



1. Zoran PANDILOV, 2. Vladimir DUKOVSKI

## COMPARISON OF THE CHARACTERISTICS BETWEEN SERIAL AND PARALLEL ROBOTS

University "Sv. Kiriil I Metodij", Faculty of Mechanical Engineering-Skopje,  
Karpos li B.B., P.O.Box 464, Mk-1000, Skopje, Republic of MACEDONIA

**Abstract:** This paper gives survey of the position analysis, jacobian and singularity analysis, stiffness analysis, dynamics and applications of serial and parallel robots. Also a detailed comparison of the characteristics of serial and parallel robots and their advantages and disadvantages are presented.

**Keywords:** serial robots, parallel robots, comparison

### INTRODUCTION - Introduction to robotics

Robotics is a field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their applications requires knowledge of mechanical engineering, electrical engineering, systems and industrial engineering, computer science, economics, and mathematics.

The term robot was first introduced into vocabulary by the Czech playwright Karel Capek in his 1920 play *Rossum's Universal Robots*, the word "robota" being the Czech word for work. Since then the term has been applied to a great variety of mechanical devices, such as teleoperators, under-water vehicles, autonomous land rovers, etc. Virtually anything that operates with some degree of autonomy, usually under computer control, has at some point been called a robot.

There are many definitions for what a robot is and this often leads to discrepancies between statistics quoted about robots. An official definition for a robot comes from the Robot Institute of America (RIA): A robot is a reprogrammable multifunctional manipulator designed to move material, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks.

The commonly accepted definition in the UK is that provided by the British Robot Association which is as follows: An industrial robot is a re-programmable device designed to both manipulate and transport parts, tools, or specialised

manufacturing implements through variable programmed motions for the performance of specific manufacturing tasks.

The definition of a robot used by the Japanese Industrial Robot Association widens these definitions in order to include arms controlled directly by humans and also fixed sequence manipulators which are not re-programmable.

The key element in the above definitions is the re-programmability of robots. It is the computer brain that gives the robot its utility and adaptability. The so-called robotics revolution is, in fact, part of the larger computer revolution. Even this restricted version of a robot has several features that make it attractive in an industrial environment. Among the advantages often cited in favour of the introduction of robots are decreased labour costs, increased precision and productivity, increased flexibility compared with specialized machines, and more humane working conditions as dull, repetitive, or hazardous jobs are performed by robots.

The robot, as it is defined, was born out with integration of two earlier technologies: teleoperators and numerically controlled milling machines. Teleoperators, or master-slave devices, were developed during the Second World War to handle radioactive materials. Computer numerical control (CNC) was developed because of the high precision required in the machining of certain items, such as components of high performance aircrafts.

The first robots essentially combined the mechanical linkages of the teleoperator with the autonomy and programmability of CNC machines. The first successful applications of robot manipulators generally involved some sort of material transfer, such as injection moulding or stamping, where the robot merely attends a press to unload and either transfer or stack the finished parts. These first robots could be programmed to execute a sequence of movements, such as moving to a location A, closing a gripper, moving to a location B, etc., but had no external sensor capability. More complex applications, such as welding, grinding, deburring, and assembly require not only more complex motion but also some form of external sensing such as vision, tactile, or force-sensing, due to the increased interaction of the robot with its environment.

It should be pointed out that the important applications of robots are by no means limited to those industrial jobs where the robot is directly replacing a human worker. There are many other applications of robotics in areas where the use of humans is impractical or undesirable. Among these are undersea and planetary exploration, satellite retrieval and repair, the defusing of explosive devices, and work in radioactive environments. Finally, prostheses, such as artificial limbs, are themselves robotic devices requiring methods of analysis and design similar to those of industrial manipulators.

One modern robotics system usually consists of a mechanical manipulator, an end-effector, a microprocessor-based controller, a computer and internal and external sensors/sensing devices (contact or noncontact).

### Classification of robots (robotic manipulators)

Robots can be classified according various criteria, such as degrees of freedom, kinematic structure, drive technology, workspace geometry, motion characteristics, control.

#### Degrees of freedom

One obvious classification scheme is to categorize robots according to their degrees of freedom. In ideal case, a manipulator should possess 6 degrees of freedom in order to manipulate an object freely in three-dimensional space. From this point of view,

we call a robot a general-purpose robot if it possesses 6 degrees of freedom, a redundant robot if it possesses more than 6 degrees of freedom, and a deficient robot if it possesses less than 6 degrees of freedom

#### Kinematic structure

Another classification of robots is according to their structural topologies. A robot is said to be a serial robot (fig.1.1 a) or serial (open-loop) manipulator if its kinematic structure takes the form of an open loop-chain, a parallel manipulator (fig.1.1 b) if it is made of a closed-loop chain, and hybrid manipulator if it consists of both open- and closed-loop chains.

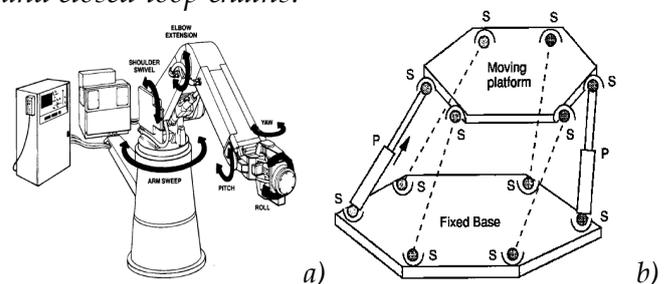


Figure 1.1 a) Serial robot (manipulator) b) parallel manipulator

#### Drive technology

Typically, robots (manipulators) are electrically, hydraulically, or pneumatically driven. Most robots use DC- or AC-servo motors or stepper motors, because they are cleaner, cheaper, quieter and relatively easy to control.

Hydraulic drives have no rival in their speed of response and torque producing capability. Therefore hydraulic robots are used primarily for lifting heavy loads. The drawbacks of hydraulic robots are that they tend to leak hydraulic fluid, require much more peripheral equipment (such as pumps, which require more maintenance), and they are noisy. Pneumatic robots are inexpensive and simple but cannot be controlled precisely, because air is a compressible fluid. As a result, pneumatic robots are limited in their range of applications and popularity.

#### Workspace geometry

The workspace of a manipulator is defined as the volume of space the end effector can reach. A reachable workspace is the volume of space within which every point can be reached by the end effector in at least one orientation. A dextrous workspace is the volume of space within which

every point can be reached by the end effector in all possible orientations. Dextrous workspace is a subset of the reachable workspace.

Robot	Axes		Wrist (DOF)			
	Principle	Kinematic Chain	Workspace	1	2	3
cartesian robot						
cylindrical robot						
spherical robot						
SCARA robot						
articulated robot						

Figure 1.2 Five most common types of robots geometry [5]

Most industrial robots (manipulators) at the present time have six or fewer degrees-of-freedom. These robots are usually classified kinematically on the basis of the first three joints of the arm (R-revolute or P-prismatic) used for manipulating the position, while the rest of joints associated with the wrist are for controlling the orientation.

The majority of these robots (manipulators) fall into one of five geometric types: Cartesian (PPP), cylindrical (RPP), spherical (RRP), SCARA (selective compliance assembly robot arm) (RRP), articulated (RRR) (fig.1.2). Each of these five manipulator arms are serial link robots. A sixth distinct class of manipulators consists of the so-called parallel robot. In a parallel manipulator, as we mentioned before, the links are arranged in a closed rather than open kinematic chain.

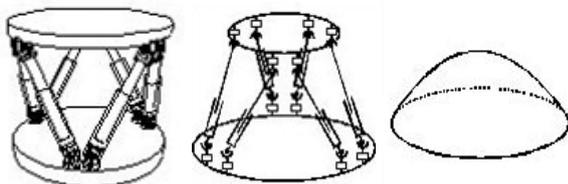


Figure 1.3 Principle, kinematic chain and workspace of parallel robot

### Motion characteristics

Robot manipulators can also be classified according to their nature of motion in planar, spherical and spatial.

A manipulator is called a planar manipulator if its mechanism is a planar mechanism (fig.1.4 a). A manipulator is called a spherical manipulator if it is made of a spherical mechanism (fig.1.4 b). A manipulator is called a spatial manipulator if at

least one of the moving links in the mechanism possesses a general spatial motion (fig.1.1 b).

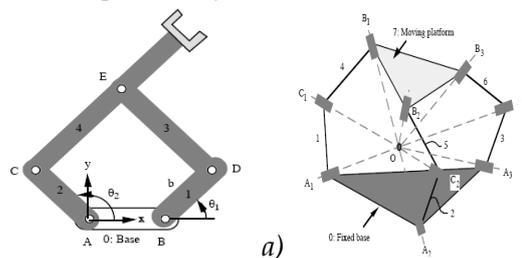


Figure 1.4 a) Planar parallel manipulator b) spherical parallel manipulator

### Control

Robots are classified by control method into servo and non-servo robots.

