ACTA TEHNICA CORVINIENSIS – Bulletin of Engineering
Tome VII [2014] Fascicule 1 [January – March]
ISSN: 2067 – 3809

[1.] Filip VITÁLOŠ, [2.] Marcel VLČEK, [3.] Cyril BELAVÝ

# THE ROBUST CONTROL OF DISTRIBUTED PARAMETER SYSTEMS

[1-3.] Slovak University of Technology, Faculty of Mechanical Engineering in Bratislava, Námestie Slobody 17, 812 31 Bratislava, SLOVAKIA

**Abstract:** This contribution describes dynamic properties of the temperature fields in casting die, as the system with distributed parameters. The dynamics of systems with distributed parameters with lumped-input/distributed-output are solved by finite element numerical method, which is conducted in a COMSOL Multiphysics software environment. In the theoretical part of this work are introduced the basic properties of systems with distributed parameters, IMC controllers and explained the programs, which are applied during this work. The last part of this presented work deals with control of the temperature field in the casting die with robust IMC controllers.
**Keywords:** modeling, robust control, distributed parameter systems

## INTRODUCTION

In the world wide scale the numerical analysis of the machines and casting processes are highly used. The performed numerical software environments such as MATLAB, COMSOL Multiphysics and others take place nowadays.

The aim of these numerical analysis to investigate technological and production processes as a dynamic systems, involved in the prescribed definition domain, to secure maximum productivity, without distortion under the highest outside and inside quality in the means of lower costs. The main core of these numerical analysis is to find the solution for the nonlinear partial differential equations tasks located on geometrical complex space definition domain.

From the system and control theory point of view, distributed parameter systems (DPS) here are involved. Acquired dynamic characteristics could define and resolve the controlling tasks of these distributed systems, what open new possibilities of technical innovations, new design approaches by the creation of new technologies in the matter of machines construction.

## DPS CONTROL LOOP WITH IMC CONTROLLERS

A robust control system for LDS can be designed, for example using the Internal Model Control (IMC) structure see Figure 1, [1]. This well-known structure is incorporated into the time synthesis (TS) block of DPS feedback control system, see Fig. 2. The relation between feedback controller R and IMC controller Q for the nominal model of the system $\tilde{S}$ is prescribed by the formula:

$$Q = \frac{R}{1+\tilde{S}\,R}\ ; \qquad R = \frac{Q}{1-\tilde{S}\,Q} \tag{1}$$

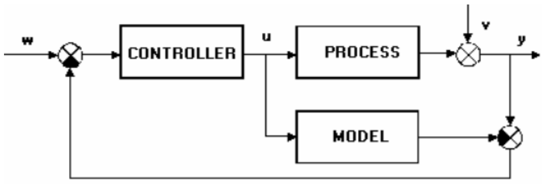


Figure 1 . Internal model control structure

The use of IMC structure at the DPS controling synthesis is interesting in the way of good control loop stability, dynamic response and robustness. Many practical design approaches of IMC controlers has been developed. These approaches further describe how to transform them into the classical feedback controllers, respect with the aplication to the different class of dynamic systems, including DPS [2],[3].

The structure of the control loop DPS based on IMC is depicted on the Figure 2.

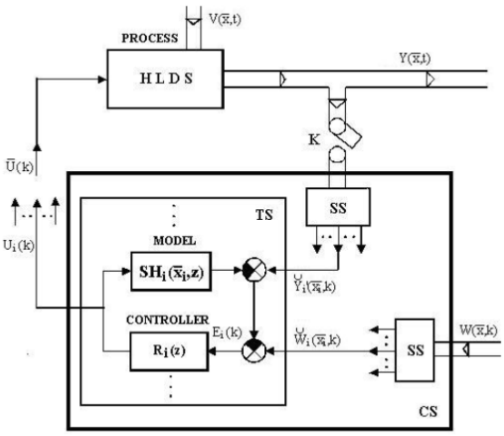


Figure 2. Distributed parameter feedback robust control system

In the continued section, the design method on the control loops with IMC structure in the time control synthesis block of IMC $Q_i^*(z)$ controllers is presented. As a nominal transfer functions, representing the process model, are in the particular control loops considered the identified transfer functions $\{SH_i(\bar{x}_i,z)\}_{i=1,n}$.

