ACTA TEHNICA CORVINIENSIS – Bulletin of Engineering
Tome VIII [2015] Fascicule 3 [July – September]
ISSN: 2067 – 3809

[1.] Nicoleta UNGUREANU, [2.] Sorin Ştefan BIRIŞ, [3.] Valentin VLĂDUŢ, [4.] Gheorghe VOICU, [5.] Gigel PARASCHIV

STUDIES ON THE MATHEMATICAL MODELING OF ARTIFICIAL SOIL COMPACTION

[1,2,4,5.] Politehnica University of Bucharest, ROMANIA
[3.] National Institute of Research - Development for Machines and Installations Designed to Agriculture and Food Industry (INMA), ROMANIA

Abstract: The compaction of agricultural soil is a phenomenon of degradation, occuring from natural or artificial causes and is defined by the increase in soil density and the decrease in soil porosity, with negative consequences for the environment and for agriculture. Artificial soil compaction is generated by the contact between soil and tires or tracks of tractors and agricultural machinery. This paper presents some general mechanical concepts regarding the behavior of soil under compressive stresses and a review of some mathematical models which can be applied to describe the behavior of soil at compaction, under the influence of various parameters.
Keywords: soil; artificial compaction; tire; contact area; stress; strain

INTRODUCTION

The phenomenon of soil compaction can be assimilated with the compressive strain and can be represented by a criterion of soil flow. A first compressive behavior of soil occurs when the compressive forces applied to the soil produce a certain degree of compaction, which, however, disappears after the removal of forces. In other terms, the volume decreases as stress increases, and viceversa. From an mathematical point of view, input and output variables are uniquely correlated to the entire field of values. Another behavior occurs when, after termination of the action of compressive forces, the soil does not return to its original state. In this case, input and output variables are uniquely correlated only to a certain set of values. The soil has both plastic (the most common situation) and elastic behavior. In order to develop and interpret equations that describe the behavior of soil to compaction, the circumstances in which the phenomenon occurs must be known [5].

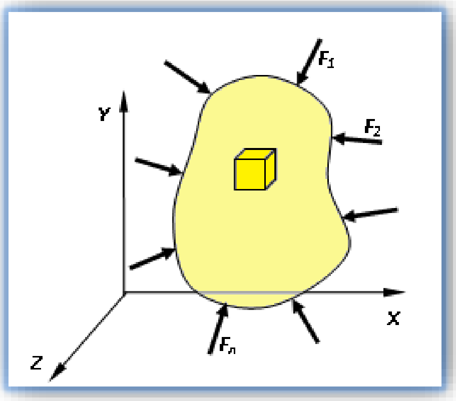

Figure 1. Action of a system of forces on an elastic body [3, 5]

A cubic element of volume, having the sides equal to the unit, is isolated from a linear-elastic, homogeneous and isotropic body upon which is applied a spatial system of forces (figure no 1). Stresses emerged on the sides of this element of volume can be decomposed after the directions of the three axes of the reference system (figure no 2).

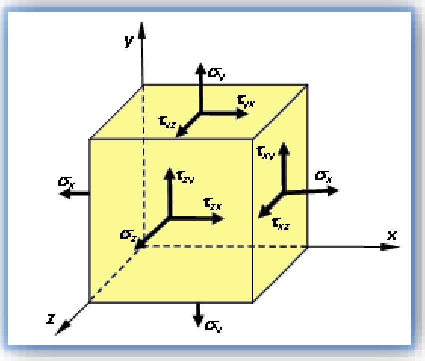

Figure 2. Stresses emerged on the sides of the element of volume [5]

MATERIAL AND METHOD

Stresses and strains in agricultural soil

Since the time of Archimedes, the distribution os stresses in the soil is a subject of scientific and engineering interest, culminating with the concept proposed by Cauchy (Truesdell, 1961; Davis şi Selvadurai, 1996), who assumed that the nature of reaction forces generated during the transmission of external loads appplied to the interior of a solid aren't different from the tractive forces applied to the boundary of a loaded solid [14].

