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MHD FREE CONVECTION FLOW WITH VARIABLE VISCOSITY AND THERMAL DIFFUSIVITY ALONG A MOVING VERTICAL PLATE EMBEDDED IN A POROUS MEDIUM

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ABSTRACT: This paper investigates a study of the flow of a viscous incompressible fluid along a heated vertical porous plate, taking into account the variation of the viscosity and thermal diffusivity in the presence of the magnetic field. The governing partial differential equations of the flow field are transformed into ordinary differential equations by means of similarity transformation. The resultant equations are solved numerically using Runge-Kutta fourth order method along with shooting technique. The effects of variable thermo-viscous parameters, magnetic parameter, permeability parameter and suction parameter on the velocity, temperature, skin-friction coefficient and Nusselt number are obtained and discussed in detail.
KEYWORDS: Variable viscosity, thermal diffusivity, magnetic field and porous medium

INTRODUCTION

Natural convection flows driven by temperature differences are of great interest in a number of industrial applications. Buoyancy is also of importance in an environment where differences between land and air temperature can give rise to complicated flow patterns, and in enclosures such as ventilated and heated rooms and reactor configurations. Natural convection flows driven by temperature differences have been studied extensively. For example, Pohlhausen [11] first studied the steady free convection flow past a semi-infinite vertical plate by integral method. But the similarity solution to steady free convection flow past a semi-infinite vertical plate was presented by Ostrach [10], who solved the ordinary non-linear equations by a numerical method. Siegel [17] was the first to study the transient free convection flow past a semi-infinite vertical plate by integral method. The same problem was studied by Gebhart [3] by an approximate method.

In all the above studies, the free convection flow along a vertical flat plate was restricted, in general, to the case where the temperature difference between the plate and the fluid is small, so that the fluid properties may be taken as constant. For the fluids, which are important in the theory of lubrication, the heat generated by the internal fiction and the corresponding rise in temperature do affect the viscosity and thermal diffusivity of the fluid and they can no longer be regarded as constant. The physical properties of fluids such as viscosity and thermal diffusivity may change significantly with temperature. The temperature dependent property problem is further complicated by the fact that the properties of different fluids behave differently with temperature. Different relations between the physical properties of fluids and temperature are given by Kays and Grawford [8]. Mehta and Sood [9] have shown that when this effect is included, the flow characteristics may be substantially changed compared to the constant viscosity case. The influence of variable viscosity on the laminar boundary layer flow and heat transfer due to a continuously moving flat plate is examined by Pop et al. [12]. Kafoussias and Williams [7] investigated the effect of temperature-dependent viscosity on free-forced convective laminar boundary layer flow past a vertical isothermal plate. Hossain and Munir [4] analyzed a two-dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity past a vertical plate. Elbashbeshy and Ibrahim [2] investigated the steady free convection flow with variable viscosity and thermal diffusivity along a heated vertical plate.

Hydromagnetic flows and heat transfer have become more important in recent years because of its varied applications in agriculture, engineering and petroleum industries. The free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field has been studied by Elbashbeshy [1]. The effect of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi-infinite vertical porous plate was studied by Seddek and Salama [16].

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The heat transfer problem from different geometries embedded in porous media has many practical applications in industrial and technological fields such as geothermal reservoirs, drying of porous solids, thermal insulation, and enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors and under ground energy transport. Raptis [13] considered mathematically the case of time-varying two-dimensional natural convection flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate through a porous medium. Raptis et al. [14] analyzed hydromagnetic free convection flow through a porous medium between two parallel plates.

However the impact of variable viscosity and thermal diffusivity of a hydromagnetic free convection flow along a vertical porous plate embedded in a porous medium has received little attention. Hence an attempt is made to study the effects of variable viscosity and thermal diffusivity on a steady two-dimensional free convection flow of a viscous incompressible electrically conducting fluid along a vertical porous plate embedded in a porous medium.

The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, skin-friction coefficient and Nusselt number are shown in figures and tables and analyzed in detail.

