[1.] A. NIKKAR

A NEW APPROACH FOR SOLVING GAS DYNAMIC EQUATION

[1.] Department of Civil Engineering, University of Tabriz, Tabriz, IRAN

ABSTRACT: In this paper, a new analytical technique called the Reconstruction of Variational Iteration Method (RVIM) is suggested for finding the exact solution of gas dynamic equation. The solution procedure explicitly reveals the complete reliability and simplicity of the proposed algorithm. The RVIM technique has been successfully applied to many nonlinear problems in science and engineering. All of these verify the great potential and validity of the RVIM technique in comparison with Variational Iteration Method (VIM) for strongly nonlinear problems in science and engineering.
KEYWORDS: Reconstruction of Variational Iteration Method (RVIM), Gas dynamic equation; Analytical solution, Adomian decomposition method (ADM)

INTRODUCTION

Gas dynamics is a science in the branch of fluid dynamics concerned with studying the motion of gases and its effects on physical systems, based on the principles of fluid mechanics and thermodynamics. The science arises from the studies of gas flows, often around or within physical bodies, especially at speeds comparable to the speed of sound or beyond, and sometimes with a significant change in gas and objects temperatures [1].

Some examples of these studies include but not limited to choked flows in nozzles and valves, shock waves around jets, aerodynamic heating on atmospheric reentry vehicles and flows of gas fuel within a jet engine. At the molecular level, gas dynamics is a study of the kinetic theory of gases, often leading to the study of gas diffusion, statistical mechanics, chemical thermodynamics and non-equilibrium thermodynamics [2].

Gas dynamics is synonymous with aerodynamics when the gas field is air and the subject of study is flight. It is highly relevant in the design of aircraft and spacecraft and their respective propulsion systems. Progress in gas dynamics coincides with the developments of transonic and supersonic flight. As aircraft began to travel faster, the density of air began to change, considerably increasing the air resistance as the air speed approached the speed of sound.

The phenomenon was later identified in wind tunnel experiments as an effect caused by the formation of shock waves around the aircraft. Major advances were made to describe the behavior during and after World War II, and the new understandings on compressible and high speed flows became theories of gas dynamics.

Most phenomena in real world are described through nonlinear equations and these kinds of equations have attracted lots of attention among scientists. A wide range of nonlinear equations do not have a precise solution, so analytical methods have been used to handle these equations.

Many different new methods have recently presented some techniques to eliminate the small parameter; for example, Hirota's bilinear method [3], the homogeneous balance method [4], inverse scattering method [5], Adomian's decomposition method ADM [6], the Variational Iteration Method [7-10], Homotopy Perturbation Method [11-14] Energy Balance Method [15], as well as Homotopy Analysis Method (HAM)[16].

One of the newest analytical methods to solve nonlinear equations is Reconstruction of variational Iteration Method (RVIM) which is an accurate and a rapid convergence method in finding the approximate solution for nonlinear equations.

By applying Laplace Transform, RVIM overcomes the difficulty of the perturbation techniques and other variational methods in case of using small parameters and Lagrange multipliers, respectively. Reducing the size of calculations and omitting the difficulty arising in calculation of nonlinear intricately terms are other advantages of this method.

In this paper RVIM has been applied for finding the exact solution of following equation:

$$\frac{\partial u}{\partial t} + 1/2\frac{\partial(u^2)}{\partial x} = u(1-u) + g(x,t); \quad 0 \le x \le 1, t > 0 \quad (1)$$

BASIC IDEA OF RVIM

To clarify the basic ideas of our proposed method in [17, 18], we consider the following differential equation same as VIM based on Lagrange multiplier [19]:

$$Lu(x_1, \cdots, x_k) + Nu(x_1, \cdots, x_k) = f(x_1, \cdots, x_k) \quad (2)$$

By suppose that

$$Lu(x_1, \cdots, x_k) = \sum_{i=0}^{k} L_{xi}u(x_i) \quad (3)$$

where L is a linear operator, N a nonlinear operator and $f(x_1, \cdots, x_k)$ an inhomogeneous term.

