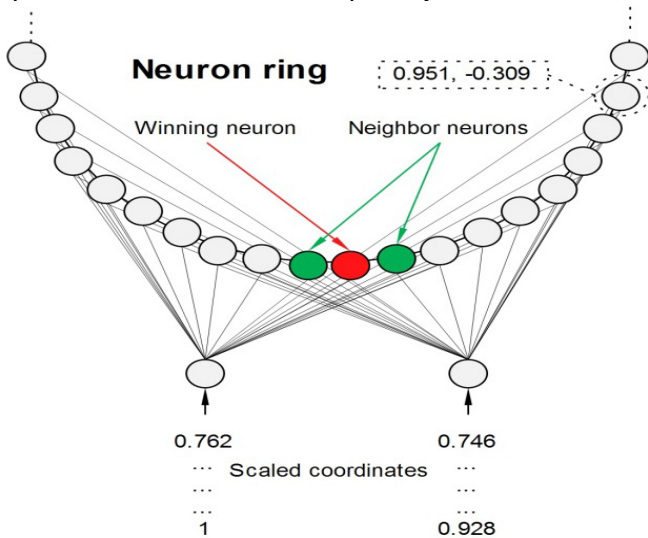


Figure 2. Kohonen's SOM with output neurons in ring architecture

Figure 2 represents the ring structure proposed. The ring can be considered as a route for an ideal problem. The weights of the neurons, which define the position of the neuron in the ring, are initially set as follows. Assuming a ring of m neurons, the neurons are equally positioned on a circle of radius equal to 1 using the angle position of given neuron that is equal to  $360^\circ$  divided by m.

The input data (a set of n WDS) are presented to the SOM in a random order and a competition based on Euclidian distance is performed between neurons in the ring. The winner neuron is the neuron  $I^*$  with the minimum distance to the presenting city.  $I^* = \text{argmin}_i \{ \|x_i - w_j\|_2 \}$ , where  $x_i$  is the coordinate of the i-th WDS,  $w_j$  is the position of the neuron j and  $\| \cdot \|_2$  is the Euclidian distance. Hence, the winner neuron, as well as neighboring neurons, moves toward the presenting i-th WDS using the neighborhood function:

$$f(\sigma, d) = e^{-\frac{d^2}{\sigma^2}} \tag{2}$$

According to the following updated function:

$$y_j^{new} = y_j^{old} + \alpha \cdot f(\sigma, d) \cdot (x_i - y_j^{old}) \tag{3}$$

where  $\alpha$  and  $\sigma$  are learning rate and neighborhood function variance, respectively. And  $d = \min\{ |j - J|, m - |j - J| \}$  is the cardinal distance measured along the ring between neurons j and J, where  $| \cdot |$  represents absolute value [8]. Learning rate has dynamic characteristic i.e. it decreases normally during training and usually takes values from 0 to 1. Similarly, the neighborhood function  $f(\sigma, d)$  is set large at very beginning of training, and slowly decreases in size with the progress of the training. After many iterations of training, the neurons tend to move closer to the WDS and finally are attached in WDS. Once all neurons are attached in the WDS, simply walk around the neurons connections and read the WDS coordinates in the order that they appear. The resulting sequence constitutes a TSP solution.

Figure 3 illustrates the evolution of the algorithm starting from the initial state of the ring, (a) reaching an intermediate stage after some iteration (b) and stopping at the final state (c).

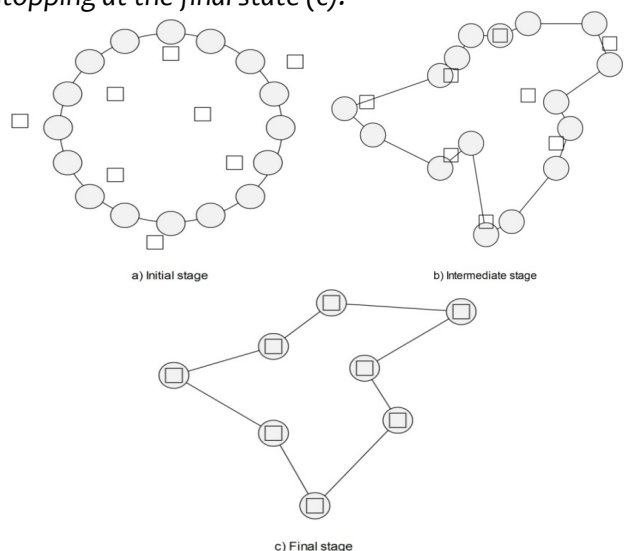


Figure 3. Evolution of Kohonen's SOM for TSP: (O) neurons and (□) WDS's coordinates

The number of neurons should be greater than the number of cities to avoid the oscillation of a neuron between different neighboring WDSs. In the present study  $m$  is set to  $2n$ . An extensive analysis of SOM algorithm and its parameters could be found in [6] [7] [8] [9] [10].

**CASE STUDY**

In this paper, SOM was applied for solving the problem of 20 waste disposal sites (WDS). The container is disposed of plastic waste. WDS are not randomly chosen but they are existed in the city of Niš, i.e. in the paper solves a real problem. Waste collection is done by company JKP Mediana. This company has a large transport costs for emptying the containers due to lack of optimal route of the vehicle. Therefore, vehicles are moving in the opinion of the driver or by a previous practice. In the process of determining the optimal route of the vehicle for collection plastic waste in the city of Niš, the first issue was to define location of containers. Locations of containers are defined with geo coordinates (table 1) so that each site has its own coordinate.

