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ADOPTION OF OPTIMAL TEETH PARAMETERS OF GEROTOR PUMP

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Abstract: Adoption of optimal teeth parameters for gerotor pumps is of great importance and it has a large influence on their operational characteristics. This is the reason why a lot of attention is directed towards their optimization. The working flow of a gerotor pump has been chosen as the topic of optimization in this paper. Taguchi method for ranking gearing parameters related to pump flow has been presented here. Based on teeth parameters influence rank, an optimization is performed by method of linear programming. As a result of optimization optimal teeth parameters are chosen.

Keywords: gerotor pump, Taguchi method, optimization, linear programming

INTRODUCTION

One of the new types of teeth, which could largely replace the involute profile, is the trochoid tooth. The introduction of this type of gearing would greatly enhance functional features such as needed efficiency and service life with minimal weight and overall dimensions. This type of tooth may lead to a reduction in material consumption that would lead to big savings. Another big advantage is the simultaneous trochoidal tooth gear contact resulting in better carrying capacity compared to the gear pairs with involute gears.

Lately, due to the advantages of trochoidal teeth a greater interest of mechanical engineers for their implementation in mechanical systems has appeared. Applications of trochoidal teeth are present in a large number of rotating machines, which are used in a variety of applications: rotary pumps, rotary motors, rotary compressors etc. An excellent application of trochoidal gear teeth has been found in cycloid drives, which belong to a special group of gearing in planetary gears. Considering the characteristics of trochoidal teeth, their development is cost effective [1].

This paper will present the choice of optimal gearing parameters for gerotor pumps. The optimization criterion is the operational displacement of the gerotor pump. Optimal choice of parameters is performed by using the hybrid optimization method on two levels. The first level of optimization is performed by using the Taguchi method, which is determined by the most influential parameter for the teeth on the working displacement of the gerotor pump. The second level is derived by the method of linear programming, where the optimal gear parameters have been derived. At the end of this paper, conclusions and directions of possible research have been made. Both directions of research have been given, to improve the construction of the pump, and to develop a hybrid optimization method.

LITERATURE OVERVIEW

The optimization process is a way to obtain the best solution to the stated problem [2]. Selection of an optimization method for a specific problem involves a solid knowledge of optimization and its methods [3]. Through deliberation of specific methods on the problem of the operational displacement of a gerotor pump, it can be concluded that for the solution of this problem it is necessary to conduct optimization in two iterations.

In order to determine which parameters have the most influence on the operational displacement of the pump the Taguchi method is chosen. The wide spectrum of the Taguchi method's application on optimization problems are given in detail in [4] and [5]. By selecting the Taguchi method it has become possible to attain a hybrid method. In [6-7], the Taguchi method and the evolutionary optimization algorithm have been used on the problem of an automobile door bracket. In this paper, optimization is performed by the criterion of minimum displacement, in order to save on material. Paper [8] presents a procedure for the use of the optimization method that is based on a combination of Taguchi method and an evolutionary algorithm. This paper gives an explanation of why the hybrid method gives better results than the conventional optimization methods.

Hybrid optimization methods are one of the youngest optimization methods. In order to implement hybrid methods, a solid knowledge of the methods themselves as well as the possibilities of their efficient computerized utilization is paramount. Using the Taguchi method, these papers determined the influence of parameters on the objective function. Determining the influence of parameters significantly reduces the spectrum of solutions. By reducing the field of solutions and using another optimization method, the most correct solutions for a given problem can be obtained.

To solve the problem of maximizing the operational displacement of a gerotor pumps, unlike the aforementioned hybrid methods, in addition to the Taguchi method, a method of linear programming has been added [9]. In order to confirm the results of the applied hybridization an experimental comparison with existing data was performed [1].

IDENTIFICATION OF THE MOST INFLUENTIAL TOOTH PARAMETER

Identification of the most influential tooth parameter for gerotor pumps is very important. The reason is the ranking of parameters from the most to least influential on the operational displacement, for choosing a parameter variation degree from which a choice of optimal parameter values can be chosen by further optimization.

Setting the objective function for maximizing displacement

The objective function in finding the most influential tooth parameter of a gerotor pump is the formula for determining operational displacement:

$$q = be^2 \frac{z-1}{z}\left[s^2 - (\lambda z - c)^2\right] \tag{1}$$

Distances from the centers of circular profiles of the center of the trochoidal gears, are calculated using the ratio:

$$s = \frac{r_s}{e} \tag{2}$$

With all the figuring variables explained in the function for determining pump displacement, the next step is determining functional parameters from which the maximal operational displacement of the gerotor pump will be determined.

Determination of functional parameters

Determination of functional parameters that will be subjected to optimization depends on the parameters that will be chosen to be unchallengeable and predetermined before optimization. For fixed-parameter optimization selected values are given in the table 1.