MATHEMATICAL ANALYSIS

A steady two-dimensional laminar free convection flow of a viscous incompressible electrically conducting fluid along a moving semi infinite vertical flat plate embedded in a porous medium is considered. The flow is assumed to be in the x-direction, which is taken along the plate and y-axis normal to the plate. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that the electric field is absent. All the physical properties of the fluid are assumed to be constant except for the fluid viscosity, which varies exponentially with the fluid temperature, the thermal conductivity which varies linearly with the fluid temperature and the density variation in the body force term in the momentum equation where the Boussinesq's approximation is invoked. Under these assumptions, the conservation equations of the laminar boundary layer flow under consideration are

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\rho g\left(T-T_\infty\right)-\sigma B_0^2 u-\frac{\mu}{K'}u \qquad (2)$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right) \qquad (3)$$

where u and v are the velocity components in the x- and y- directions respectively, ρ - the density of the fluid, μ - the variable dynamic coefficient of viscosity, g - the gravitational acceleration, T - the temperature of the fluid, T_∞ - the temperature far away from the plate, σ - the electrical conductivity of the fluid, B_0 - the magnetic induction, K' - the permeability of the porous medium and α - the variable thermal diffusivity of the fluid.

The boundary conditions for the velocity and temperature fields are

$$u=U_0\,,\ v=v_0(x)\,,\ T=T_w \ \text{ at }\ y=0$$
$$u\to 0\,,\ T\to T_\infty \ \text{ as }\ y\to\infty \qquad (4)$$

where U_0 is the uniform velocity, $v_0(x)$ - the velocity of suction at the plate and T_w - the temperature of the plate.

The mass conservation equation (1) is satisfied by the stream function $\psi(x,y)$

$$u=\frac{\partial\psi}{\partial y}\,,\ v=-\frac{\partial\psi}{\partial x} \qquad (5)$$

To transform equations (2) and (3) into a set of ordinary differential equations, the following dimensionless variables are introduced:

$$\eta=y\sqrt{\frac{U_0}{2\upsilon x}}\,,\ \psi=\sqrt{2\upsilon xU_0}\,f(\eta)\,,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\,,$$
$$Gr=\frac{g\beta(T_w-T_\infty)2x}{U_0^2}\,,\ M=\frac{\sigma B_0^2\,2x}{\rho U_0}\,,$$
$$K=\frac{2\upsilon x}{K'U_0}\,,\ Pr=\frac{\upsilon}{\alpha_0}\,,\ \upsilon=\frac{\mu_0}{\rho}\,,\ \alpha=\frac{k}{\rho c_p} \qquad (6)$$

where θ is the non-dimensional temperature function, Gr - the thermal Grashof number, M - the magnetic field parameter, K - the permeability parameter, Pr - the Prandtl number, υ - the kinematic viscosity, α_0 - the thermal diffusivity at temperature T_w, μ_0 -the viscosity at temperature T_w, k - the thermal conductivity and c_p - the specific heat at constant pressure.

The variations of viscosity and thermal diffusivity with the dimensionless temperature are written in the form (Ibrahim and Ibrahim [5], Slattery [18])

$$\frac{\mu}{\mu_0}=e^{-\beta\theta} \qquad (7)$$

and

$$\frac{\alpha}{\alpha_0}=1+\gamma\theta \qquad (8)$$

where β and γ are the parameters depending on the nature of the fluid.

In view of equations (6)-(8), the equations (2) and (3) transform into

$$f'''+f''\left[e^{\beta\theta}f-\beta\theta'\right]+Gr\theta e^{\beta\theta}-(Me^{\beta\theta}+K)f'=0 \qquad (9)$$

$$\theta'' + \left(\frac{Pr\,f}{1+\gamma\theta}\right)\theta' + \left(\frac{\gamma}{1+\gamma\theta}\right)(\theta')^2 = 0 \qquad (10)$$

The corresponding boundary conditions are

$$f = f_w\,,\ f' = 1\,,\ \theta = 1 \text{ at } \eta = 0$$

$$f' = 0\,,\ \theta = 0 \text{ as } \eta \to \infty \qquad (11)$$

where f is the dimensionless stream function, $f_w = -v_0\sqrt{\frac{2x}{\upsilon U_0}}$ is the dimensionless suction velocity and prime denotes differentiation with respect to the variable η.

SOLUTION OF THE PROBLEM

The non-linear governing boundary layer equations (9) and (10) together with the boundary conditions (11) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential equations (9) and (10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et al. [6]).