The understanding of strain processes of arable soil is still limited. One reason for this fact could be that most research studies related to soil *compaction focused on the agronomic impact of compaction (and, more recently, on the environmental impact of compaction), at the expense of the strain process itself. External mechanical stresses applied to the soil (by the agricultural machinery) are related to the physical functions of soil or to crops reaction. To understand the impact of compaction on soil functions (reaction) is necessary to know the process of soil strain (cause). Strain is the response of soil to a mechanical or hydraulic stress applied on it. Soil stresses and strains are coupled phenomena: soil strain depends on soil stress and soil resistance, and the distribution of stresses in the soil depends on soil resistance and soil strain [14].*

Stress tensor for the soil

Stress state of a cubic and infinitely small soil element can be descrbed by the normal stress σ_i - perpendicular to the side of the element of volume and by shear stresses τ_{ij} – tangential to the sides of the element [5, 12].

Stress state can be described by the matrix of stress tensor (Koolen & Kuipers, 1983) [12]. To describe the stress state in point A found in the interior of the element of volume on which is applied an external load, is chosen a system of axes x, y, z and is assumed an infinitesimal cube around point A, with the sides parallel to the axes of the coordinate system (figure no 3) [5].

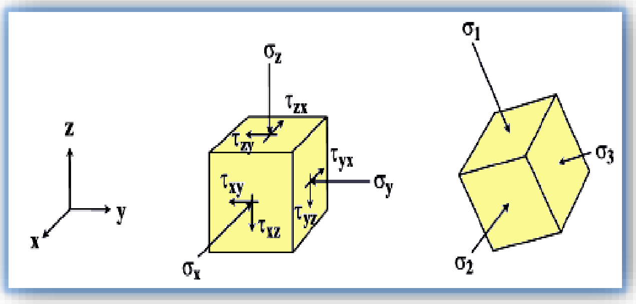

Figure 3. Components of stress [5, 6, 12, 16]

„Mohr's stress diagram" allows the plotting of stresses σ and τ for given σ_1 and σ_3 and for an variable θ angle. The steps required for this method are:

1. *drawing a rectangular system of axes with coordinates σ-τ;*
2. *positioning of stresses σ_1 and σ_3 on axis σ;*
3. *drawing a circle through the diametrically-opposite points σ_1 and σ_3 with the centre on axis σ;*
4. *drawing of A-B line from point σ_3 which makes an angle θ to the horizontal;*
5. *determining the coordinates of point B located on the circle, that represents the values of stresses σ and τ.*

This method is useful to find the values of stresses σ and τ for any θ angle, if the values of main stresses σ_1 and σ_3 are known (figure no 4).

Stresses theory for infinitesimal cubes can be extended to bodies (volumes of soil). This extension is verified experimentally using devices that measure the geometric changes of the volume of soil subjected to loadings, considering the soil samples as finite bodies rather than infinitesimal cubes. Next are analyzed the cases of a finite cube, a finite cylinder and a finite sphere.

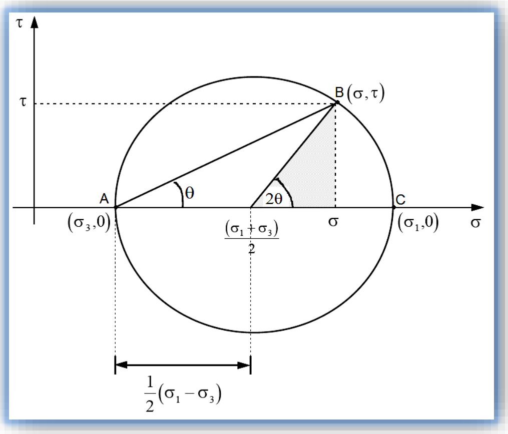

Figure 4. Mohr's stress diagram [5]