We can rewrite equation (2) down a correction functional as follows:

$$L_{xj}u(x_j) = \underbrace{f(x_1, \cdots, x_k) - Nu(x_1, \cdots, x_k) - \sum_{\substack{j=0 \\ i \ne j}}^{k} L_{xi}u(x_i)}_{h((x_1, \cdots, x_k), u(x_1, \cdots, x_k))} \quad (4)$$

therefore

$$L_{xj}u(x_j) = h((x_1, \cdots, x_k), u(x_1, \cdots, x_k)) \quad (5)$$

with artificial initial conditions being zero regarding the independent variable x_j.

By taking Laplace transform of both sides of the equation (5) in the usual way and using the artificial initial conditions, we obtain the result as follows

$$P(s).U(x_1,\cdots,x_{i-1},s,x_{i+1},x_k)=H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u) \quad (6)$$

where $P(s)$ is a polynomial with the degree of the highest derivative in equation (6), (the same as the highest order of the linear operator L_{x_i}). The following relations are possible;

$$\ell[h] = H \quad (7\text{-}a)$$

$$B(s) = \frac{1}{P(s)} \quad (7\text{-}b)$$

$$\ell[b(x_i)] = B(s) \quad (7\text{-}c)$$

which that in equation (7-a) the function $H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u)$ and $h((x_1,\cdots,x_{i-1},x_i,x_{i+1},x_k),u)$ have been abbreviated as H, h respectively.

Hence, rewrite the equation (6) as;

$$U(x_1,\cdots,x_{i-1},s,x_{i+1},x_k) =$$
$$= H((x_1,\cdots,x_{i-1},s,x_{i+1},x_k),u).B(s) \quad (8)$$

Now, by applying the inverse Laplace Transform on both sides of equation (8) and by using the (7-a) - (7-c), we have;

$$u(x_1,\cdots,x_{i-1},x_i,x_{i+1},x_k) =$$
$$= \int_0^{x_i} h((x_1,\cdots,x_{i-1},\tau,x_{i+1},x_k),u).b(x_i-\tau)d\tau \quad (9)$$

Now, we must impose the actual initial conditions to obtain the solution of the equation (2). Thus, we have the following iteration formulation:

$$u_{n+1}(x_1,\cdots,x_{i-1},x_i,x_{i+1},x_k)$$
$$= u_o(x_1,\cdots,x_{i-1},x_i,x_{i+1},$$
$$\int_0^{x_i} h((x_1,\cdots,x_{i-1},\tau,x_{i+1},x_k),u).b(x_i-\tau)d\tau \quad (10)$$

where u_o is initial solution with or without unknown parameters. Assuming u_o is the solution of Lu, with initial/boundary conditions of the main problem, In case of no unknown parameters, u_o should satisfy initial/ boundary conditions. When some unknown parameters are involved in u_o, the unknown parameters can be identified by initial/boundary conditions after few iterations, this technology is very effective in dealing with boundary problems. It is worth mentioning that, in fact, the Lagrange multiplier in the He's variational iteration method is $\lambda(\tau) = b(x_i-\tau)$ as shown in [18].

The initial values are usually used for selecting the zeroth approximation u_o. With u_o determined, then several approximations $u_n \; n>0$, follow immediately. Consequently, the exact solution may be obtained by using:

$$u(x_1,\cdots,x_{i-1},x_i,x_{i+1},x_k) =$$
$$= lim_{n\to\infty} u_n(x_1,\cdots,x_{i-1},x_i,x_{i+1},x_k). \quad (11)$$

In order to assess the advantages and the accuracy of RVIM for solving gas dynamic equation, we will consider the following two examples. For the sake of comparison, we take the same examples as used in [20].