The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta = 0.05$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence.

From the process of numerical computation, the skin-friction coefficient and the Nusselt number are also obtained and are presented in a tabular form.

RESULTS AND DISCUSSIONS

The parameters of the flow β, γ and Pr can be taken as follows (H. Schlichting [15], E.M.A. Elbashbeshy [1]): for air: $-0.7 \le \beta \le 0$, $0 \le \gamma \le 6$, $Pr = 0.733$.

The effects of magnetic field parameter M, permeability parameter K, β, γ, thermal Grashof number Gr and suction parameter f_w on the velocity are shown in Figures 1-6.

It is observed that the velocity decreases as the magnetic parameter increases (Figure 1). It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter.

The parameter K as defined in equation (6) is inversely proportional to the actual permeability K' of the porous medium. An increase in K will therefore increase the resistance of the porous medium (as the permeability physically becomes less with increasing K') which will tend to decelerate the flow and reduce the velocity. This behavior is evident from Figure 2.

From Figure 3, it is clear that the velocity near to the vertical plate ($\eta =$ constant) increases as β increases (the viscosity of air decreases). But an opposite effect is noticed at a certain distance from the plate ($\eta_0 \cong 1$). Figure 4 shows that the velocity in the fluid increases as γ increases (the thermal diffusivity of air increases) for fixed values of β. Moreover, the rise in the magnitude of the velocity is quite significant in the present case, showing that the volume rate of flow at a section perpendicular to the plate increases with an increase in γ.

The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer.