The earliest robots were non-servo robots. These robots are essentially open-loop devices whose movement is limited to predetermined mechanical stops, and they are useful primarily for materials transfer. In fact, according to the definition given previously, fixed stop robots can hardly qualify as robots.

Servo robots use closed-loop computer control to determine their motion and are thus capable of being truly multifunctional, reprogrammable devices. Servo controlled robots are further classified according to the method that the controller uses to guide the end-effector. The simplest type of robot in this class is the point-to-point robot. A point-to-point robot can be taught with a discrete set of points, but there is no control on the path of the end-effector in between taught points. Such robots are usually taught a series of points with a teach pendant. The points are then stored and played back. Point-to-point robots are severely limited in their range of applications. In continuous path robots, on the other hand, the entire path of the end-effector can be controlled. For example, the robot end-effector can be taught to follow a straight line between two points or even to follow a contour such as a welding seam. In addition, the velocity and/or acceleration of the end-effector can often be controlled. These are the most advanced robots and require the most sophisticated computer controllers and software development.

### Accuracy and repeatability

The accuracy of a manipulator is a measure of how close the manipulator can come to a given point within its workspace. Repeatability is a measure of

how close a manipulator can return to a previously taught point.

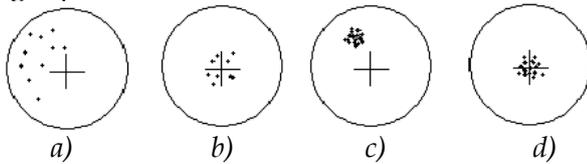


Figure 1.5. a) low accuracy, low repeatability b) high accuracy, low repeatability c) low accuracy, high repeatability d) high accuracy, high repeatability

In this target analogy each dot represents an attempt to get to the central cross. The size of the cluster shows the spread in the result, and the closeness of the centre of the cluster to the cross is measure of accuracy [44].

The primary method of sensing positioning errors in most cases is with position encoders located at the joints, either on the shaft of the motor that actuates the joint or on the joint itself. There is typically no direct measurement of the end-effector position and orientation. One must rely on the assumed geometry of the manipulator and its rigidity to calculate the end-effector position from the measured joint positions. Accuracy is affected therefore by computational errors, machining accuracy in the construction of the manipulator, flexibility effects such as the bending of the links under gravitational and other loads, gear and joint backlash, and an existing of other static and dynamic effects. It is primarily for this reason that robots are designed with extremely high rigidity. Without high rigidity, accuracy can only be improved by some sort of direct sensing of the end-effector position, such as with vision.

Once a point is taught to the manipulator, however, say with a teach pendant, the above effects are taken into account and the proper encoder values necessary to return to the given point are stored by the controlling computer. Repeatability therefore is affected primarily by the controller resolution. Controller resolution means the smallest increment of motion that the controller can sense. The resolution is computed as the total distance traveled by the tip divided by  $2^n$ , where  $n$  is the number of bits of encoder accuracy. In this context, linear axes, that is, prismatic joints typically have higher resolution than revolute joints, since the straight line distance traversed by the tip of a linear axis between two points is less

than the corresponding arc length traced by the tip of a rotational link. In addition rotational axes usually result in a large amount of kinematic and dynamic coupling among the links with a resultant accumulation of errors and a more difficult control problem. One may wonder then what the advantages of revolute joints are in manipulator design. The answer lies primarily in the increased dexterity and compactness of revolute joint designs. For example, Figure 1.6 shows that for the same range of motion  $d$ , a rotational link can be made much smaller than a link with linear motion. Thus manipulators made from revolute joints occupy a smaller working volume than manipulators with linear axes. This increases the ability of the manipulator to work in the same space with other robots, machines, and people. At the same time revolute joint manipulators are better able to maneuver around obstacles and have a wider range of possible applications.

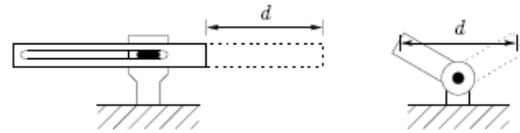


Figure 1.6 Linear vs. rotational link motion.

Accuracy and repeatability are usually of the same order, typically millimetre for very large robots, tenths of millimetre for general purpose robots and hundredths of millimetre for the most accurate assembly robots.

### SERIAL ROBOTS - Position analysis

A serial robot consists of several links connected in series by various types of joints, typically revolute and prismatic. One end of the robot is attached to the ground and the other end is free to move in space. The fixed link is called base, and the free end where a gripper or a mechanical hand is attached, the end effector.

For a robot to perform a specific task, the location of the end effector relative to the base should be established first. This is called position analysis problem. There are two types of position analysis problems: direct position or direct kinematics and inverse position or inverse kinematics problems. For direct kinematics, the joint variables are given and the problem is to find the location of the end effector. For inverse kinematics, the location of the end effector is given and the problem is to find the

joint variables necessary to bring end effector to the desired position. For a serial robot direct kinematics is fairly straightforward, whereas inverse kinematics becomes very difficult. For a deficient robot the end effector can not be positioned freely in space, and for the redundant robot there may be several infinitudes of inverse kinematics solutions corresponding to a given end-effector location, depending on the degrees of redundancy. In solving the inverse kinematics problem, we are often interested in obtaining a closed-form solution, that is, in reducing the problem to an algebraic equation relating the end-effector location to a single joint variable. In this way, all possible solutions and manipulator postures can be accounted for.

To achieve this goal, various methods of formulation have been proposed: vector algebra method, geometric method, 4x4 matrix method (Denavit-Hartenberg), 3x3 dual matrix method, iterative method, screw algebra method and quaternian algebra method [45].

The number of possible inverse kinematics solutions depends on the type and location of a robot manipulator. In general, closed-form solutions can be found for robot manipulators with simple geometry, such as manipulators with three consecutive joint axes intersecting at a common point or three consecutive joint axes parallel to one another. For manipulator of general geometry, the inverse kinematics problem becomes an extremely difficult task.

Two commonly used methods for kinematics analysis of serial robot manipulators are Denavit and Hartenberg's method and the method of successive screw displacements.

### **Jacobian and singularity analysis**

For some applications, such as spray painting (fig. 2.1), it is necessary to move the end effector of the robotic manipulator along some desired paths with prescribed speed. To achieve this goal, the motion of the individual joints must be carefully coordinated. There two types of velocity coordination problems, called direct velocity and inverse velocity problems. For the direct velocity problem, the input joint rates are given and the objective is to find the velocity state of the end effector. For the inverse velocity problem, the velocity state of the end

effector is given, and the input joint rates required to produce desired velocity are to be found.



Figure 2.1 Spray painting robots [57]

Vector space spanned by the joint variables is called joint space, and the vector space spanned by the end-effector location, the end-effector space. For robot manipulators, the Jacobian matrix, or simply Jacobian, is defined as the matrix that transforms the joint rates in the actuator space to velocity state in the end effector state. The Jacobian matrix is a critical component for generating trajectories of prescribed geometry in the end effector-space. Most coordination algorithms used by industrial robots avoid numerical inversion of the Jacobian matrix by deriving analytical inverse solutions on an ad hoc basis. Therefore, it is important that efficient algorithm be developed.

Since the velocity state of the end-effector can be defined in various ways, a variety of Jacobian matrices and consequently, different methods of formulation have appeared in the literature. The most frequently used in practice are a conventional Jacobian and screw-based Jacobian [45].

For a serial robot solving direct velocity problem is relatively easy, whereas inverse velocity problem becomes very difficult, especially for robots of general geometry.

The Jacobian matrix is also useful in other applications. For some manipulator configurations, the Jacobian matrix may lose its full rank. Such conditions are called singular conditions or singular configurations. Physically this implies that the instantaneous screws spanning the  $n$ -dimensional space of the Jacobian matrix became linearly dependent. Therefore, at a singular condition, a serial robot manipulator may lose one or more degrees of freedom, and it will not be able to move in some directions in the end-effector space.