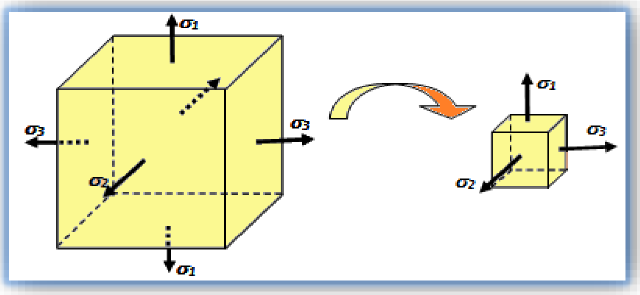

Figure 5. Finite cube [5]

Finite cube consists of small infinitesimal cubes, loaded with main stresses σ_1, σ_2 and σ_3. The large cube will have the following stresses: on the upper and on the lower side - normal stress σ_1, on the left and on the right side - normal stress σ_3, and on the front side and rear side – normal stress σ_2. So, if a cubic body is loaded by stresses σ_1, σ_2 and σ_3, then the stress state of each point will be given ny the matrix:

$$[\sigma]=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix} \qquad (1)$$

having the axex parallel to the sides of the cube.

Finite cylinder is composed of small cubes, respectively small prisms in the walls area. For small cubes is assumed that $\sigma_2 = \sigma_3$. For the upper and lower sides of the cylinder, the stress will be σ_1 and for stresses near the wall can be used equations 16 in section or Mohr's diagram, and it can be inferred that in figure no 6 the stress σ is equal to $\sigma_2 = \sigma_3$ and $\tau=0$. If a cylinder is loaded with σ_1 in the upper and lower sides, and with σ_m to the wall, then the stress in each point is given by the following matrix:

$$[\sigma_c]=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2=\sigma_m & 0\\ 0 & 0 & \sigma_3=\sigma_m\end{bmatrix} \qquad (2)$$

with σ_1 acting along the simetry axis of the cylinder.

For a finite sphere, loaded with σ_h after all directions, it can be shown similarly that stress state in any point is given by the matrix:

$$[\sigma_s]=\begin{bmatrix} \sigma_1=\sigma_h & 0 & 0 \\ 0 & \sigma_2=\sigma_h & 0 \\ 0 & 0 & \sigma_3=\sigma_h \end{bmatrix} \quad (3)$$

Strain theory *[5]*

Theory of strain state in a point, with limitation in the area of small strains. At each point of the body (volume) of soil loaded there is a stress state and a strain state. Strains can be described comparably to that for the description of stress. Stress state has been described by normal and shear stress, and similarly the components of strain are: normal strain and shear strain.

Normal strain can be defined considering an infinitesimal line segment, with length L before the strain, respectively lenght L+ΔL after strain, thus:

$$\varepsilon=\frac{\Delta L}{L} \quad (4)$$

Shear strain is defined based on an angle which is straight before strain occurs for an infiniesimal volume of soil and for which, after strain, is rotated clockwise with an small α angle, while the horizontal side is rotated counterclockwise with an small β angle, then the shear strain of the initial right angle is given by the following equation:

$$\gamma=\frac{1}{2}\cdot(\alpha+\beta) \quad (5)$$

RESULTS

Probabilistic and deterministic mathematical models to express the behavior of soil at compression

Strategies and recommendations to prevent soil compaction are often based on simulation models (models of soil compaction), that allow the calculus of stress distribution and soil failure in the profile of soil for a certain mechanical loading (made by agricultural machinery) and for certain soil conditions (e.g., soil moisture and bulk density) and which can be of real benefit to farmers, in planning and deciding on the specific conditions of traffic on agricultural soils [13, 24].

Bekker (1956) and Söhne (1951, 1953) used the concepts of mechanics developed by Boussinesq (1885), Terzaghi (1925, 1943) and Fröhlich (1934) to obtain stress-strain soil models of soil under the traffic of agricultural vehicles [13, 14, 18].