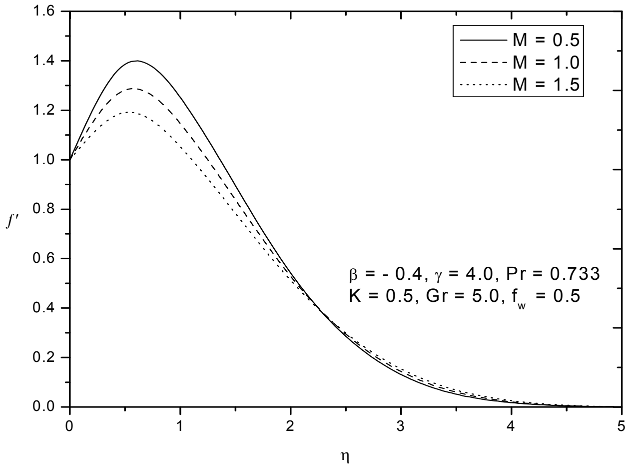

Figure 1. Velocity profiles for different values of M

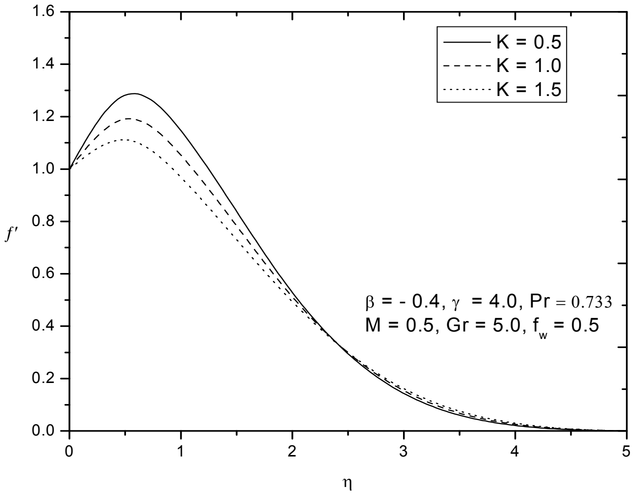

Figure 2. Velocity profiles for different values of K

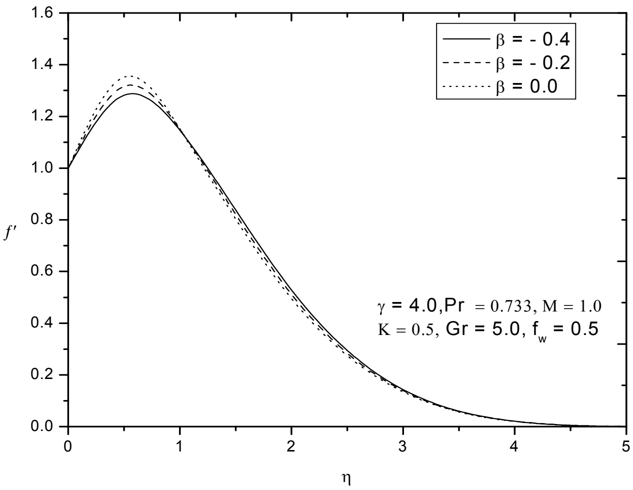

Figure 3. Velocity profiles for different values of β

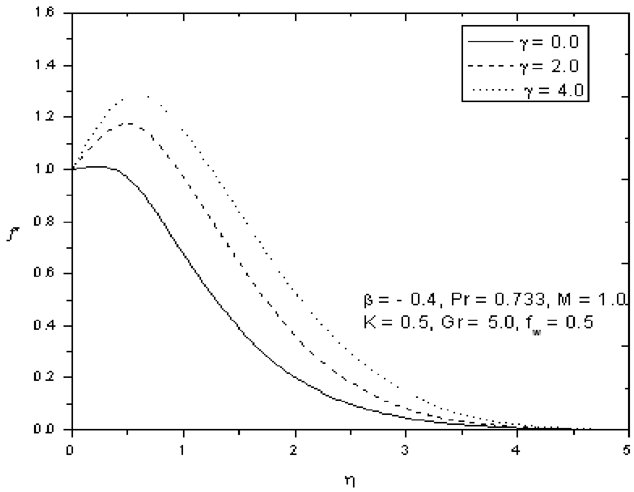

Figure 4. Velocity profiles for different values of γ

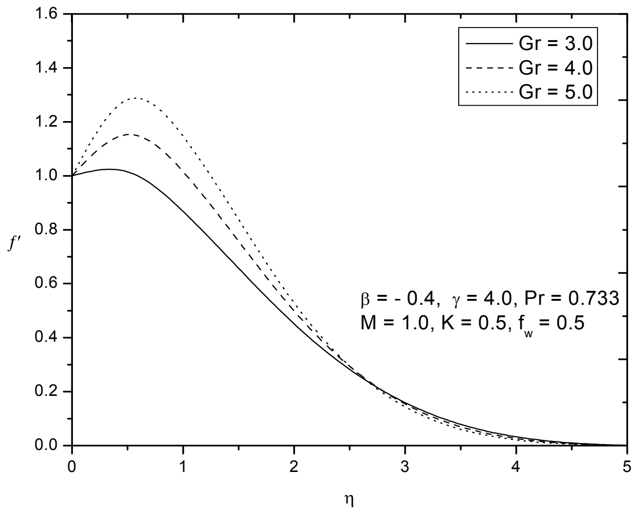

Figure 5. Velocity profiles for different values of Gr

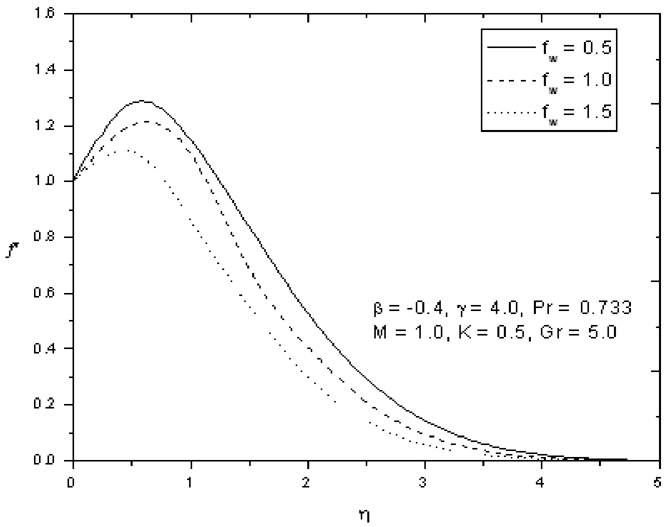

Figure 6. Velocity profiles for different values of f_w

As expected, from Figure 5, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

From Figure 6, it is noticed that an increase in the suction parameter results in a decrease in the velocity.

The effects of magnetic field parameter M, permeability parameter K, β, γ, thermal Grashof number Gr and suction parameter f_w on the temperature are shown in Figures 7-12.