Singularity configurations can be found by setting the determinant of the Jacobian matrix to zero. In

general, this will result in a single algebraic equation. For serial robot manipulators, the singular condition is a function of the intermediate joint variables, not of the first and the last joint variables. This is because the presence of the singularity depends solely on the relative locations of the joint axes. Rotations of the entire manipulator about the first axis not change the relative locations of the joint axes. Similarly, rotation of the end effector about the last joint axis does not affect the location of any joint axis. Therefore, the first and the last joint variables do not appear in the determinant of Jacobian matrix. There two types of singularities for a serial robot manipulator: boundary singularity and interior singularity. A boundary singularity occurs when the end effector is on the surface of the workspace boundary, and it usually happens when the manipulator is either in a fully stretched-out or a folded-back configuration. Boundary singularity can also occur when one of its actuators reaches its mechanical limit. An interior singularity occurs inside the workspace boundary. Several conditions may lead to an interior singularity. For example, when two or more joint axes line up on a straight line, the effects of a rotation about one joint axis can be cancelled by counterrotation about another joint axis. Thus the end effector remains stationary even though the intermediate links of the robot manipulator may move in space. Another example of interior singularity occurs when four revolute joint axes are parallel to one another or intersect in common point.. For a manipulator of general geometry, the problem of identifying interior singularities becomes a much more complex problem. Basically, an interior singularity occurs whenever the screws of two or more joint axes become linearly dependent. Boundary singularities are not particularly serious, since they can always be avoided by arranging the task of manipulation far away from the workspace boundary. Interior singularity is more troublesome because it is more difficult to predict during the path planning process.

#### Stiffness analysis of serial robots

When a robot manipulator performs a given task, the end effector exerts some force and/or moment on its environment. This contact force and/or

moment will cause the end effector to be deflected away from its desired location. Intuitively, the amount of deflection is a function of the applied force and the stiffness of the manipulator. Thus the stiffness of a robot manipulator has a direct impact on the position accuracy. Furthermore some advanced control strategies use the stiffness characteristics for feedback control of a robot manipulator.

The overall stiffness of a robot manipulator depends on several factors, including the size of and material used for the links, the mechanical transmission mechanisms, the actuators and the controller. As the links become longer and more slender. Link compliance becomes the major source of deflection. This is particularly true for space robots, for which light weight and compactness are the major concern. (fig. 2.2)

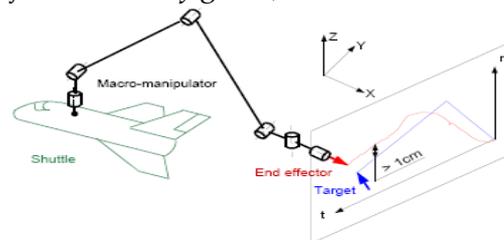


Figure 2.2. Serial space robotic manipulator [1]

Most of the modern serial industrial robots are constructed with fairly rigid links, and the major sources of compliances come from the mechanical transmission mechanisms and control system.

For a serial robot manipulator, each joint is typically driven by an actuator through a multiple-stage speed reducer along several drive shafts. The speed reducer and the drive shafts may deflect when torque or force is transmitted. Further, the drive torque or force generated by a servo system usually depends on the position and error signals and its feedback gains. The stiffness of the speed reducer, the drive shafts, and the servo system may be combined into an equivalent stiffness.

#### Dynamics of serial robots

For some applications, such as arc welding, (fig.2.3), it is necessary to move the end effector of manipulator from point to point rapidly.

The dynamics of the robot manipulator plays an important role in achieving such high-speed performance. The development of dynamical model is important in several ways. First, a dynamical model can be used for computer simulation of a

robotic system. By examining the behaviour of the model under various operating conditions, it is possible to predict how a robotic system will behave when it is built. Various automation tasks can be examined without the need of real system. Second, it can be used for the development of suitable control strategies. A sophisticated controller requires the use of a realistic dynamical model to achieve optimal performance under high-speed operations. Some control schemes rely directly on a dynamic model to compute actuator torques required to follow a desired trajectory. Third, the dynamic analysis of the manipulator reveals all the joint reaction forces (and moments) needed for the design and sizing of links, bearings and actuators. There are two types of dynamical problems: direct dynamics and inverse dynamics. The direct dynamic problem is to find the response of a robot arm corresponding to some applied torques and/or forces. That is, given a vector of joint torques or forces, we wish to compute the resulting motion of the robot manipulator as a function of time. The inverse dynamic problem is to find the actuator torques and/or forces required to generate a desired trajectory of the manipulator. The problem can be formulated in joint space, or the end effector space. The two formulations are related by the Jacobian matrix and its time derivative. In general, the efficiency of computation for direct dynamics is not as critical since it is used primarily for computer simulations of a manipulator. On the other hand an efficient inverse dynamical model becomes extremely important for real-time feedforward control of a robot manipulator.



Figure 2.3. Arc welding robot [57]

The dynamical equations of motions can be formulated by several methods. The most frequently used are the application of the Newton and Euler laws and the Lagrange's equations of motion.

#### Applications of serial robots

Robots, basically serial robots, are used in applications that require repetitive tasks over long

periods of time, operations in hazardous environments (like nuclear radiation, under water, space exploration, etc.), and precision work with high degree of reliability. They can also be used by handicapped persons to overcome some of their physical disabilities.

Some examples of use of industrial robots are following: machine loading and unloading (fig 2.4), palletizing, die casting, forging, press work, arc welding and spot welding (fig.2.5), heat treatment, spraying (paint, enamel, epoxy resin and other coatings), deburring, grinding, polishing, injection moulding, cutting (laser, plasma), inspection, assembly (fig.2.6), packaging (fig.2.7), material handling (fig.2.8), etc.

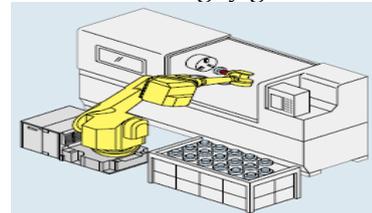


Figure 2.4 Robot application in machine loading and unloading [56]

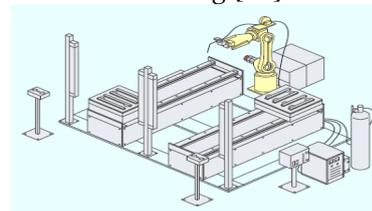


Figure 2.5. Application of robot in welding process [56]

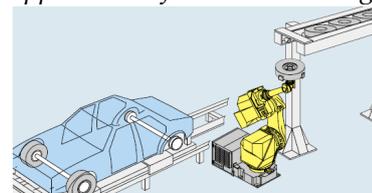


Figure 2.6 Robot application in assembly [56]

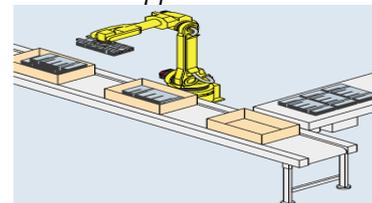


Figure 2.7 Application of robot in packaging [56]

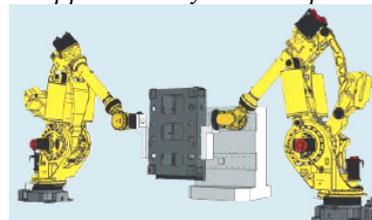


Figure 2.8 Application of two robots in handling heavy objects (materials) [56]

According to the International Federation of Robotics (IFR) of all installed industrial robots, approximately 33% are in assembly, 25% are used in different welding applications, 2,8% in packaging/palletizing, but with intention to grow-up, etc.

Main characteristics of the serial robots are given in the table below:

Table 2.1. Characteristics of serial robots

Feature	Serial robot
Workspace	Large
Solving forward kinematics	Easy
Solving inverse kinematics	Difficult
Position error	Accumulates
Force error	Averages
Maximum force	Limited by minimum actuator force
Stiffness	Low
Dynamics characteristics	Poor, especially with increasing the size
Modelling and solving dynamics	Relatively simple
Inertia	Large
Areas of application	A great number in different areas, especially in industry
Payload/weight ratio	Low
Speed and acceleration	Low
Accuracy	Low
Uniformity of components	Low
Calibration	Relatively simple
Workspace/robot size ratio	High

### PARALLEL ROBOTS - Position analysis

A parallel robot manipulator is composed of two or more closed-loop kinematic chains in which the end-effector (mobile platform) is connected to the fixed base platform by at least two independent kinematic chains. Between the base and end-effector platforms are serial chains (called limbs or legs). (fig.3.1)

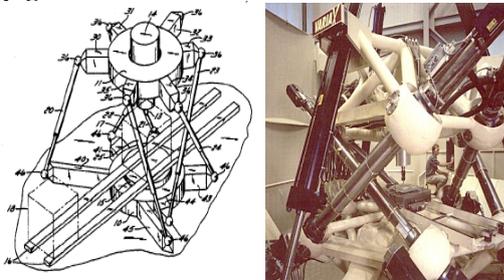


Figure 3.1 Example of parallel robot manipulator, Patent US 5388935: VARIAX machining center,

(Courtesy: Giddings & Lewis, Inc., Fond du Lac, WI)

Typically, the number of limbs is equal to the number of degrees of freedom such that every limb is controlled by one actuator and all actuators can

be mounted at or near the fixed base. For this reason, parallel manipulators are sometimes called platform manipulators. Because the external load can be shared by the actuators, parallel manipulators tend to have a large load-carrying capacity.