Soil compaction models can be divided in three parts [8, 12, 13]:

1) *boundary conditions at soil surface, which refers to the contact area and the distribution of stress on soil surface;*
2) *distribution of equivalent stress in the soil;*
3) *soil behavior at stresses-strains.*

Among the three parts of soil compaction models there are various relationships, for example, the stress-strain behavior can influence the stresses on soil surface. First, must be defined the stresses at soil surface, then stress distribution is calculated, and soil strain is calculated by applying a stress-strain relationship to the calculated stress, or the state of compacted soil is estimated by comparing the calculated stress to the critical stress (for example, the precompression stress) [13]. It is neccessary to determine soil behavior in the field and then to correlate this behavior with in situ (in laboratory) behavior by mechanical tests, because [13]:

- *the duration of mechanical load in the field (for example, by agricultural tire) is much smaller compared to that of laboratory tests for the determination of soil mechanical properties;*
- *in the fied, loads are dynamic (so, the direction of main stresses is not constant in time), while in laboratory the loads are static;*
- *unlike filed loads, soil samples used in laboratory tests are often loaded under controlled conditions.*

Mechanical models simulate the processes taking place in a system, and empirical models define the relationship between input and output data, without defining the dynamic processes. The most common empirical model is the concept of soil resistance-soil stresses, in which soil resistance is considered similar to the preconsolidation pressure, and stress distribution in the soil can be estimated using a concentration factor [13].

Numerical models for soil compaction were proposed by Bailey et. al. (1986), Gupta and Larson (1982), Larson et. al. (1986), et. al. (1977a, 1977b), Smith (1985), Gupta and Allmaras (1987). Smith's model (1985) is based on the prediction of specific soil volume change due to changes in spherical stress produced by wheel load and contact area between tire and soil. Soil depth is divided into elements of layers, then is estimated the increase in spherical stress in the center of each layer under the wheel. The model can be used to compare the influence of different types and arrangements of wheels on the compaction. Gupta and Larson (1982) developed a model for soil compaction based on compression equations and on Boussinesq's equations modified by Söhne (1953), by introducing a concentration factor to describe different types and conditions of soil. Raghavan et (1977a; 1977b) developed an equation that describes the maximum change of loamy soil density after several passes of a tractor tire [10].

Mathematical models using bulk density to describe soil behavior at compression

The process of soil compaction can be described mathematically by relationships between soil bulk density (ρ) and the stress applied on the soil (σ).

✓ *Bailey's multiplicative model with three parameters (1986) [1]*

$$ln(\rho)=ln(\rho_0)-(a+b\cdot\sigma)\cdot(1-e^{-c\cdot\sigma}) \quad (6)$$

where: ρ – soil density; ρ_0 – soil bulk density when the stress applied on the soil σ = 0; a, b, c – empirical parameters.

Logarithmic models predict that soil bulk density increases as the stress applied on the soil increases [1].

✓ *Assouline's model (1986) [1]*

Experimental observations based on Bailey's model indicate that the soil can be compacted to a maximum bulk density which depends on its initial state. In this regard, Assouline proposed the following logarithmic model:

$$ln(\sigma) = \rho_0 + (\rho_{max} - \rho_0) \cdot (1 - exp[-\xi \cdot \sigma]) \quad (7)$$

where: ρ_{max} and ξ – connection parameters.

Improvements to Bailey's model consist in the need of only two commection parameters, and the value of ρ_{max} can be determined in laboratory conditions using compression tests [1].

- ✓ *Fritton's model (2001) [1].*

Since the models proposed by Bailey and Assouline are not flexible enough to represent the variation of the shape of compression curves, from almost linear to S-shaped curves, expressed on logarithmic scale, Fritton proposed the following model:

$$\rho = \rho_m - (\rho_m - \rho_0) \cdot \{1 + [\alpha \cdot (\sigma + 1)]^n\}^{-m} \quad (8)$$

where: ρ_m – particle density; ρ_0 – initial bulk density; α, n, m – empirical parameters of connection.

