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APPLICATION OF ALGEBRAIC INVERSE METHOD TO SURFACE WAVE TESTING OF PAVEMENTS USING FREE PLATE SOLUTION

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ABSTRACT: The use of surface waves of the Rayleigh type enables the properties of the component materials of a layered structure, such as a pavement, to be determined. The method has the advantage that the measurements are performed dynamically, thus making an allowance for the inertial and frequency dependent response of the pavement. The velocity of the waves is not a constant, but exhibits dispersion. The manner of the variation of the wave velocity is used to determine the properties of the surface and subsurface materials of the structure. Inverse problems are involved in interpreting the results. Wave propagation in elastic plates is analogous to propagation in layered spaces and therefore the free plate system can be used to model the surface layer of a single layered pavement structure. A direct algebraic inverse technique has been developed and used to calculate the thickness of the surface layer of a pavement system employing free plate analysis. To assess the effectiveness and reliability of the proposed technique, the application of the method to a published set of experimental data obtained in the field is presented and discussed. It is shown that the experimental results and the algebraic solutions are in good agreement, indicating that the proposed method can be used to determine the thickness of the pavement surface layer without resorting to excavation and in a very quick and economic manner.

KEYWORDS: Algebraic inverse, Free plate, Pavement, Surface wave testing

INTRODUCTION

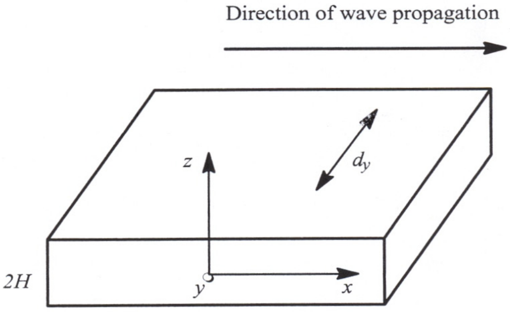
The determination of in-situ elastic properties of pavement materials along with layers thicknesses is of great importance in pavement management system. The information is needed in order to (1) design the constructed layers in such a manner that the imported materials are strained only to within acceptable limits, and (2) locate and to characterize zones of weakness. The use of surface waves of the Rayleigh type enables this information to be determined, with the advantage that the measurements are conducted dynamically, thus making an allowance for the inertial and frequency dependent response of the pavement. The use of Rayleigh wave enables an estimate to be formed of the thickness of the surface layer of a pavement. The precision of the estimate depends upon the contrast between the elastic properties of the materials in the surface and the underlying layers. The greater the contrast, the more precise is the estimate. The measurement of the deflections at the surface of a pavement, in the neighborhood of a known load, can also be used to estimate the elastic moduli of the subsurface materials. It is preferable, although not essential, to make separate measurements of the thicknesses, and to supply the thicknesses of the component media as data. The measurement of surface deflections is widely used as a means of pavement investigation. The interpretation of the results yields information on the elastic moduli of the component materials.

WAVE METHODS

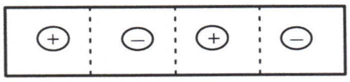
Waves of the Rayleigh type are generated by applying an impulsive load, a hammer blow, at right angles to the surface of the pavement. SH-waves are generated by applying an oscillatory torque to the free surface: the axis of the torque is normal to the free surface. Sensors are placed at suitable distances from the source of the vibration. It is sufficient to perform measurements at two sensors. The velocity of the generated wave is determined as a function of the frequency, enabling a frequency-dispersion curve to be plotted. This curve shows the phase velocity of a wave at the surface of the structure. The frequency-dispersion data can be used to determine the properties of the media of which the structure is composed. Graphical methods sometimes suffice for approximate results, although most systems must be analyzed with the aid of relevant solutions of the wave equation of the particular system studied.

FREE PLATE SOLUTION

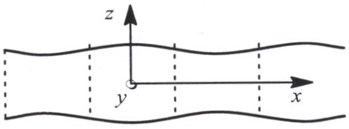
The simplest assumption is that the structure consists of a free plate. Free plate solutions for the propagation of plane stress waves can be used to determine the engineering properties of the surface layer of a layered medium from the frequency-dispersion data of the system obtained in the field. A number of different types of vibrations can exist in a solid plate (or layer) with free boundaries. In a free isotropic elastic plate of thickness 2H, three types of particle motions can occur. These types of particle motions are often called wave modes and depend on *the characteristics and stiffness of the transmission medium, existing boundary conditions and the frequency of the wave. Figure 1 shows these different wave modes in a free isotropic plate.*