Example 2.1. Homogeneous gas Dynamic equation:

To apply the RVIM, first we rewrite Eq. (1) with $g(x, t) = o$ in the following form:

$$\frac{\partial u}{\partial t} = -1/2\frac{\partial(u^2)}{\partial x} + u - u^2, \quad (12)$$

With initial condition:

$$u_o(x,t) = e^{-x}, \quad (13)$$

At first rewrite eq. (1) based on selective linear operator as

$$\ell\{u(x)\} = u_t = \overbrace{-1/2\frac{\partial(u^2)}{\partial x} + u - u^2}^{h(x,t,u)} \quad (14)$$

Now Laplace transform is implemented with respect to independent variable x on both sides of eq. (14) and by using the new artificial initial condition (which all of them are zero) we have

$$sU(x,t) = \ell\{h(x,t,u)\} \quad (15)$$

$$U(x,t) = \frac{\ell\{h(x,t,u)\}}{s} \quad (16)$$

And whereas Laplace inverse transform of 1/s is as follows

$$\ell^{-1}[\frac{1}{s}] = 1 \quad (17)$$

Therefore by using the Laplace inverse transform and convolution theorem it is concluded that

$$u(x,t) = \int_0^t h(x,\varepsilon,u)d\varepsilon \quad (18)$$

Hence, we arrive the following iterative formula for the approximate solution of subject to the initial condition (13).

So, in exchange with applying recursive algorithm, following relations are achieved

$$u_{n+1} = u_o + \int_0^t (-1/2\frac{\partial(u^2)}{\partial x} + u - u^2)d\varepsilon \quad (19)$$

Now we start with an arbitrary initial approximation $u_o(x,t) = e^{-x}$, that satisfies the initial condition and by using the RVIM iteration formula (19), we have the following successive approximation

$$u_o(x,t) = e^{-x},$$

$$u_1(x,t) = te^{-x},$$

$$u_2(x,t) = \frac{t^2}{2}e^{-x},$$

$$u_3(x,t) = \frac{t^3}{3!}e^{-x},$$

$$u(x,t) = u_o + u_1 + u_2 + \dots = e^{-x}(1+t+\frac{t^2}{2}+\frac{t^3}{3!}+\dots) = e^{t-x}$$

Obtained upon using the Taylor expansion for e^t. Which is exactly the same as obtained by Adomain decomposition method [20], homotopy perturbation method [21] and the variational iteration method coupled with He's polynomials [22].

Example 3.2. Inhomogeneous gas Dynamic equation:

Now we rewrite Eq. (1) in the following form:

$$\frac{\partial u}{\partial t} = -1/2\frac{\partial(u^2)}{\partial x} + u - u^2 - e^{t-x}, \qquad (20)$$

With initial condition:

$$u_o(x,t) = 1 - e^{-x}, \qquad (21)$$

At first rewrite eq. (20) based on selective linear operator as

$$\ell\{u(x)\} = u_t = \overbrace{-1/2\frac{\partial(u^2)}{\partial x} + u - u^2 - e^{t-x}}^{h(x,t,u)} \qquad (22)$$

RVIM's iteration formulae in t-direction can be readily obtained.

$$u_{n+1} = u_o + \int_o^t (-1/2\frac{\partial(u^2)}{\partial x} + u - u^2 - e^{t-x})d\varepsilon \qquad (23)$$

Now we start with an arbitrary initial approximation $u_o(x,t) = 1 - e^{-x}$, that satisfies the initial condition and by using the RVIM iteration formula (23), we have the following successive approximation

$$u_o(x,t) = 1 - e^{-x},$$
$$u_1(x,t) = -e^{t-x} + e^{-x},$$
$$u_n(x,t) = 0, n \geq 2$$
$$u(x,t) = u_o + u_1 + u_2 + \ldots =$$
$$= 1 - e^{-x} \quad e^{t-x} + e^{-x} + 0 + 0 + \quad \ldots = 1 - e^{t-x}$$

which is exactly the same as obtained by Adomain decomposition method [20], homotopy perturbation method [21] and the variational iteration method coupled with He's polynomials [22].

CONCLUSIONS

In this paper, we successfully apply Reconstruction of Variational Iteration Method (RVIM) for finding the exact solution of gas dynamic equation.

The obtained solutions are compared with those of ADM, HPM and VIMHP. Simplicity and requiring less computation, rapid convergence, and high accuracy are advantages of this technique.

Moreover, the RVIM reduces the size of calculations by not requiring the tedious Adomian polynomials, and hence the iteration is direct and straightforward.

The results reported here provide further evidence of the usefulness of RVIM for finding the analytic and numeric solutions for the linear and nonlinear diffusion equations and, it is also a promising method to solve different types of nonlinear equations in mathematical physics.

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