These measures were selected as fixed input parameters because they define the dimensions of trochoidal gears. By defining them at the start the required gross measurements of the pump housing can be obtained. Other dimensions define the operational displacement of the pump, work-life, reliability and other pump parameters. Optimizing the other three parameters leads to a maximization of the operational displacement of the gerotor pump.

Table 1. Fixed parameters

Trochoidal gear width, b	*16,46 [mm]*
Pump eccentricity, e	*3,56 [mm]*
Distance of the center of the circular profile to from the center of the trochoidal gear, s	*7,57 [mm]*
The radius of the root of the external gear, r_S	*26,94 [mm]*

The table show parameters which will be optimized, are number of teeth and profile coefficient of the trochoid gear: z – number of teeth of the external gear, λ – trochoid radius coefficient, c – equidistant radius ratio.

The variation of the number of teeth z, trochoid radius coefficient λ, and equidistant radius coefficient depend on the limit function.

The methodology developed here can be carried out for optimization of the displacement of any gerotor pump. For the gerotor pump from the paper [1], with which our results will be compared, the restricting functions are as follows:

1. *Number of gear teeth will be included in the set of integers z = {4, 5, 6, 7}. Number of gear teeth is a discrete variable.*
2. *The trochoid radius coefficient, λ, will be in the interval $1 < \lambda < 2$, and will be divided into five parts,*
3. *The equidistant radius coefficient, c, will also be placed at the interval of $2 < c < 6$, and it's limit will be discussed later in this paper.*

After defining all the parameters that affect the gearing gerotor pumps, access to determining the type of orthogonal matrices.

Selection of an orthogonal matrix to maximize operational displacement

Selection of an orthogonal matrix for the Taguchi method is based on the number of effective parameters of the objective function. Since the objective function is affected by six parameters matrix selection is restricted to a choice between matrices L8, L18, L32 and L25. Because of the six influential parameters on the gerotor pump displacement a matrix of six parameters with five levels is chosen. This is the L25 matrix. By selecting these orthogonal matrix greater variations in relation to the matrix of lower rank is enabled, between trochoid radius coefficients, λ, and equidistant radius, c, and the number of teeth that was previously defined.

Application of Taguchi method to maximize the displacement

After determining all functional parameters and their limits, they form an orthogonal matrix L25. In an extreme case, a small variation of fixed parameters value can be performed within acceptable tolerances of trochoid. However, due to the nature of the objective function this would not have a significant impact on the result. In the case that the first three columns are not considered in the orthogonal matrix, due to the methods functioning reliable results would not be possible.

After the formation of the basic L25 orthogonal matrix, it is necessary to determine the value of the objective function to five levels of the Taguchi method. Once the objective function S/N ratio is calculated according to the formula that is used to maximize the objective function:

$$S/N_i = -10\log\left(\frac{1}{N_i}\sum_{j=1}^{Ni}\frac{1}{T_j^2}\right) \tag{3}$$

Parameter ranking according to influence

The final operation before finding the influence of variables on the objective function is completing the matrix for ranking parameters. Calculating the matrix elements for ranking parameters is done by calculating average values of elements compared to the position variation of matrix elements [4].

Table 2. Parameter ranking by the impact of gerotor pump operational displacement

Level	*λ*	*c*	*z*
1	*45.95*	*42.61*	*65.28*
2	*51.98*	*48.58*	*56.27*
3	*46.21*	*51.28*	*47.99*
4	*55.40*	*53.55*	*36.56*
5	*52.21*	*55.74*	*45.66*
Δ	*9.46*	*13.13*	*28.72*
	3	*2*	*1*

Since only three variables are included in, ranking the objective function of the matrix will be 3x5. The matrix with ranked elements is given in Table 2. Table 2 shows that the greatest impact on the change in displacement, when choosing gearing parameters of gerotor pumps, has the number of teeth of the trochoid gear, z. The following parameter, according to impact, is the equidistant radius coefficient, c, while the smallest impact is from the trochoid radius coefficient, λ. Parameter, z, is a discrete value and is located in the set of previous values {4, 5, ... , 13}, while for this particular problem it is in the subset of the set of values {4, 5, 6, 7}, as was noted in the beginning of this chapter. These kind of variables greatly facilitats further optimization, because the objective function can be viewed as independent of each value of the parameter z.

IDENTIFICATION OF THE MOST INFLUENTIAL TOOTH PARAMETER

Optimization according to its general definition pertains to the best and most acceptable solution to a defined problem. Mathematically speaking optimization implies finding a maximum or minimum of a function f(x), which depends on the vector parameters $\{X_1, X_2, \dots, X_n\}$ while satisfying the constraint function g(x). Many methods have been developed for solving optimization problems. Depending on the type of problem a method for solving this problem is chosen or a new method is developed. For the task of optimizing the gerotor pump, operational displacement linear programming has been applied.