Parallel manipulators have been used in applications like airplane simulators [43], adjustable articulated trusses [40], mining machines [4], pointing devices [22], walking machines [46], machining centres [20], etc.

The development of parallel manipulators can be dated back to the early 1960's when Gough and Whitehall [23], first devised a six-linear jack system for use as a universal tire testing machine. Later, Stewart [43] developed a platform manipulator for use as an aircraft simulator. Hunt [26] first made a systematic study of the structural kinematics of parallel manipulators.

Since then, parallel manipulators have been studied by numerous researches [45]. More than 100 different mechanical architectures of parallel robots have already been proposed.

Most of the 6-DOF parallel manipulators studied to date consist of six extensible limbs. These parallel manipulators possess the advantages of high stiffness, low inertia and large payload capacity. However, they suffer the problems of relatively small useful workspace, design difficulties and difficult control.

For parallel robot manipulators two position analysis problems have to be solved: direct kinematics and indirect kinematics. A parallel robot indirect kinematics is fairly straightforward, whereas direct kinematics is very difficult problem. Perhaps, the only six limbed 6 DOF parallel manipulators for which closed-form direct kinematics solutions have been reported in the literature are special forms of the Stewart-Gough platform. As to the general Stewart-Gough platform, research has to resort to numerical techniques for the solutions.

Parallel robot manipulators can be classified as planar (fig.3.2 a), spherical (fig. 1.4 b), or spatial (fig.3.2 b) manipulators in accordance with their motion characteristics.

Position analysis of planar and spherical parallel robot manipulators is easier than position analysis

of parallel robot manipulators, or if the spatial manipulator has less than 6 DOF, or if the parallel manipulator is symmetrical.

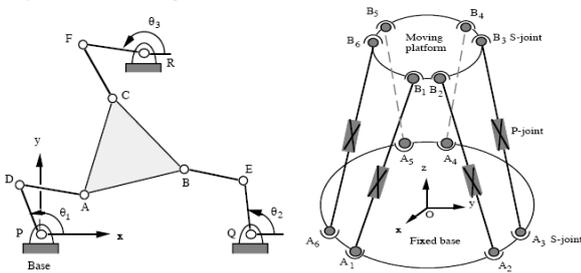


Figure 3.2 a) planar parallel robot manipulator b) spatial parallel robot manipulator

The parallel manipulator is symmetrical if it satisfied the following conditions:

1. The number of limbs is equal to the numbers of degrees of freedom of the moving platform.
2. The type and number of joints in all the limbs are arranged in an identical pattern.
3. The number and location of actuated joints in all the limbs are the same.

When the conditions above are not satisfied, the manipulator is called asymmetrical.

For position analysis (direct and indirect kinematics) for parallel manipulators, both vector and algebraic techniques are used.

Details about position analysis for different types of planar parallel robot manipulators are given by [49, 48, 12, 24] and for different types of spatial parallel robot manipulators [37, 15, 35, 65, 66, 13, 8, 31, 67, 27, 11, 19, 41] etc.

### Jacobian and singularity analysis

The Jacobian analysis of parallel manipulators is a much more difficult problem than that of serial manipulators because they are many links that form a number of closed loops.

An important limitation of parallel manipulator is that singular configurations may exist within its workspace where the manipulator gains one or more degrees of freedom and therefore loses its stiffness completely. This property has attracted the attention of several researches. For example, Gosselin & Angeles [21] studied the singularities of closed-loop mechanisms and suggested a separation of the Jacobian matrix into two matrices: one associated with the direct kinematics and the other with the inverse kinematics. Depending on which matrix is singular, a closed-loop mechanism may be at a direct kinematic

singular configuration, an inverse kinematic singular configuration, or both.

The most widely used methods for Jacobian analysis for parallel robot manipulators are the method of velocity vector-loop equations and the method of reciprocal screws.

A parallel manipulator such as VARIAX machining center shown in fig 3.1 typically consists of a moving platform and a fixed base connected by several limbs. This moving platform serves as the end effector. Because of the closed-loop construction, not all joints can be controlled independently. Thus some of the joints are driven by actuators, whereas others are passive. In general, the number of actuated joints should be equal to the number of degrees of freedom of the manipulator.

Let the actuated joint variables be denoted by a vector  $q$  and the location of the moving platform be described by vector  $x$ . Then the kinematic constrains imposed by limbs can be written in the general form

$$f(x,q)=0 \quad (3.1)$$

where  $f$  is an  $n$ -dimensional implicit function of  $q$  and  $x$  and  $0$  is  $n$ -dimensional zero vector.

Differentiating equation (3.1) with respect to time, we obtain a relationship between the input joint rates and the end-effector output velocity as follows:

$$J_x \dot{x} = J_q \dot{q} \quad (3.2)$$

where  $J_x = \frac{\partial f}{\partial x}$  and  $J_q = -\frac{\partial f}{\partial q}$

The derivation above leads to two separate Jacobian matrices. Hence the overall Jacobian matrix,  $J$ , can be written as,

$$\dot{q} = J \dot{x} \quad (3.3)$$

where  $J = J_q^{-1} J_x$ . Jacobian matrix defined in equation 3.3 for a parallel manipulator corresponds to the inverse Jacobian of a serial manipulator.

Due to the existence of two Jacobian matrices, a parallel robot manipulator is said to be at singular configuration when either  $J_x$  or  $J_q$  or both are singular.

An inverse kinematic singularity occurs when the determinant of  $J_q$  goes to zero, namely,

$$\det(J_q)=0 \quad (3.4)$$

When  $J_q$  is singular ant the null space of  $J_q$  is not empty, there exist some nonzero  $\dot{q}$  vectors that

result in zero  $\dot{x}$  vectors. Infinitesimal motion of the moving platform along certain directions cannot be accomplished. On the other hand, at the inverse kinematic singular configuration, a parallel manipulator can resist forces or moments in some directions with zero actuator forces or torques.

Inverse kinematic singularities usually occur at the workspace boundary, where different branches of the inverse kinematic solutions converge. It is similar to that of serial manipulator.

A direct kinematic singularity occurs when the determinant  $J_x$  is equal to zero, namely

$$\det(J_x) = 0 \quad (3.5)$$

Assuming that in presence of such a singular condition the null space of  $J_x$  is not empty, there exist some nonzero  $\dot{x}$  vectors that result in zero  $\dot{q}$  vectors. That is, the moving platform can possess infinitesimal motion in some directions while all actuators are completely locked. Hence the moving platform gains one or more degrees of freedom. This is in contradiction with the serial manipulator, which loses one or more degrees of freedom [47]. In other words, at a direct kinematic singular configuration, the manipulator cannot resist forces or moments in some directions. In those directions the stiffness is zero. Direct kinematic singularities usually occur where different branches of direct kinematic solutions meet.

A combined singularity occurs when the determinants of  $J_x$  and  $J_q$  are both zero. Generally, this type of singularity can occur only for manipulators with special kinematic architecture. At a combined singular configuration, equation (3.1) will degenerate. The moving platform can undergo some infinitesimal motions while all the actuators are locked. On the other hand, it can also remain stationary while actuators undergo some infinitesimal motions.

Singularity analyses for different types of parallel robot manipulators are presented by [30, 13, 65, 66, 63, 64, 62, 2, 3, 31, 68, 69, 33, 28] etc.

### **Dynamics of parallel robots**

While the kinematic of parallel robot manipulators have been extensively studied during the last two decades, fewer papers can be found on the dynamic of parallel manipulators [45]. The dynamic analysis of parallel manipulators is complicated by

the existence of multiple closed loop chains. Several approaches have been proposed, including the Newton-Euler formulation the Lagrangian formulation and the principle of virtual work. Details about dynamic modelling of parallel robots are given by [14, 34, 29, 36, 63, 64, 32, 25, 18] etc. The traditional Newton-Euler formulation requires the equations of motion to be written once for each body of the manipulator, which inevitably leads to a large numbers of equations and results in poor computational efficiency. The Lagrangian formulation eliminates all of the unwanted reaction forces and moments at the outset. It is more efficient than the Newton-Euler formulation. However, because of the numerous constraints imposed by closed loops of a parallel manipulator, deriving explicit equations of motion in terms of a set of independent generalized coordinates becomes a prohibitive task. To simplify the problem additional coordinates along with set of Lagrangian multipliers are often introduced. In some cases, limbs are approximated by point masses by arguing that such approximation does not introduce significant modelling errors. In this regard, the principle of virtual work appears to be the most efficient method of analysis.