$H_2$ optimal controllers $Q_i^*(z)$ for the input $\breve{W}_i(k)$ in the form of step function $\gamma^*(z)=\dfrac{z}{z-1}$ are defined by minimization the criterion:

$$\min_{Q_i^*(z)}\left\| E_i(\bar{x}_i,z)\right\|_2=\min_{Q_i^*(z)}\left\|\left(1-SH_i(\bar{x}_i,z)Q_i^*(z)\right)\gamma^*(z)\right\|_2 \quad (1.1)$$

Discrete IMC controller $Q_i^*(z)$ is then in the form:

$$Q_i^*(z)=SH_{iM}(\bar{x}_i,z)^{-1} \quad (1.2)$$

Final result of the discrete IMC controller $Q_{iF}^*(z)$ with the filter is then in the form:

$$Q_{iF}^*(z)=Q_i^*(z)F_i^*(z)=Q_i^*(z)\frac{(1-\alpha_i)}{z-\alpha_i} \quad (1.3)$$

For $SH_i(\bar{x}_i,z)$ = $SH_{iM}(\bar{x}_i,z)$ and discrete filter $F_i^*(z)$ will be $R_i^*(z)$ in the simple feedback control loop of time element synthesis in the discrete form:

$$R_i^*(z)=\frac{SH_{iM}(\bar{x}_i,z)^{-1}F_i^*(z)}{1-SH_i(\bar{x}_i,z)SH_{iM}(\bar{x}_i,z)^{-1}F_i^*(z)}=$$

$$=\frac{1}{SH_{iM}(\bar{x}_i,z)}\cdot\frac{F_i^*(z)}{1-F_i^*(z)} \quad (1.4)$$

And then:

$$R_i^*(z)=\frac{1}{SH_{iM}(\bar{x}_i,z)}\cdot\frac{1-\alpha_i}{z-1} \quad (1.5)$$

We establish the filter parameters $\alpha_i$ by solving the optimization problem, in order to accomplish the robust stability and robust performance conditions.

## MODELING OF TEMPERATURE FIELDS IN CASTING DIE

A significant software tool for numerical modeling based on finite element method (FEM) is the COMSOL Multiphysics environment. The casting die is one of the critical factors affecting on result quality of the cast as a product, mainly from the view of the die distributed temperature thermal filed. For the thermal fields analyze purposes in the die body the thermal modeling of the casting die field in the COMSOL Multiphysics environment was performed. Layout diagram of heating elements, cooling bodies, and thermocouples in the lower part of the mould is shown in Figure 3.