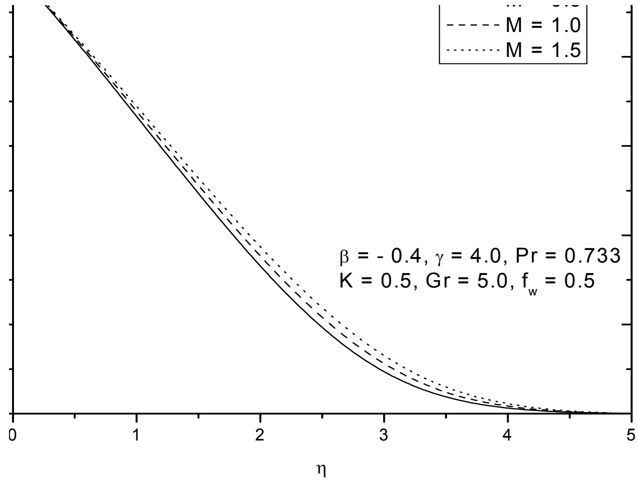

Figure 7. Temperature profiles for different values of M

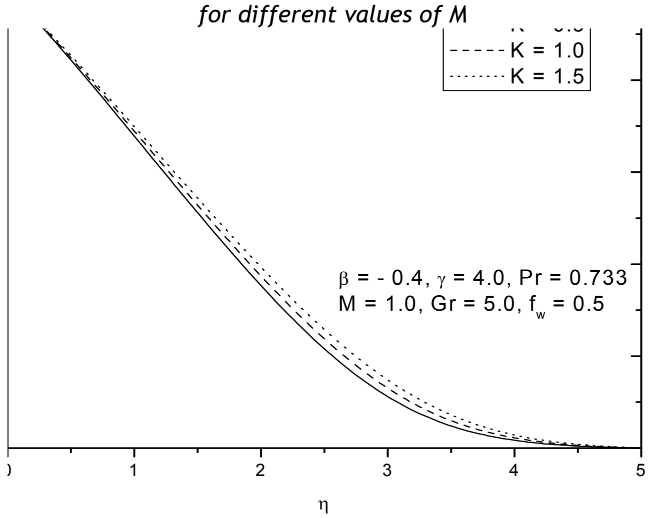

Figure 8. Temperature profiles for different values of K

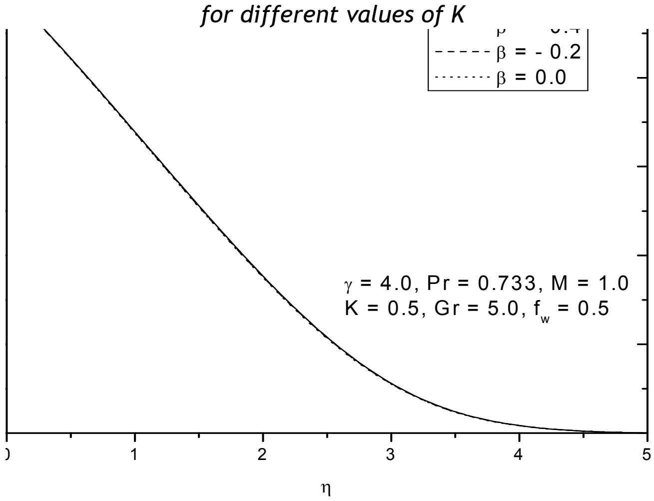

Figure 9. Temperature profiles for different values of Β

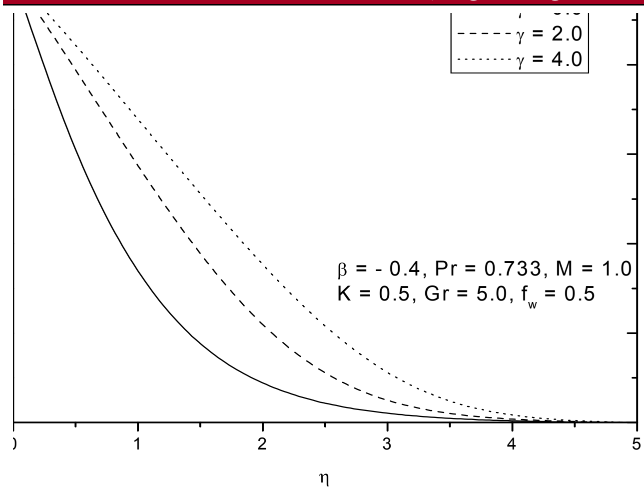

Figure 10. Temperature profiles
for different values of γ

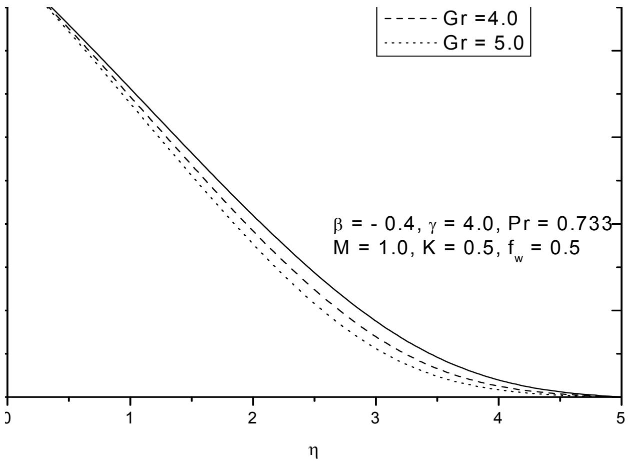

Figure 11. Temperature profiles
for different values of Gr

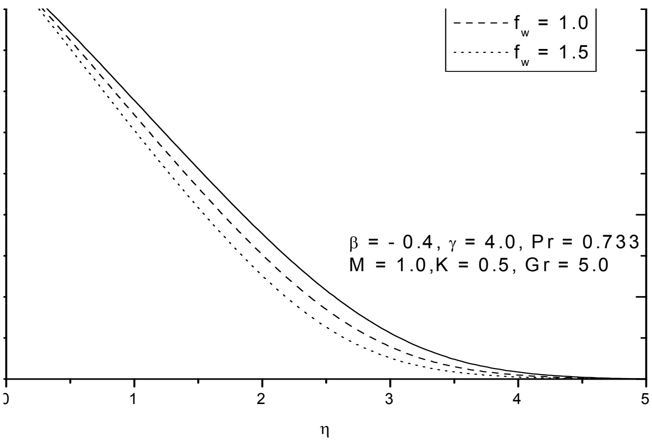

Figure 12. Temperature profiles
for different values of f_w

It is observed that greater magnetic parameter causes a rise in the temperature (Figure 7). An increase in the permeability parameter causes a rise in the temperature. This is obvious from Figure 8. From Figure 9, it is noticed that as β increases, the temperature decreases and the decrement is very small.

Figure 10 reveals that the temperature in the fluid increases as γ increases (the thermal diffusivity of air increases) for fixed values of β. Moreover, in this case, the rise in the magnitude of the temperature is quite significant. It is seen that an increase in the Grashof number results in a decrease in the temperature (Figure 11).