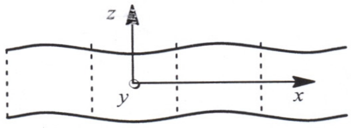
(a) – Undisturbed



(b) – Shear Wave



(c) – Symmetrical (Longitudinal) Wave



(d) – Antisymmetrical (Flexural) Wave

Figure 1. Wave modes in free isotropic elastic plate

Plane waves in plates can be described as symmetrical (longitudinal) and antisymmetrical (flexural) waves depending upon the symmetry with respect to the median plane of the plate. The antisymmetrical wave corresponds approximately with the Rayleigh waves [4]. For symmetrical and antisymmetrical modes, the wavelength (or phase velocity) and frequency of the wave are related by the corresponding characteristic equations. The characteristic equation for the antisymmetrical mode is as follows [2]:

$$\frac{\tanh qH}{\tanh sH} = \frac{4k^2 qs}{(s^2+k^2)} \qquad (1)$$

where

$$q^2 = \omega^2 \left(\frac{1}{c^2} - \frac{1}{V_P{}^2}\right) \qquad (2\text{-a})$$

$$s^2 = \omega^2 \left(\frac{1}{c^2} - \frac{1}{V_S{}^2}\right) \qquad (2\text{-b})$$

$$k = \frac{\omega}{c}; \quad k_P = \frac{\omega}{V_P}; \quad k_S = \frac{\omega}{V_S} \qquad (2\text{-c})$$

INVERSE PROBLEM

The problem is to determine the unknown parameters of the component materials of the surface layer of a pavement structure, based on the wave propagation theory in the free plate. The unknown parameters are the elastic modulus (whether Young's or shear modulus) and thickness of the uppermost layer in a given pavement system.

Most of the inverse techniques developed thus far are based on the numerical methods, which have been used as a criterion for back calculation of the shear wave velocities or shear moduli and thicknesses of the pavement layers. With the numerical inverse method, an initial layered model is assumed for the system, and the theoretical dispersion curve is obtained by employing the theory of wave propagation in elastic layered media. Then the necessary structural changes are iteratively made until the experimental and theoretical dispersion curves are matched adequately. Thus the user must be familiar with surface wave dispersion theory in order to know what structural parameters to adjust, whether to decrease or increase the magnitudes and if so by how much. Therefore this process is time consuming and considerably dependent on the experience of the user. Also it requires engineering judgment and finally the solution is not a unique one. Other possible solutions must be sought and developed.

Graphical methods of interpretation can be applied to the results of measurements obtained from field measurements conducted on a layered structure [1]. The reciprocal of the wavelength is plotted against the frequency, from which the elastic parameters of the surface layer can be derived. The plot of reciprocal of wavelength against the frequency consists of data points which fall on a straight line and the reciprocal of the slope of line gives the shear wave velocity. By employing the graphical inverse method for determining the elastic shear modulus of the surface layer, the thickness of the layer is the only unknown parameter which can be calculated directly from the characteristic equation of the system.

ALGEBRAIC INVERSE

With algebraic inverse method, the characteristic equation of the system is expanded as a polynomial and is solved for the unknown parameters of the system. The characteristic equation which corresponds with Rayleigh waves can be represented by power series. By making power series, the equation is expanded symbolically in terms of the layer thickness parameter. Thickness of the layer, as illustrated in Fig. 1, is 2H.

The following file shows the necessary input to Mathematica [5]:

```
(* Expansion of the free plate equation, Mathematica file *)
k = w / c ;   kP = w / VP ;   kS = w / VS
q = Sqrt [ k ^ 2 - kP ^ 2 ]
s = Sqrt [ k ^ 2 - kS ^ 2 ]
Do [ fH [ i ] := Series [ Tanh [ q H ] / Tanh [ s H ] , {H , 0 , i } ] , { i , 2 , l , 1 } ]
Do [ gH [ i ] := fH [ i ] - q / s , { i , 2 , l , 1 } ]
h := 4 k^ 2 q s / ( s ^ 2 + k^ 2 ) ^ 2 - q / s
Do [ aH [ i ] := Normal [ InverseSeries [ gH [ i ] , H ] ] , { i , 2 , l , 1 } ]
Do [ bH [ i ] := aH [ i ] / . H -> h , { i , 2 , l , 1 } ]
Do [ H [ i ] := bH [ i ] / . { VP -> V_P , VS -> V_S , c -> c , w-> ω } , { i , 2 , l , 1 } ]
```