### **Applications of parallel robots**

After spending almost 20 years in the laboratories for preliminary studies parallel robots are now used in real-life applications. This interest for parallel robots come from the potentially interesting features of parallel mechanisms: high accuracy, rigidity, speed and large load carrying capability, which in a very large number of cases may overcome the drawbacks of the more complex kinematics, dynamics and smaller workspace.

But a fact is that these advantages are only potential and any real parallel robot will present in practice impressing performances only if all its components (either hardware or software) present a high level of performance.

The current applications of parallel robots are in domains such as fine positioning devices (fig.3.3 and fig.3.4), simulators (fig. 3.5), motion generators (platforms) (fig. 3.6), ultra-fast pick and place robots (fig.3.7), machine-tools (fig. 3.8, fig.3.9 and fig.3.10), medical applications (fig.3.11, fig.3.12), haptic devices (fig.3.13), entertainment,

force sensors, micro-robots (fig. 3.14), articulated trusses, etc.



Figure 3.3 Application of parallel robots for fine positioning UKIRT (United Kingdom Infrared Telescope), collaboration between Royal Observatory Edinburgh and Max-Planck-Institut für Astronomie Heidelberg [61]



Figure 3.4 Parallel robots for fine positioning [60]

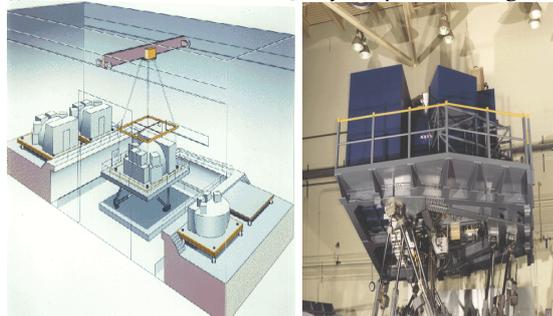


Figure 3.5 Application of parallel robots as simulators NASA LARC-simulator [61]



Figure 3.6 Parallel robots as motion platforms [55],[53]



Figure 3.7 Ultra-fast pick and place robot ABB-Flex Picker IRB 340 [50]

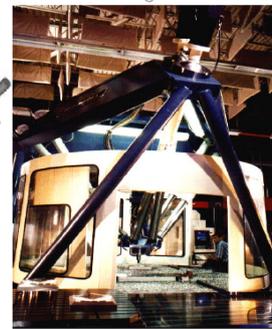
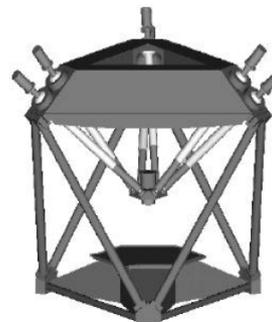
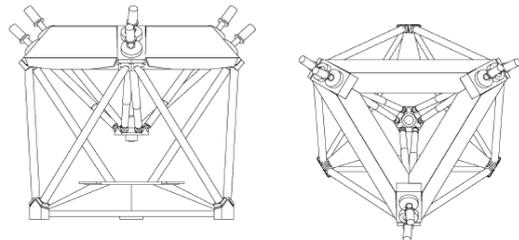


Figure 3.8 Side and top view, solid model and photo of Ingersoll Octahedral Hexapod machine tool installed at NIST [16]



Figure 3.9 Hexapod parallel robot based machine tool [6]

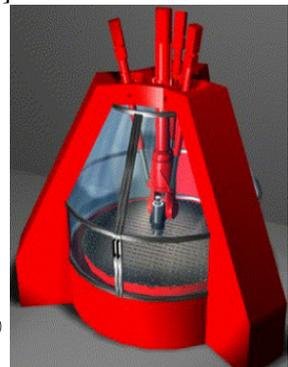
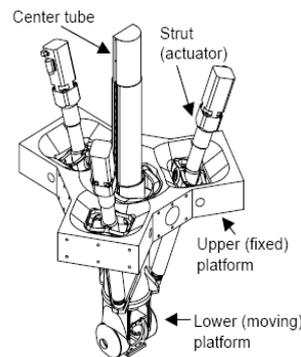


Figure 3.10, Hybrid parallel-serial robot Tricept 805 tripod and a complete machining center [42]



Figure 3.11 Hexapod for brain surgery. Photo courtesy of IPA [59]



Figure 3.12 Parallel robot SurgiScope in action at the Surgical Robotics Lab, Humboldt-University at Berlin (courtesy of Prof. Dr. Tim C. Lueth) [58]

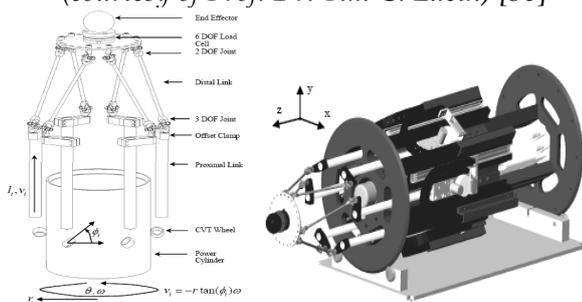


Figure 3.13 Cobotic parallel platform [17]



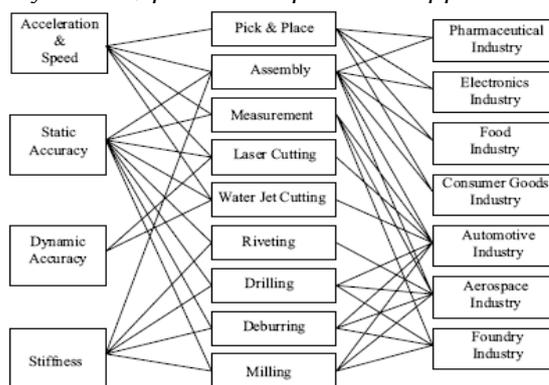
Figure 3.14 Parallel micro robot [52]

But in spite of above given examples and high performance potential of parallel robots, this technology has not yet made a dramatic impact on industrial automation. However, there is an interesting trend towards the use of general purpose industrial serial robots for applications with higher demands on accuracy, stiffness, natural frequency, cycle time etc.

Thus, significant efforts are now being made to use industrial robots for such applications as measurements, laser cutting, laser welding, high precision assembly, grinding, deburring, milling etc. Because of the inefficient robot performance for these applications, several compensation methods

are used, which add cost and make installation, programming, maintenance etc., difficult [9]. Moreover, in most cases the industrial serial robots of today probably will never reach the application requirements for high performance applications. One way to solve these problems could be using of robots based on parallel kinematics. But it is not easy to challenge and change the mature industrial robot technology, even if some successful structures find increasing market shares today, too. Parallel kinematic structures provide such high performance potential, but it is very important for the research community to come up with concepts and technologies which will make parallel kinematic robots a natural choice when flexible automation systems are designed.

One example of a successful parallel kinematic robot structure is the Delta structure (fig. 3.7), designed in 80's from Prof. Reymond Clavel (professor at EPFL – École Polytechnique Fédérale de Lausanne). The reason for this success is that the features of this structure fit into applications requiring very fast handling of light weight products, for example in the consumer goods, food and electronics industries. Thus, to be successful with the transfer of results from parallel kinematics robots research to industrial product development, it is very important to understand the application requirements. Moreover, it is important to understand what advantages parallel kinematics robots features, provide in potential applications.



Parallel robot features Applications End Users  
Figure 3.15 Diagram exemplifying the relations between potential performance features of a parallel kinematic robot and the applications and industries needing this performance for improved flexible automation [9]

For example, parallel kinematic robot structures may give higher speed and acceleration, higher

static and dynamic accuracy and higher stiffness than what is possible with the serial industrial robots used today. Starting with these competitive features, potential applications and end users can be evaluated, like the example diagram given in fig. 3.15. For each application and for each type of installation in the manufacturing plants of the end users, a detailed study is needed to find out if the parallel kinematic robot will satisfy all requirements.

### Several examples of successful parallel robots

#### - Delta parallel robot

It is in the early 80's when Reymond Clavel (professor at EPFL – École Polytechnique Fédérale de Lausanne) comes up with the brilliant idea of using parallelograms to build a parallel robot with three translational and one rotational degree of freedom. Latter called his creation the Delta robot (fig.3.16), without suspecting that at the turn of the century, it will establish itself as one of the most successful parallel robot designs, with several hundreds active robots worldwide.