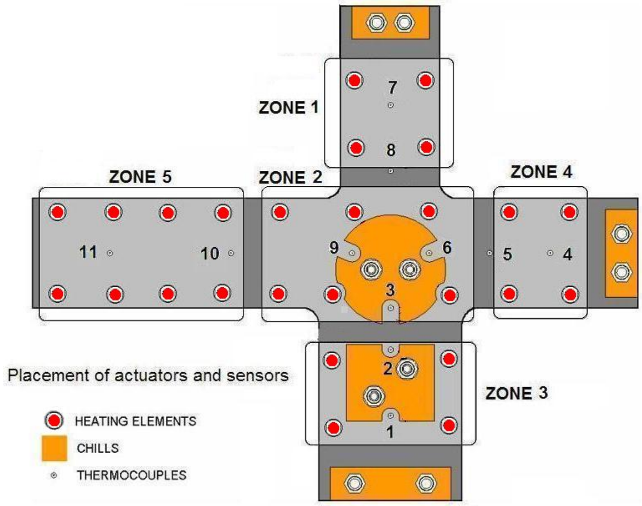


Figure 3. Bottom side of the steel casting mould

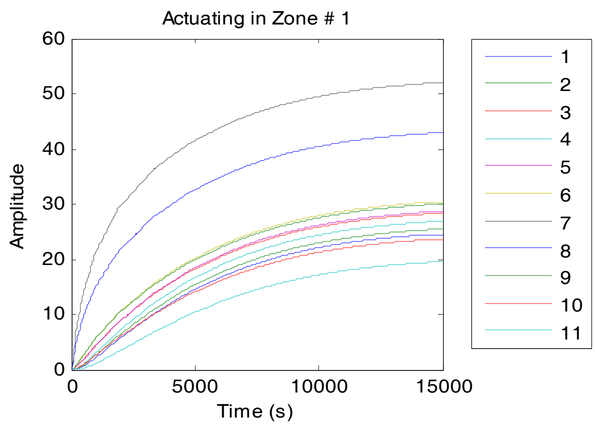


Figure 4. Temperature courses at given points of the mould

*3D geometry of the casting die model was modeled in CAD environment and after that imported to COMSOL Multiphysics. PDR parameters, boundary condition specifications, heat sources at individual zones, FEM conditions, plot results and many other result animations performed by GUI were set in the menu Physics, Mesh, Solve, Postprocessing, File and so on. Temperature fields were analyzed in 11-points where thermocouples are placed in molds physical model. Temperatures of FEM modeling are obtained in the above 11-points are shown in Figure 4.*

## CONTROL OF TEMPERATURE FIELDS IN CASTING DIE WITH IMC CONTROLLERS

*For the zone heating in casting die as DPS, the feedback control loops circuit with 5 time-discrete robust controllers* $\left\{R_i^*(z)\right\}_{i=1,5}$ *with blocks of DPS Blockset , [4], [5] was created, Figure 5.*

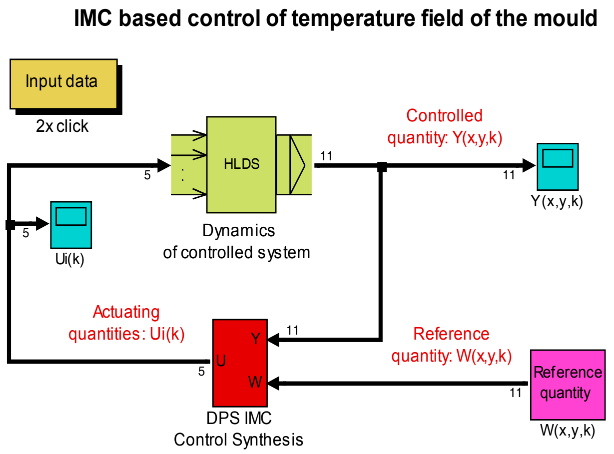


*Figure 5. Feedback control loop of the casting die preheating simulation in DPS Blockset*

*Approximation result for 5 DPS control loops are the approximation coefficients of distributed output quantity and distributed reference quantity* $\left\{\breve{Y}_i(k)\right\}_{i=1,5}$, $\left\{\breve{W}_i(k)\right\}_{i=1,5}$. *Their difference* $\breve{E}_i(k)=\breve{W}_i(k)-\breve{Y}_i(k)$ *input into the time control synthesis component block as a control error.*

*Time variation of quadratic norm of distributed control error* $\left\|E(\bar{x},k)\right\|$ *is total data of control quality, not in the space dependency* $\bar{x}$*, but also in the time dependency k, what can be used by the quadratic function formulation for the parameters optimization of controllers for DPS control.*

$$J=\min_{\alpha_i}\sum_{k=0}^{N}\left\|W(\bar{x},k)-Y(\bar{x},k)\right\| \qquad (3)$$

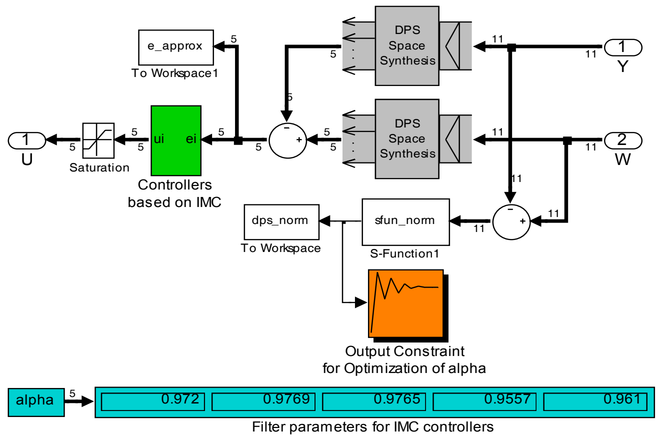


*Figure 6. Structure of IMC controlling synthesis block of DPS*

*Simulation results of the control process for distributed reference quantity W are depicted on Figure 7 and Figure 8.*