By definition, for $\sigma = 0$ is required that $\rho = \rho_0$. Thus, Fritton's model becomes:

$$\rho = \rho_m - (\rho_m - \rho_0) \cdot [1 + \alpha^n]^{-m} \quad (9)$$

This model assumes that soil can be compacted until particle density, ρ_m, but this assumption is physically unrealistic and incompatible with the experimental observations of Amir et. al. (1976) and Faure (1981).

- ✓ *Gupta and Larson's model (1982) [12]*

Change of soil volume is described by a mathematical relationship between soil bulk density and the logarithm of maximum main stress:

$$\rho = [\rho_k + \Delta_T \cdot (S_l - S_k)] + C \cdot log\left(\frac{\sigma_a}{\sigma_k}\right) \quad (10)$$

where: ρ – final density corresponding to the applied stress σ_a; ρ_k – reference bulk density corresponding to reference stress σ_k on the virgin compression line (VCL); Δ_T – slope of variation curve of bulk density depending on water saturation degree at σ_k; S_l – desired saturation degree at σ_k; S_k – saturation degree at ρ_k and σ_k; C – compression index (slope of VCL).

- ✓ *Bailey and Johnson's model (1989) [12]*

This model describes the state of cylindrical stress in the soil:

$$ln\rho = ln\rho_0 - \left[(A + B \cdot \sigma_{oct}) \cdot (1 - e^{-C \cdot \sigma_{oct}}) + D \cdot \left(\frac{\tau_{oct}}{\sigma_{oct}}\right)\right] \quad (11)$$

where: ρ_0 – initial bulk density; σ_{oct} – octaedrical normal stress; τ_{oct} – octaedrical shear stress; A, B, C, D – compactity coefficients (for $D = 0$, this model is reduced to Bailey's model, 1986).

- ✓ *Model to estimate soil density after tire passage, depending on the contact pressure [21]*

Schwanghart studied the effects of agricultural vehicle tires on soil compaction, the pressure in these tires being generally lower than that of transport tires.

$$p = 45 + 0{,}32 \cdot p_i \quad (12)$$

where: p - contact pressure (kPa); p_i – tire inflation pressure (kPa).

Soil density after the passage of tire is given by the following relationship:

$$BD = \frac{3{,}275}{1{,}86 + \frac{0{,}36}{0{,}01 \cdot p + 0{,}277}} \cdot 1000 \quad (13)$$

where: p – contact pressure (kPa); BD – soil bulk density (kg/m[3]).

Based on these two models was obtained the variation presented in figure no 6, from which it can be noticed that the variation of tire inflation pressure has low effects on soil compaction.

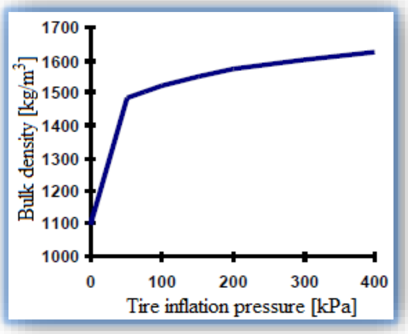

Figure 6. Influence of tire inflation pressure on soil bulk density [21]

- ✓ *Model for the calculus of soil compactity [19]*

Soil compactity is expressed as the ratio between the density of solid parts of soil ρ_s and total density of soil (voids and solid), ρ_b:

$$v = \frac{\rho_s}{\rho_b} \quad (14)$$

where: v – soil compactity; ρ_s – density of solid parts of soil; ρ_b – total density of soil.

Soil compactity is expressed as the maximum specific volume of soil at a certain given value of mean normal stress. The equation used for practical purposes is [12, 17, 19]:

$$v = N - \lambda_n \cdot lnp \quad (15)$$

where: λ_n – compression index; N – specific volume of soil if pressure p = 1 kPa.

- ✓ *Model for soil sinkage after repeated passes of a tractor (Abebe, 1998) [20]:*

$$z_n = z_1 \cdot n^{\frac{1}{a}} \quad (16)$$

where: z_n – soil sinkage after n passes (m); z_1 – soil sinkage after the first passage (m); n – number of passes; a – multipass coefficient.