This input yields a series of the power series expansion for the left hand side of the characteristic equation about the point H = 0 to the order H^{i}, where the variable I successively takes on the values 2 through 15 in steps of 1. The value of I equating to 15 is found by trial and error. The value of I is increased at each step by one and the calculation is repeated until the difference between the two successive results become negligible. Inverse Series performs reversion of the series, which gives a series for the inverse of the function represented by gH [i]. The inverse can only be calculated when the first term in the gH [i] is of order H. The normal expression aH [i] now is a polynomial representing H in terms of the velocities of longitudinal and shear waves in the surface layer and of measured values of the frequency and the corresponding surface phase velocity. The frequency and the surface phase velocity are obtained from the results of the measurements made in the field on the ground surface of the system, and are data pairs ω and c. The limiting velocity of waves of the Rayleigh type in the component material of the surface layer can be read directly from the results of the field measurements. If a value of Poisson's ratio is assumed, the parameters V_P and V_S can be estimated. The resulting explicit function representing the thickness of the layer can be directly used to calculate the layer thickness 2H.

APPLICATION TO EXPERIMENTAL RESULTS

As an application, the data shown below on Fig. 2 are based on the experimental results given by Jones [3]. His investigation was on a pavement section and the results were obtained from experiments on a concrete slab with $\nu = 0.25$ for Poisson's ratio and 9.5 inches in thickness, laid on a thick ground base.

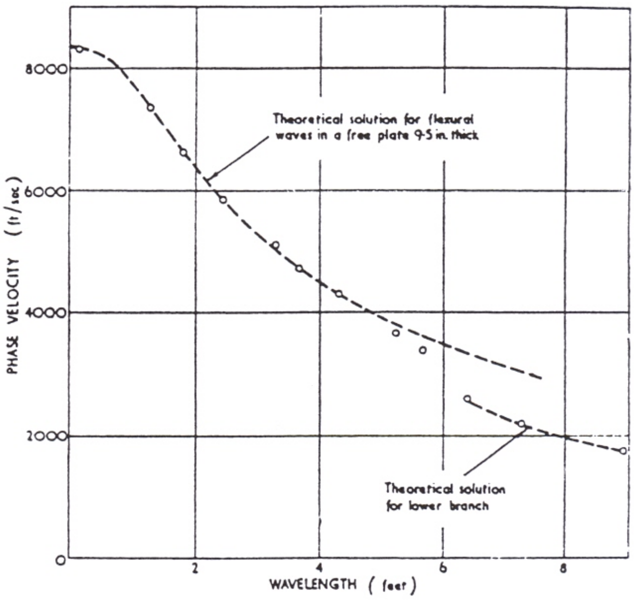


Figure 2. Results from concrete slab laid on well-compacted hoggin [3]

This example shows the results of the measurements of the phase velocities of waves of the Rayleigh type as a function of the wavelength on a practical construction of the single layered surface. The value of Rayleigh wave velocity was 8350 feet per second, from which the shear and longitudinal wave velocities are calculated to be 9100 ft/sec and 15760 ft/sec, respectively.

Table 1. Experimental results

Frequency ω (rad/sec)	Phase Velocity c (ft/sec)	$k=\frac{\omega}{c}$	$k_P=\frac{\omega}{V_P}$	$k_S=\frac{\omega}{V_S}$
37196	7400	5.03	2.36	4.09
23650	6700	3.53	1.50	2.60
15315	5850	2.62	0.97	1.68
9860	5100	1.93	0.63	1.08
8203	4700	1.75	0.52	0.90
6225	4260	1.46	0.39	0.68
4261	3650	1.20	0.27	0.47
3759	3350	1.21	0.24	0.41

Numerical values of the phase velocity and the frequency are supplied as data. These data have been summarized and shown in Tab. I. The calculated surface layer thickness for different values of expansion order, applied to the characteristic equation of the free plate for making power series in terms of H, are plotted in Fig. 3.