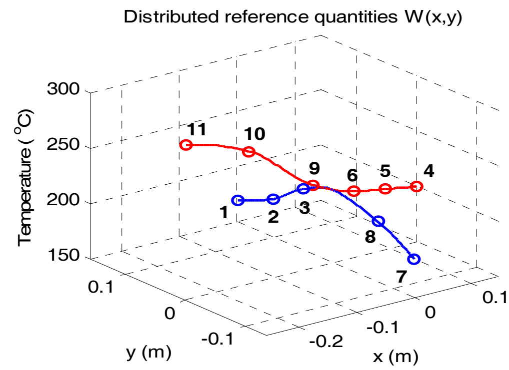


*Figure 7. Distributed reference quantity*

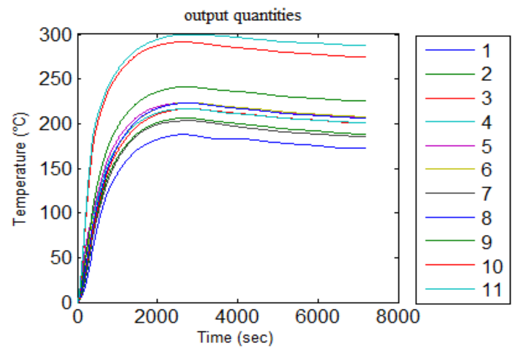


*Figure 8. Output quantities in the points 1-11*

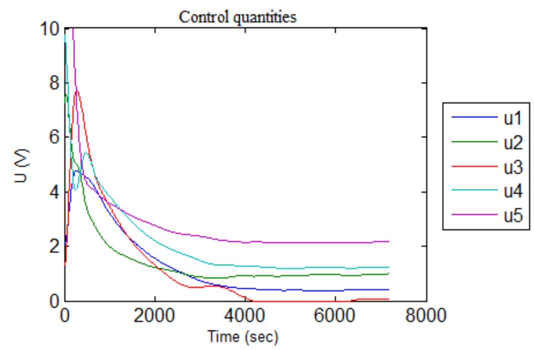


*Figure 9. Control quantities in the zones 1-5*

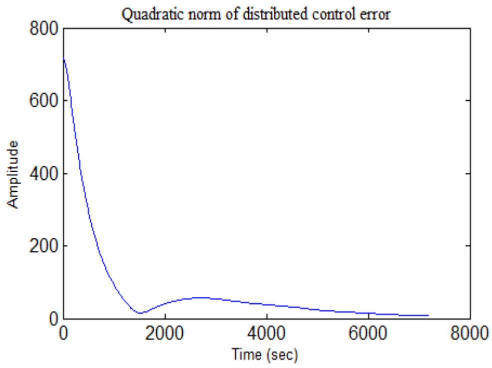


*Figure 10. Quadratic norm of distributed control error*

## 4. CONCLUSION

*In this paper, the problems of the proposal was robust controllers for robust control mould temperature field as a system with distributed parameters. Next, we examined the proposal robust control based on IMC control structure of the control loop. Identified continuous and discrete transfer functions of temperature fields obtained FEM modeling in COMSOL Multiphysics were exported to MATLAB & Simulink to determine distributed and lumped models. The presented methodology of robust control DPS-based LDS reflecting the uncertainty of dynamic models in time synthesis domain and space synthesis domain.*

### ACKNOWLEDGMENT

*Research funded by grants VEGA 1/0138/11 "Control of dynamic systems represented by numerical structures as distributed parameter systems", APVV-0131-10 „High-tech solutions for technological processes and mechatronic components as controlled distributed parameter systems"*

### REFERENCES

[1.] Morrari, M., Zafiriou, E.: Robust Processes Control, New Jersey, Prentice Hall, 1989
[2.] Wang, Qing, Guo et al.: Single-loop controller design via IMC principles. In: Automatica 37 (2001), pp. 2041-2048.
[3.] Vilanova, Ramon: IMC based robust PID design: Tuning guidelines and automatic tuning. In: Journal of Process Control, No 18 (2008), pp. 61-70.
[4.] DPS Control website: http://www.dpscontrol.sk/, 2009.
[5.] Hulkó, G. et al.: Distributed Parameter Systems Blockset for MATLAB & Simulink – third- party Math Works product. Bratislava – Natick, MA, USA, 2004.

ACTA Technica CORVINIENSIS
BULLETIN OF ENGINEERING
ISSN:2067-3809