The basic idea behind the Delta robot design is the use of parallelograms. A parallelogram allows an output link to remain at a fixed orientation with respect to an input link. The use of three such parallelograms restrain completely the orientation of the mobile platform which remains only with three purely translational degrees of freedom. The input links of the three parallelograms are mounted on rotating levers via revolute joints. The revolute joints of the rotating levers are actuated in two different ways: with rotational (DC or AC servo) motors or with linear actuators. Finally, a fourth leg is used to transmit rotary motion from the base to an end-effector mounted on the mobile platform.

The use of base-mounted actuators and low-mass links allows the mobile platform to achieve accelerations of up to 50 G in experimental environments and 12 G in industrial applications. This makes the Delta robot a perfect candidate for pick and place operations of light objects (from 10 gr to 1 kg). Ideally, its workspace is the intersection of three right circular tori. The Delta robots available on the market operate typically in a cylindrical workspace which is 1 m in diameter and 0.2 m high.

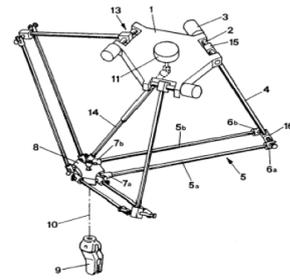


Figure 3.16 Schematic of the Delta robot (from US patent No. 4,976,582) [51]

As simple as it is, the design of the Delta robot is covered by a family of 36 patents of which the most important are the WIPO patent issued on June 18, 1987 (WO 87/03528), the US patent issued on December 11, 1990 (US 4,976,582), and the European patent issued on July 17, 1991 (EP 0 250 470). Overall, these patents protect the invention in USA, Canada, Japan, and most West European countries. The patents do not specify the way in which the Delta structure is actuated in order to incorporate the basic design as well as its variants [7].

The Delta robot is mostly used as a pick-and-place robot ( C33 and CE33 Robots, fig.3.17, from SIG pack Systems-from 2004 part of Packaging Technology division of Bosch and IRB 340 Flex Picker Robot from ABB Automation fig. 3.7), although there exist some other applications in medicine (SurgiScope fig.3.12)- and machine tools (Krause & Mauser Group Quickstep 3-axis milling machine is in fact Delta robot with linear motors, fig. 3.18).

The Delta robot was licensed to various companies. In addition, some machine tool manufacturers managed to get their own patents and have built parallel kinematics machines based on the Delta robot architecture.



Figure 3.17 Two of the three Delta robot models offered by SIG Pack Systems, C33 and CE33 (courtesy of SIG Pack Systems-now Bosch Packaging Technology division)



Figure 3.18 The Quickstep 3-axis machining center and Quickstep's kinematic structure [58]

- **FANUC parallel robot**

Another successful type of parallel robot is F-200iB (fig 3.19) a product of FANUC Robotics North America of Rochester Hills, MI. The F-200iB is a six degrees of freedom servo-driven parallel link robot designed for use in a variety of manufacturing and automotive assembly processes.

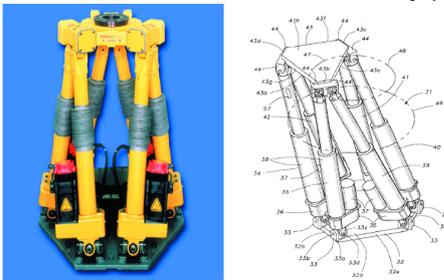


Figure 3.19 A FANUC parallel robot F-200iB [57] (US patent No. 5987726) [51]

The F-200iB is engineered for applications requiring extreme rigidity and exceptional repeatability in a compact, powerful package. F-200iB, the solution for: sub-compact robot welding, pedestal welding, part loading/positioning, nut running, vehicle lift and locate, flexible/convertible fixturing, material removal, dispense. The F-200i is very rigid when compared to serial linked robots. There is less flexing of the arms and high repeatability. With serial linked robots, the end-of-arm flexing errors are cumulative. In a parallel link structure they are averaged. Compared to a serial link machine, this type of robot has a small range of motion due to the configuration of the axes, although it has a broad mix of applications. It has motion speed in vertical z axis 300 (mm/sec), in horizontal x and y axes 1500 (mm/sec) and repeatability  $\pm 0.1$ (mm).

Other atypical applications for the F-200iB include education, medical and scientific research uses.

- **TRICEPT robot**

In 1987 a new type of robot, the 3 DOF parallel kinematic robot, was designed and built by Karl-

Erik Neumann (fig.3.20). This type of robot has three or more linear axes which function parallel to one another. It has three prismatic actuators which control two rotational and one translational degree of freedom of the mobile platform. A conventional wrist is additionally mounted on the mobile platform (fig.3.21)

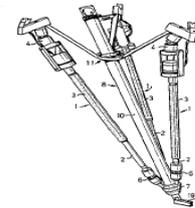


Figure 3.20 3-DOF Parallel kinematic robot Tricept (US Patent No.: US 4,732,525) [51]



Figure 3.21 Parallel kinematic robot Tricept IRB 940 [50] The initial challenge for this system was that it required computer power that was unavailable at the time. Karl-Erik Neumann, the inventor of the Tricept robot, explains: "There was no control system to run the machine until 1992 when the company Comau Pico launched the first multiprocessor controller. That, and open architecture, made it so we could adapt its complex kinematics [10].

Neumann founded Neos Robotics in Sweden. Neos Robotics has purchased another Swedish machining company, and strated to go under the name of SMT Tricept. Now SMT Tricept is in a strategic alliance with ABB robotics.

The Tricept robot is the system that greatly influenced the parallel kinematic robot phenomenon. Although initially designed as an assembly robot, the demands from the market transformed it into a machine tool. This market demand led introduction in 1999 of Tricept model 805, a larger version of Tricept. This was developed as a machine tool robot, which combined the flexibility of a robot with the stiffness of a machine tool. The last few years, the biggest application of Tricept is metal cutting. The Tricept can also be used to hold laser and saw cutting tools, as well as friction welders. Customers who use Tricept robots

include automobile makers in Europe and North America: Peugeot, Ford, Renault, Volvo, General Motors, BMW, and Volkswagen. The aerospace industry uses Tricept robots for fabricating propellers, turbine blades, impellers and any other item that requires a considerable amount of contouring. Other applications for the Tricept include assembly with thrust, deburring, polishing, woodworking, water-jet cutting and spot-welding.

In October 2002 ABB and SMT Tricept launched the last type of an exceptionally powerful and stiff Tricept robot IRB 940 - for heavy-duty cleaning and pre-machining of aluminium parts (vertical machining power of 1300 kg, horizontal machining power of 350 kg, accuracy  $\pm 0.2$  mm, and repeatability of  $\pm 0.02$  mm).

IRB 940 Tricept is designed to form an integrated part of optimized production lines, teaming up with traditional arm robots and CNC machine tools. Arm robots handle material, machine tending and light cleaning. Tricept robots then take over to do heavy-duty cleaning and pre-machining, while CNC machines put the finishing touches to cleaning and part processing.

Main characteristics of the parallel robots are given in the table below:

Table 3.1. Characteristics of parallel

Feature	Parallel robot
Workspace	Small and complex
Solving forward kinematics	Very difficult
Solving inverse kinematic	Easy
Position error	Averages
Force error	Accumulates
Maximum force	Summation of all actuator forces
Stiffness	High
Dynamics characteristics	Very high
Modelling and solving dynamics	Very complex
Inertia	Small
Areas of application	Currently limited, especially in industry
Payload/weight ratio	High
Speed and acceleration	High
Accuracy	High
Uniformity of components	High
Calibration	Complicated
Workspace/robot size ratio	Low

### Comparison of the characteristics of serial and parallel robots

Table below gives comparison between main characteristics of serial and parallel robots:

Table 4.1. Characteristics of serial and parallel robots

Feature	Serial robot	Parallel robot
Workspace	Large	Small and complex
Solving forward kinematics	Easy	Very difficult
Solving inverse kinematics	Difficult	Easy
Position error	Accumulates	Averages
Force error	Averages	Accumulates
Maximum force	Limited by minimum actuator force	Summation of all actuator forces
Stiffness	Low	High
Dynamics characteristics	Poor, especially with increasing the size	Very high
Modelling and solving dynamics	Relatively simple	Very complex
Inertia	Large	Small
Areas of application	A great number in different areas, especially in industry	Currently limited, especially in industry
Payload/weight ratio	Low	High
Speed and acceleration	Low	High
Accuracy	Low	High
Uniformity of components	Low	High
Calibration	Relatively simple	Complicated
Workspace/robot size ratio	High	Low

### CONCLUSION

If we analyse the table 4.1 we will see that the both types of robots have advantages and disadvantages. For example parallel robots offer potential advantages compared with serial, with higher overall stiffness, higher precision, low inertia, and higher operating speeds and accelerations. However these advantages could be easily relativised by reduced workspace, difficult mechanical design, and more complex kinematics and control algorithms.