Models for the calculus of the contact area

- ✓ *COMPSOIL model for contact area [8]*

O'Sullivan et. al. (1999) estimated soil bulk density on the longitudinal centerline of the wheel, starting with the tire's ruts on the soil. Contact area was calculated with the following relationship:

$$A = s_1 \cdot b \cdot d + s_2 \cdot L + s_3 \cdot \frac{L}{p_i} \quad (17)$$

where: A – contact area (m[2]); L – wheel load (kN); b – width of transversal tire section (m); d – overall tire diameter (m); p_i – tire inflation pressure (kPa); s_1, s_2, s_3 – empirical parameters depending on soil surface.

Table 1. Values of empirical parameters s_1, s_2, s_3 (O'Sullivan et.al.) [19]

Parameter	Rigid surface	Deformable surface
s_1	0,041	0,31
s_2	0	0,00263
s_3	0,613	0,239

✓ *Komandi's emprical model for contact area [7, 15, 22]*

$$A_{sol} = c \cdot F^{0.7} \cdot \sqrt{\frac{b}{D}} \cdot p_i^{-0.45} \quad (mm^2) \qquad (18)$$

where: c - constant (c = 0,3 - 0,32 for rather bearing soils; c = 0,36 - 0,38 for sandy soils; c = 0,42 - 0,44 for loose soil); F – wheel load (N); b – tire width (mm); D – tire diameter (mm); p_i – tire inflation pressure (MPa).

✓ *FRIDA model for footprint area [25]*

Is used to describe the footprint between soil and tire by an superellipse and the distribution of stress by an exponential function (perpendicular on the driving direction) or a power function (along the driving direction). The contour of footprint, in top view, can be modeled by a superellipse (Hallonborg, 1996; Febo ş.a., 2000; Keller, 2005) which, in orthogonal system with the center in the origin, has the following shape:

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1 \qquad (19)$$

where: a and b – half of small and large axes [m]; n – rectangularity. Boundary and interior of the superellipse are given by:

$$\Omega = \{(x,y) \mid |x/a|^n + |y/b|^n \leq 1\} \qquad (20)$$

***Empirical models for footprint area** (Grecenko, 1995) [22]*

$$A = 1{,}57 \cdot (d - 2 \cdot r_l) \cdot \sqrt{d \cdot b} \qquad (21)$$

$$A = \pi \cdot \delta \cdot \sqrt{d \cdot b} \qquad (22)$$

$$A = c \cdot d \cdot b \qquad (23)$$

where: A – contact area (m^2); r_l – radius of unloaded tire (m); b – tire width (m); d – tire diameter (m); δ – soil deformation (m); c – constant (c = 0,175 for rigid tire on rigid soil; c = 0,245 for flexible tire with 20% deformation on soft soil; c = 0,270 for rigid tire on soft soil).

***Söhne's model for stress distribution at soil-tire interface** [9]*

Söhne proposed equations describing three types of vertical stress distribution (uniform, square, parabollic), for three types of soil (resistant, relatively resistant and soft), if the contact area has circular shape.

$$\sigma_v = \sigma_0 \qquad (24)$$

$$\sigma_v = 1{,}5 \cdot \sigma_0 \cdot \left(1 - \frac{\rho^2}{\rho_0^2}\right) \qquad (25)$$

$$\sigma_v = 2 \cdot \sigma_0 \cdot \left(1 - \frac{\rho^2}{\rho_0^2}\right) \qquad (26)$$

where: σ_0 – mean stress applied in the contact area; ρ – distance from the center of contact area to an considered point; ρ_0 – radius of circular contact area.