Table 1: Position of WDSs

WDS	Coordinate of WDS		WDS	Coordinate of WDS	
	latitude	longitude		latitude	longitude
A	53.214	19.155	K	52.349	18.924
B	53.560	19.221	L	49.763	19.076
C	53.280	19.167	M	52.758	18.790
D	53.111	19.220	N	52.988	18.920
E	54.200	19.210	O	53.719	19.194
F	54.390	19.185	P	53.684	19.031
G	53.848	19.079	Q	54.058	19.269
H	53.721	19.170	R	54.076	19.399
I	53.603	19.027	S	54.390	19.399
J	53.494	18.969	T	49.763	18.790

The coordinates are latitude and longitude locations. To simplify the implementation, the coordinates given in table 1 are scaled using the equation 1 and then are presented to the SOM. Determining the optimal the vehicle routes significantly reduces transportation costs and reduces the total discharge time of 20 containers.

**THE EXPERIMENTAL RESULTS**

The paper presents three simulations. The first simulation (figure 4b) was performed with 1000 epochs and the learning rate 0.1. This corresponds to the first phase in which neurons tend to WDC. The second simulation was performed with 2000 epochs and the learning rate 0.1. An extension of learning can be seen that all the neurons are closer to WDC and some are in the centers of these coordinates. The third simulation (figure 4c) corresponds to the maximum number of epochs. All  $m$  neurons coincided with WDC. The route can be read from the weighting coefficients of neurons. Under this optimum vehicle route is:

**M-T-L-K-I-A-D-B-O-E-Q-R-S-F-H-C-G-P-J-N-M.**

The resulting route of vehicles using the SOM model is compared to the route which was obtained by heuristic methods Clark-Wright savings algorithm [11].

Routes are the same length only difference is that it does not start from the same node.

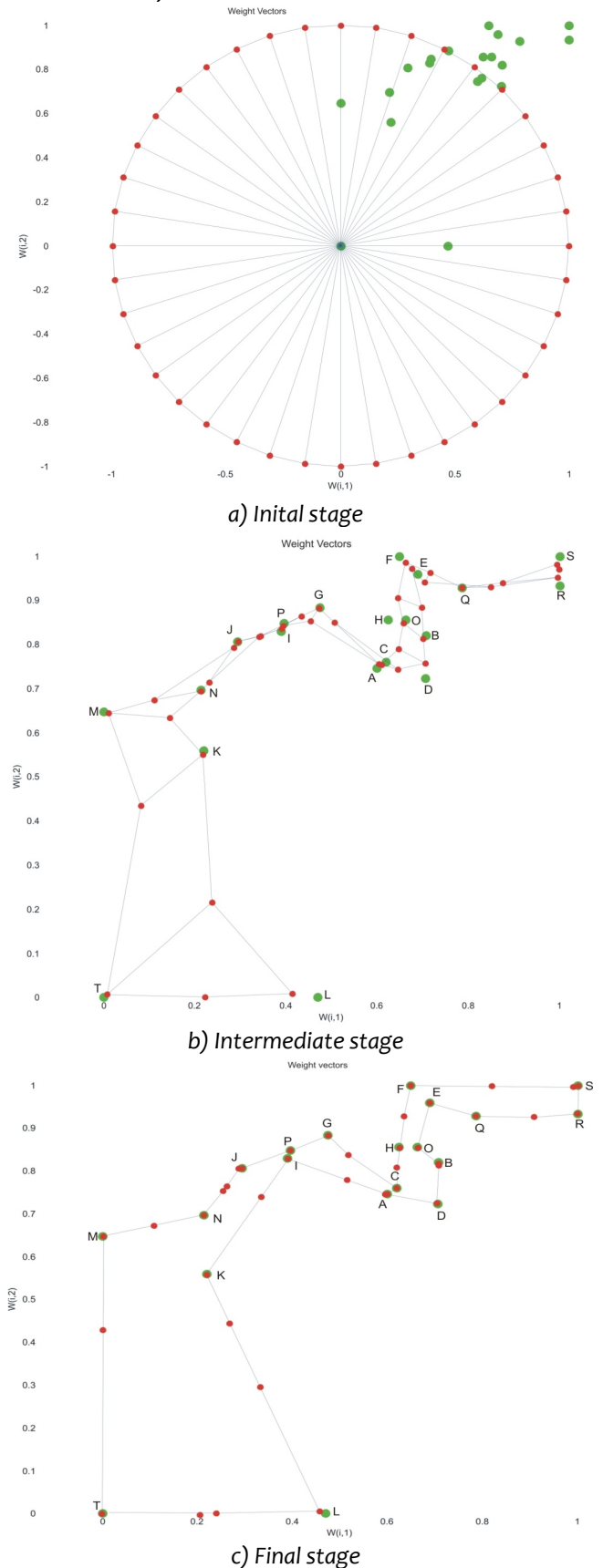


Figure 4. Evolution of Kohonen's SOM for TSP

**CONCLUSIONS**

The method of solving the TSP presented in this paper is based on Kohonen's SOM. This method represents an alternative for collection of plastics waste in the city of Niš. The advantages of this method are:



- Easy implementation and fast computation, robust applicability, production of good solutions.
- Based on our experiments it can be concluded that SOM provides flexible and quick means to obtain optimized routs for collecting plastic waste.
- Although the concept of using SOM for this task was shown to be viable, additional work must be done to obtain improved results.

But each method has some disadvantages:

- Some of the SOM parameters need to be optimized such as learning rate, neighborhood distance and number of iterations.

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