It is really very difficult to say what kind of robot is better, serial or parallel. A robot selection procedure is very difficult and complex activity. It depends on many different factors like type of application (dangerous, repetitive and boring, precise, etc.), task requirements (DOF, speed, accuracy, repeatability), load requirements, workspace, economic justification, programming time, maintaining, etc.

Parallel robots are most successful in applications like motion simulators, ultra precision positioning devices, medical applications, ultra-fast pick and

place robots and micro-robots. But serial robots dominate almost in all manufacturing applications. Probably this will change with continuously solving of the open problems in parallel robotics given in [38, 39] or using hybrid structures. Hybrid structures are in fact compromise between advantages and disadvantages of both robot structures, serial and parallel. The two most successful manufacturing applications of parallel robots are in fact hybrid structures. First one, Tricept robot is (parallel-serial) structure, 3-axis parallel machine tool-robot plus 2-axis, conventional serial wrist. The second, Sprint Z3 3-axis parallel kinematic tool head by DS Technologie (fig.5.1) that may advance in Z and tilt in all directions, and may be mounted on a conventional XY stage. The machining centre lines ECOSPEED and ECOLINER equipped with the Sprint Z3 tool head are in fact hybrid structures (serial-parallel).

This two robot structures probably will live parallel a long years. If we compare about 20 years research in parallel mechanisms and more than 200 years in research to reach the current level of knowledge for serial mechanisms, it is easy to conclude that this process of solving problems in parallel robotics will be long term.

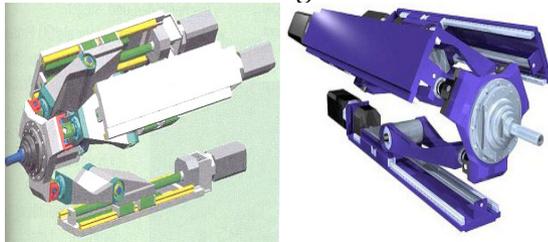


Figure 5.1. Sprint Z3 parallel kinematic tool head [54]

#### ACKNOWLEDGMENTS

This research was done during my research stay at the Department of Machine Tools and Automation, TU Hamburg-Harburg, Germany, financed by DFG (Deutsche Forschungsgemeinschaft).

#### REFERENCES/BIBLIOGRAPHY

- [1.] ALAZARD D., CHRETIEN J.P., 1994, Dexterous External Space Manipulation: Serial Or Parallel Concepts Comparison, Proceedings of the IFAC Automatic Control in Aerospace 94, Palo Alto (CA), USA, Sept. 12th-16th, 1994
- [2.] ANGELES J., YANG G., CHEN I. M., 2001, Singularity Analysis of Three-Legged, Six-DOF Platform Manipulators with RRRS Legs, 2001 IEEE/ASME International Conference on

- Advanced Intelligent Mechatronics Proceedings, 8-12 July 2001, Como, Italy, pp.32-36
- [3.] ANGELES J., YANG G., CHEN I. M., 2003, Singularity Analysis of Three-Legged, Six-DOF Platform Manipulators With URS Legs, IEEE/ASME TRANSACTIONS ON MECHATRONICS, VOL. 8, NO. 4, DECEMBER 2003, pp.469-475
- [4.] ARAI, T., CLEARY, K., HOMMA, K., ADACHI, H., AND NAKAMURA, T., 1991, Development of Parallel Link Manipulator for Underground Excavation Task, Proc. 1991 International Symposium on Advanced Robot Technology, pp. 541-548, 1991.
- [5.] BI Z., 2002, Adaptive Robot Systems for Manufacturing Applications, Ph.D. Thesis, Department of Mechanical Engineering, University of Saskatchewan Saskatoon
- [6.] BLÜMLEIN W. J., 1999, The Hexapod, Maschine + Werkzeug, 10/99
- [7.] BONEV I., 2001, Delta Parallel Robot – the Story of Success, /www.parallemic.org/
- [8.] BONEV A. I., RYU J., KIM S. G. AND LEE S. K., 2001, A Closed-Form Solution to the Direct Kinematics of Nearly General Parallel Manipulators with Optimally Located Three Linear Extra Sensors, TRANSACTIONS ON ROBOTICS AND AUTOMATION, VOL. 17, NO. 2, APRIL 2001, pp.148-156
- [9.] BROGARDH T., 2002, PKM Research - Important Issues, as seen from a Product Development Perspective at ABB Robotics, Proceedings of the WORKSHOP on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators October 3-4, 2002, Quebec City, Quebec, Canada, pp.68-82
- [10.] BRUMSON B., 2002, Parallel Kinematic Robots, /www.roboticonline.com/public/articles/
- [11.] CALLEGARI M. AND MARZETTI, P. 2004, Kinematic Characterisation of a 3-PUU Parallel Robot, Proc. Intelligent Manipulation and Grasping: IMG04 (R. Molfino Ed.), Genova, June 30-July 1, 2004, pp. 377-382
- [12.] CHABLAT D., CARO S., WENGER P. ET ANGELES J., 2002, The Isoconditioning Loci of Planar Three-DOF Parallel Manipulators, 4ème Conférence Internationale sur la Conception et la fabrication Intégrées en Mécanique, IDMME, Clermont-Ferrand, France, Mai, 2002.
- [13.] CHEN S.-L. AND YOU I.-T., 2000, Kinematic and Singularity Analyses of a Six DOF 6-3-3 Parallel Link Machine Tool, Int J Adv Manuf Technol (2000) 16:835-842
- [14.] CODOUREY A., HONEGGER M., BURDET E., 1997, A Body-oriented Method for Dynamic

- Modeling and Adaptive Control of Fully Parallel Robots, SYROCO'97, Nantes, France, September 1997
- [15.] DANIALI H. R. M., MURRAY P. J. Z., ANGELES J., 1996, Direct Kinematics of Double\_Triangular Parallel Manipulators, *Mathematica Pannonica* 7/1 (1996), pp.79-96
- [16.] FALCO, J.A., 1997, Simulation Tools for Collaborative Exploration of Hexapod Machine Capabilities and Applications, *Proceedings of the 1997 International Simulation Conference and Technology Showcase*, Auburn Hills, MI, September 29 - October 3, 1997
- [17.] FAULRING E. L., COLGATE J. E. AND PESHKIN M. A., 2004, A High Performance 6-DOF Haptic Cobot, *Proceedings of IEEE International Conference on Robotics and Automation*, 2004
- [18.] GALLARDO J., RICO J.M., FRISOLI A. , CHECCACCI D., BERGAMASCO M., 2003, Dynamics of parallel manipulators by means of screw theory, *Mechanism and Machine Theory* 38 (2003) pp. 1113-1131
- [19.] GAO X.-S., LEI D., LIAO Q., ZHANG G.-F., 2005, Generalized Stewart-Gough Platforms and Their Direct Kinematics, *IEEE TRANSACTIONS ON ROBOTICS*, VOL. 21, NO. 2, APRIL 2005, pp.141-150
- [20.] GIDDINGS AND LEWIS, 1995, *Giddings and Lewis Machine Tools*, Fond du Lac, WI, 1995.
- [21.] GOSSELIN C., ANGELES J., 1990, Singularity analysis of closed-loop kinematic chains, *IEEE Trans. on Robotics and Automation*, 6(3), June 1990, pp.281-290.
- [22.] GOSSELIN, C. AND HAMEL, J., 1994, The Agile Eye: A High - Performance Three - Degree - of - Freedom Camera-Orienting Device, *Proc. IEEE International Conference on Robotics and Automation*, pp. 781-786, 1994.
- [23.] GOUGH V. E., WITEHALL S. G., 1962, Universal Tire Test Machine, *Proceedings of the 9th International Automobile Technical Congress FISITA*, London (UK), ImechE (pp. 117 - 137) 1962.
- [24.] HAYES M. J. D., MURRAY P. J. Z., CHEN C., 2004, Unified Kinematic Analysis of General Planar Parallel Manipulators, *Journal of Mechanical Design* 2004 by ASME, September 2004, Vol. 126, pp.1-10
- [25.] HUANG Q., HADEBY H. AND SOHLENIUS G., 2002, Connection Method for Dynamic Modelling and Simulation of Parallel Kinematic Mechanism (PKM) Machines, *Int J Adv Manuf Technol* (2002) 19: pp.163-173
- [26.] HUNT, K.H., 1983, Structural Kinematics of In-Parallel-Actuated Robot Arms, *ASME Journal of Mechanisms, Transmissions, and Automation in Design*, Vol. 105, pp. 705-712.
- [27.] JAKOBOVIC D., BUDIN L., 2002, Forward Kinematics of a Stewart Platform Mechanism, *INES 2002, 6th International Conference on Intelligent Engineering Systems 2002*, Opatija, Croatia
- [28.] JIN Y., CHEN I-M., YANG G., 2004, Structure synthesis and singularity analysis of a parallel manipulator based on selective actuation, *Proceedings of IEEE Int. Conf. on Robotics and Automation*, pages 4533-4538, New Orleans, 28-30 April 2004
- [29.] KHALIL W., GUEGAN S., 2001, A New Method for the Dynamic Formulation of Parallel Manipulators, *Journées Franco-Mexicaines d'automatique appliquée*, 12-14 Septembre, 2001
- [30.] KIM D. AND CHUNG, W., 1999, Analytic Singularity Equation and Analysis of Six-DOF Parallel Manipulators Using Local Structurization Method, *IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION*, VOL. 15, NO. 4, AUGUST 1999, pp.612-622
- [31.] KONG X., GOSSELIN C. M., 2002, Kinematics and Singularity Analysis of a Novel Type of 3-CRR 3-DOF Translational Parallel Manipulator, *The International Journal of Robotics Research*, Vol. 21, No. 9, September 2002, pp. 791-798,
- [32.] KOVECSSES J., PIEDBOEUF J.-C., LANGE C., 2002, Methods for Dynamic Models of Parallel Robots and Mechanisms, *Proceedings of the WORKSHOP on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators* October 3-4, 2002, Quebec City, Quebec, Canada, pp.339-347
- [33.] LIU G., LOU Y., AND LI, Z. 2003, Singularities of Parallel Manipulators: A Geometric Treatment, *IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION*, VOL. 19, NO. 4, AUGUST 2003, pp.579-594
- [34.] McAREE P. R., SELIG J. M., 1999, Constrained Robot Dynamics II: Parallel Machines, *Journal of Robotic Systems* 16(9), 487-498 (1999)
- [35.] MERLET J-P., 1999, Forward kinematics of parallel robots, *Proceedings of IMACS Conf. on Applications of Computer Algebra*, El Escorial, 24-27 June 1999
- [36.] MILLER K., 2001, Dynamics of the New UWA Robot, *ACRA 2001, Australian Conference on Robotics and Automation*, Sydney, 14 - 15 November 2001, pp.1-6
- [37.] NANUA P., WALDRON K.J., 1989, Direct kinematic solution of a Stewart platform,