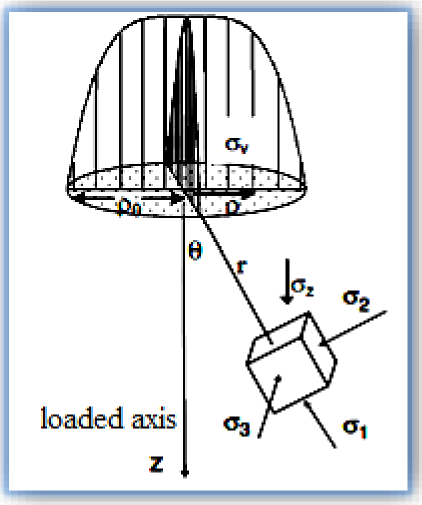

Figure 7. Stress on an elementary soil volume (Söhne) [9]

***Johnson and Burt's model for the estimation of stress in the footprint** (1990) [2, 11]*

Radial normal stress (maximum main stress) in any point of an elastic environment, due to a load applied in a point concentrated on a surface is given by Fröhlich's equation:

$$\sigma_r = \frac{v \cdot P \cdot \cos^{v-2}\phi}{2 \cdot \pi \cdot R^2} \qquad (27)$$

where: v – Fröhlich's concentration factor ($v \geq 3$); P – normal loaded point; R – radial distance between the concentrated load and the considered point; Φ – the angle between the normal load vector and the position vector from the concentrated load to the considered point (figure no 8).

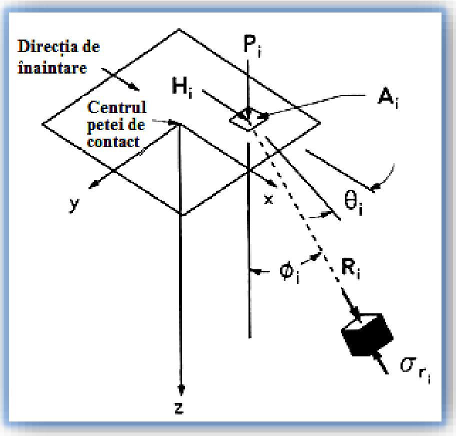

Figure 8. Geometrical relations for stress in a point of soil [11]

The concentration factor (v) can be: 3 – for very hard soil, the ideal case, 4 – for hard soil, 5 – for stable soil, 6 – for soft soil [19].

The equation proposed by Cerruti estimates the normal stress in a radial point of a semi-elastic medium, due to a single shear stress on the surface, taking into account the increase of the modulus of elasticity with increasing depth:

$$\sigma_r = \frac{v \cdot H \cdot \sin^{v-2}\phi \cdot \cos\theta}{2 \cdot \pi \cdot R^2} \qquad (28)$$

where: H_i – loaded shear point; θ – the angle between the vector of shear stress and the vertical plane in which is found the position vector from the shear stress to the considered point.

Johnson and Burt (1990) estimated the normal radial stress in an elastic half-space, due to a single concentrated shear load, using Fröhlich's equation. The stress for each concentrated stress applied on soil surface is given by the equation:

$$\sigma_{r_i} = \frac{\nu \cdot (P_i \cdot \cos^{\nu-2}\Phi_i + H_i \cdot \sin^{\nu-2}\Phi_i \cdot \cos\theta_i)}{2 \cdot \pi \cdot R_i^2} \qquad (29)$$

Keller's model for stress distribution in the soil *(2004) [12]*

The distribution of stress σ_1 under the action of a concentred vertical load P on a semi-infinite, homogeneous, isotropic and ideal elastic medium is given by:

$$\sigma_1 = \frac{3 \cdot P}{2 \cdot \pi \cdot r^2} \cdot \cos^3\theta \qquad (30)$$

where: r – radial distance from the application point of load to a certain point; θ – angle between the normal direction of load vector and the direction of the position vector of the desired point towards the load application point (figure no 9).