Optimization of pump gearing using linear programming method

Equation (1) could be subjected to optimization methods using quadratic programming, however in order to simplify the problem only part of the equation will be used (1), depending on which the objective function has a minimum or maximum. Therefore, the new objective function will have the following form:

$$f(x)_{min} = \lambda z - c \tag{4}$$

In this function, the parameter z will belong to a set of values z = {4, 5, 6, 7}. After this declaration, it can be seen that the linear programming optimization process has four iterations. In addition to the objective function for optimization process function limitations must also be defined [1]:

$$c > 2 \tag{5}$$

$$c \le z\lambda \sin\frac{\pi}{z} \tag{6}$$

$$c = z\lambda + 2 - S_{fa} \tag{7}$$

$$c \le z\sqrt{\left(\frac{3}{z+1}\right)^3 \left(\lambda^2 - 1\right)\left(z-1\right)} \tag{8}$$

To perform linear programming functions suiting labels must be adapted to symbols which are used in linear programming. The way this is done is described in the following chapter.

Linear programing for z=6

Before the start of linear programming, all functions must be translated into a form that corresponds to the used method. Translating the objective function and the limitation to perform linear programming, for z = 6, is given by expressions:

$$f(x)_{min} = 6x_1 - x \tag{9}$$

$$g(x)_1 \Rightarrow 1 < x_1 < 2 \tag{10}$$

$$g(x)_2 \Rightarrow x_2 > 2 \tag{11}$$

$$g(x)_{1,26} \Rightarrow x_2 \le 6x_1 \sin\frac{\pi}{6} \tag{12}$$

$$g(x)_{1,26} \Rightarrow x_2 = 6x_1 - 5{,}5 \tag{13}$$

$$g(x)_{1,26} \Rightarrow x_2 \le 6\sqrt{\left(\frac{3}{6+1}\right)^3 \left(x_1^2 - 1\right)\left(6-1\right)} \tag{14}$$

After transforming to the form which is suitable for the linear programming approach, linear programming diagrams are drawn. The diagram of linear programming for z=6, is shown in Figure 1.

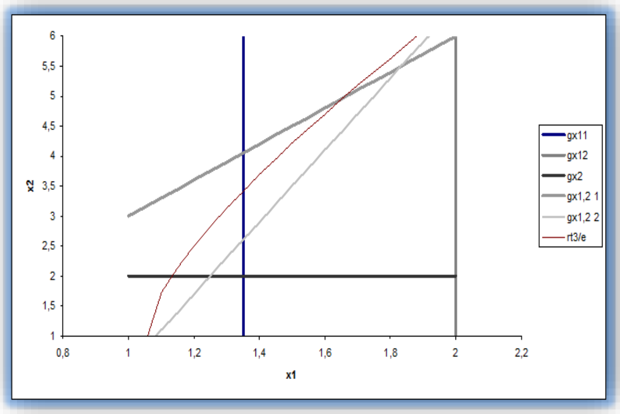

Figure 1. Diagram of linear programming for z=6

From figure 1 the middle of the line which is limited by the function (7) is found. Since the value, Sfa (Sfa=7,5), is also a constant for gear dimensions which are optimized, the function is as shown in (13). By calculating the line the midpoint, obtained optimal values are given in Table 3:

Table 3. Optimal parameters for z=6

z	λ	c
6	1.591	4.049

Optimal solution choice

The last step in the whole process, after four iterations of linear programming, is the choice of an optimal solution according to operational displacement criteria. Based on certain optimal values a table for comparison of working displacement has been formed according to the numbers of teeth (Table 4).

Table 4 shows that according to the criterion of maximum displacement the optimal solution is the combination of teeth number parameters for z = 6 teeth.

Table 4. Optimal solution choice

No. teeth	λ	c	q [mm^3]
z=4	1,937	2,250	13290
z=5	1,749	3,247	14181
z=6	1,591	4,049	14772
z=7	No solution		

CONCLUSION

In this study, the two known are methods presented as new hybrid optimization method. The obtained results are compared with the results from paper [1]. By applying, the methods presented here a significant improvement, in terms of increasing displacement has been achieved. The methodology after determining the most influential parameter of the process the Taguchi method can be changed in various directions, that is, choosing other optimization methods for continuing the research.

Further research could include determining, using flow measuring methods, and the operational displacement of a gerotor pump with the newly attained gearing parameters. Experimental verification would confirm these methods.

NOMENCLATURE

Letter symbols:

b	Width of trochoidal gear, mm
c	equidistant radius ratio
e	pump eccentricity, mm
N	number of calculated objective functions
q	operational displacement of gerotor pump mm^3
r	invert of the outer radius of the circle gear, mm
s	distance from the center of the circular profile to the center of trochoidal gear, mm
S/N	signal to noise ratio
T	value of the objective function
z	number of external gear teeth

Greek symbols

λ	trochoid radius coefficient

Subscripts and superscripts

i	trial number
j	number of calculated objective function
o	outer

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