Figure 12 illustrates that an increase in the suction parameter results in a decrease in the temperature.

Table 1. Skin-friction coefficient C_f and Nusselt number Nu for Pr = 0.733.

B	γ	M	K	Gr	f_w	C_f	Nu
-0.4	4	1.0	0.5	5.0	0.5	1.1398	0.3009
-0.2	4	1.0	0.5	5.0	0.5	1.3508	0.3022
-0.4	2	1.0	0.5	5.0	0.5	0.8967	0.4102
-0.4	4	2.0	0.5	5.0	0.5	0.6051	0.2832
-0.4	4	1.0	1.0	5.0	0.5	0.8579	0.2917
-0.4	4	1.0	0.5	4.0	0.5	0.7108	0.2875
-0.4	4	1.0	0.5	5.0	1.0	1.0135	0.3415

The effects of the β , γ , M , K, Gr and f_w on the skin-friction coefficient and Nusselt number are shown in Table 1. It is seen that, as β or K or Gr increases, the skin-friction coefficient as well as the Nusselt number increases.

Also, as γ increases, the skin-friction coefficient increases, whereas the Nusselt number decreases. Further, as M increases, there is a fall in both the skin-friction coefficient and Nusselt number. It is observed that as f_w increases the skin-friction coefficient decreases, whereas the Nusselt number increases.

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