- Proceedings of the IEEE Int. Conf. on Robotics and Automation, Scottsdale, 14-19 May 1989, pp. 431-437
- [38.] PANDILOV Z., V. DUKOVSKI V., 2011, *Several open problems in parallel robotics*, ACTA TECHNICA CORVINIENSIS-Bulletin of Engineering, Tome IV (Year 2011), Fascicule 3 (July-September), pp. 77-84, ISSN 2067-3809
- [39.] PANDILOV Z., RALL K., 2008, *Open problems in parallel robotics*. Mechanical Engineering-Scientific Journal, Published by Faculty Mechanical Engineering, Ss. Cyril and Methodius University, Skopje, Vol.27, No.1, (2008), pp.31-41. CODEN: MINS5-392, ISSN 1857-5293.
- [40.] REINHOLTZ, C. AND GOKHALE, D., 1987, *Design and Analysis of Variable Geometry Truss Robot*, Proc. 9<sup>th</sup> Applied Mechanisms Conference, Oklahoma State University, Stillwater, OK, USA.
- [41.] SADJADIAN H., TAGHIRAD H.D., FATEHI A., 2005, *Neural Networks Approaches for Computing the Forward Kinematics of a Redundant Parallel Manipulator*, International Journal of Computational Intelligence Volume 2 Number 1 2005, pp.40-47
- [42.] SELLGREN U. AND PETTERSSON J., 2001, *Modeling of conformal mechanical interfaces in technical systems*, Proc. of NAFEMS World Congress, April 24-28, Lake Como, Italy, pp. 943-954.
- [43.] STEWART, D., 1965, *A Platform with Six Degrees of Freedom*, Proc. Inst. Mech. Eng. London, Vol 180, 1965, pp. 371-386.
- [44.] TODD, D.J., 1986, *Fundamentals of Robot Technology*, Kogan Page, 1986.
- [45.] TSAI L. W., 1999, *Robot Analysis: The Mechanics of Serial and Parallel Manipulators*, New York: John Wiley & Sons, Inc., 1999.
- [46.] WALDRON, K. J., VOHNOUT, V.J., PERY, A., AND MCGHEE, R.B., 1984, *Configuration Design of the Adaptive Suspension Vehicle*, Int. J. Robot. Res., Vol. 3, 1984, pp. 37-48
- [47.] WALDRON K.J. ET HUNT K.H., 1987, *Series-parallel dualities in actively coordinated mechanisms*, Proceedings of 4th ISRR, Cambridge, MA, pp. 175-182
- [48.] WANG J., TANG X., DUAN G., LI J., 2001, *Design Methodology For a Novel Planar Three Degrees of Freedom Parallel Machine Tool*, Proceedings of the 2001 IEEE International Conference on Robotics & Automation Seoul, Korea . May 21-26, pp.2448-2453
- [49.] WILLIAMS II R. L., JOSHI, A. R. 1999, *Planar Parallel 3-RPR Manipulator*, Proceedings of the Sixth Conference on Applied Mechanisms and Robotics, Cincinnati OH, December 12-15, 1999
- [50.] [www.abb.com](http://www.abb.com)
- [51.] [www.delphion.com](http://www.delphion.com)
- [52.] [www.alioindustries.com/parkin\\_robotics.htm](http://www.alioindustries.com/parkin_robotics.htm)
- [53.] [www.cmttec.com/contmeas/motion.htm](http://www.cmttec.com/contmeas/motion.htm)
- [54.] [www.ds-technologie.de](http://www.ds-technologie.de)
- [55.] [www.moog.com](http://www.moog.com)
- [56.] [www.fanuc.co.jp](http://www.fanuc.co.jp)
- [57.] [www.fanucrobotics.com](http://www.fanucrobotics.com)
- [58.] [www.parallemic.org](http://www.parallemic.org)
- [59.] [www.physikinstrumente.de/products](http://www.physikinstrumente.de/products)
- [60.] [www.physikinstrumente.com/micropositioningsystems](http://www.physikinstrumente.com/micropositioningsystems)
- [61.] [www-sop.inria.fr](http://www-sop.inria.fr)
- [62.] YANG G., CHEN I.-M., LIN W. AND ANGELES J., 2001, *Singularity Analysis of Three-Legged Parallel Robots Based on Passive-Joint Velocities*, IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, VOL. 17, NO. 4, AUGUST 2001, pp. 413-422
- [63.] YIU Y.K., AND LI Z.X., 2001-1, *Modeling Configuration Space and Singularities of Parallel Mechanisms*, International Conference on Mechatronics Technology, 6 - 8 June 2001, Singapore pp. 298-303
- [64.] YIU Y.K., AND LI Z.X., 2001-2, *Dynamics of a Planar 2-dof Redundant Parallel Robot*, International Conference on Mechatronics Technology, 6 - 8 June 2001, Singapore, pp.339-344
- [65.] ZHAO X. AND PENG S., 2000-1, *Uncertainty configurations of parallel manipulators*, Robotica (2000) volume 18, pp. 209-211.
- [66.] ZHAO X., PENG S., 2000-2, *Direct Displacement Analysis of Parallel Manipulators*, Journal of Robotic Systems 17(6), (2000), pp.341-345
- [67.] ZHOU K., MAO D. AND TAO Z., 2002, *Kinematic Analysis and Application Research on a High-Speed Travelling Double Four-Rod Spatial Parallel Mechanism*, Int J Adv Manuf Technol (2002) 19: pp.873-878
- [68.] ZLATANOV, D., BONEV, I.A., GOSSELIN, C.M., 2002, *Constraint Singularities as C-Space Singularities*, 8th International Symposium on Advances in Robot Kinematics (ARK 2002), Caldes de Malavella, Spain, 24-28 June, 2002.
- [69.] ZOPPI M., BUZZONE L. E., MOLFINO R. M., NICHELINI R. C., 2003, *Constraint Singularities of Force Transmission in Nonredundant Parallel Robots With Less Than Six Degrees of Freedom*, Journal of Mechanical Design 2003 SEPTEMBER 2003, Vol. 125, pp. 557-563