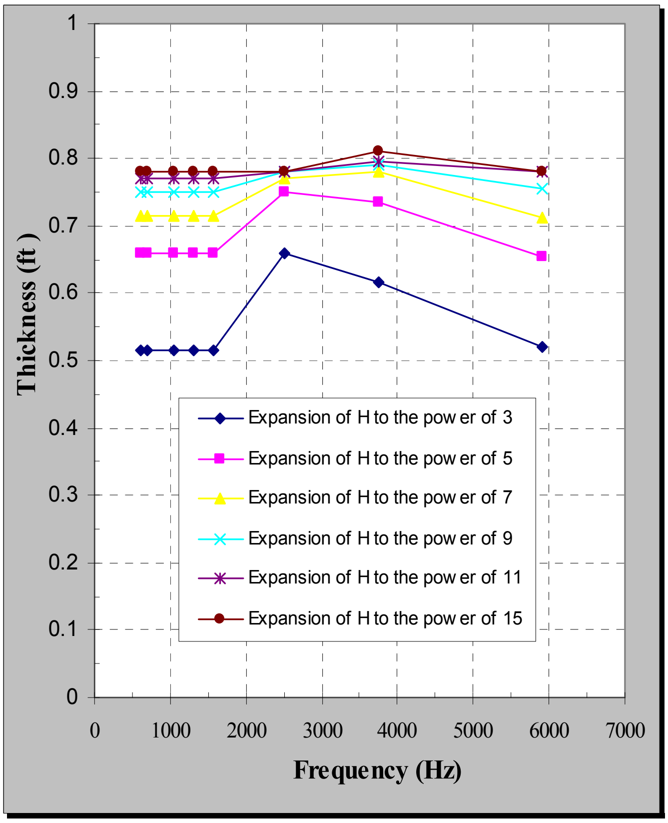


Figure 3. Variation of free plate thickness at different expansion powers

Figure 3 shows the calculated depth as a function of the frequency. It can be seen that as the expansion order increases, the calculated thickness becomes convergent towards the measured value. The average layer thickness obtained from the thicknesses calculated for each pair of dispersion data is also shown in Fig. 4. This figure shows that the average thickness increasingly becomes convergent as expansion order increases. In other words, if the average thickness approaches the real existing value, it does not change with increase in the expansion order. It is evident from Fig. 4 that the average H remains constant at expansion order equal to and/or *greater than 13. According to the calculated results shown in Fig. 4, the solution of the thickness at I = 13 is equal to 0.79 ft which is comparable to measured depth of 0.80 ft. It can be concluded that the experimental results and the algebraic inverse solutions are in best agreement.*

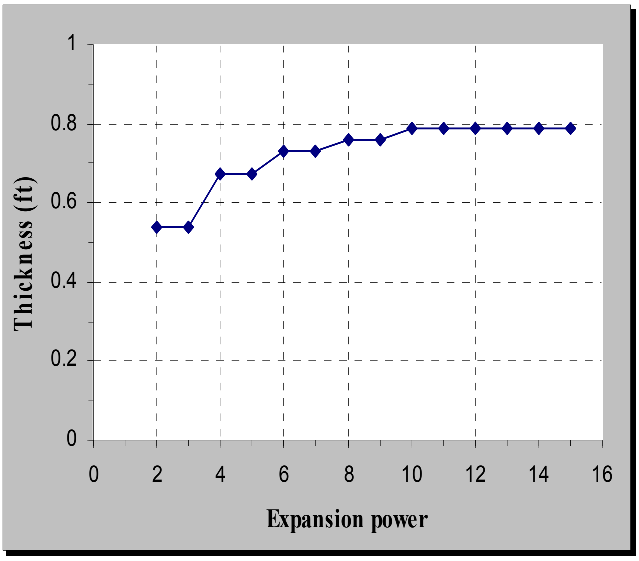


Figure 4. Variation of free plate average thickness against expansion power

SUMMARY AND CONCLUSIONS

Surface waves can be used to determine the mechanical and physical properties of the materials of a layered structure such as soil sites, highway pavements and airport runways. The elastic properties and thicknesses of the component materials are derived from measurements of the wavelength and the phase velocity of vibrations generated along the surface of the ground. The dispersion curve of Rayleigh waves can be determined in the field with the aid of measurements of ground motions induced by a hand hammer using transducers placed on the ground surface. The surface wave method has been successfully used to construct the dispersion curve of the waves of the Rayleigh type. The phase velocity is not constant, but varies with the frequency. The manner of the variation is used to determine the required parameters of the system (the velocity of wave propagation in each medium).

For many years, the numerical inverse procedure has been extensively employed for the determination of the layers moduli and thicknesses of a layered structure. However, this technique is complex time consuming and requires experienced person and engineering judgment. Ideally, the inverse should be obtained by means of a true mathematical inverse. The frequency equation for the system is expanded as a polynomial, and is solved for the unknown parameters of the system. A direct symbolic solution for the determination of the thickness of the free plate model, using mathematical software, has been developed. It is used to calculate the thickness of the uniform free plate corresponding with pairs of the values of phase velocity and frequency as data. The wave propagation in elastic plates is analogous to propagation in layered spaces and therefore this system can be used to model the surface layer of a single layered pavement structure. To assess the effectiveness and reliability of the proposed technique, the application of the method to a published set of data obtained in the field is presented and discussed. The comparison between the experimental results and the developed algebraic solutions show very good agreement, indicating that the proposed method can be used as an economic and effective technique to determine the thickness of the pavement surface layer.

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