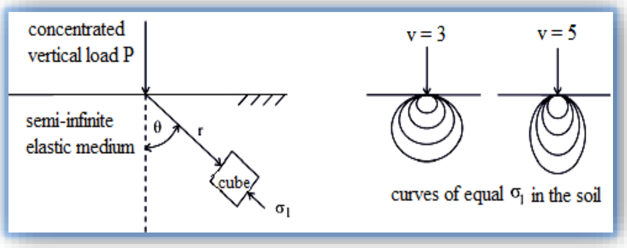

Figure 9. Soil stresses at the application of a vertical load (left) and soil behavior depending on agricultural conditions (right) [12, 16]

Model for stress distribution under the agricultural tire *[12]*

It is considered that the shape of stress distribution in the forward direction and perpendicular to this direction is variable, the parameters used to generate the contact area and stress distribution being directly calculated based on the available parameters of the tire. For the law of stress variation in a direction perpendicular to the advance direction of the vehicle it can be used a Decay function, [12]:

$$\sigma(y) = C \cdot \left(\frac{w(x)}{2} - y\right) \cdot e^{-\delta\left(\frac{w(x)}{2} - y\right)}, \quad 0 \le y \le \frac{w(x)}{2} \qquad (31)$$

where: C and δ – empirical parameters depending on the tire; w(x) – contact width.

The value of δ increases from 1,4 to 9 if tire width decreases from 1,15 m to 0,28 m. Depending on the value of δ, the maximum position of stress moves from the center of tire to its outline (from parabollic distribution to U-shapedd distribution) [9].

Stress distribution on the forwarding direction is given by:

$$\sigma(x) = \sigma_{x=0,y} \cdot \left[1 - \frac{\left(\frac{x}{l(y)}\right)^{\alpha}}{2}\right], \quad 0 \le x \le \frac{l(y)}{2} \qquad (32)$$

where: $\sigma_{x=0,y}$ – stress under the center of the wheel; l(y) – contact lenght; α – parameter.

Parameters in equations 31 and 32 are calculated knowing the values of wheel load, tire inflation pressure, the reccomended tire inflation pressure at a certain wheel load, tire width and the overall diameter of the unloaded tire. This model offers significantly useful input data for soil compaction models (figure no 10), increasing the accuracy of stress estimation and the precision of estimation of soil compaction due to the traffic of agricultural vehicles.

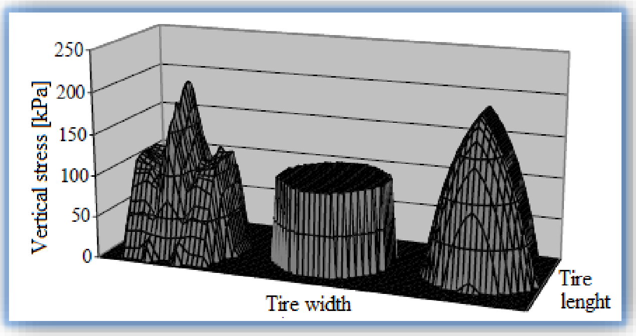

Figure 10. Measured stress (left), uniform stress distribution (center) and stress distribution obtained by modeling (right), under 1050/50 R32 tire, for tire inflation pressure of 100 kPa and wheel load of 86 kN on a wet clay soil [4, 12]

Karafiath and Nowatsky's model for the calculus of contact pressure between tire and rigid surface *(1978) [22]:*

$$p = c_i \cdot p_i + p_c \qquad (33)$$

where: p – tire contact pressure (kPa); c_i – constant of tire stifness (for high pressures $c_i = 0{,}6$ and for low pressures $c_i = 1$); p_i – tire inflation pressure (kPa); p_c – contact pressure of the non-inflated tire, when $p_i = 0$ kPa.

Koolen's model for the calculus of pressure in the footprint *(1992) [22]:*

Koolen studied the stresses in the soil, produced by wheels of agricultural vehicles, and found that stress in the contact area (contact pressure) is twice larger than tire inflation pressure:

$$p = 2 \cdot p_i \qquad (34)$$

Helenelund's model for stress distribution in the soil *(1974) [23]:*

Stress distribution under a circular plate (figure no 11) is given by the following equation:

$$\sigma_z = p \cdot (1 - \cos^{\alpha}\beta) \qquad (35)$$

where: σ_z – vertical stress in the soil at depth z (kPa); z – soil depth (m); a – concentration factor; β – angle depending on plate's radius at depth z.

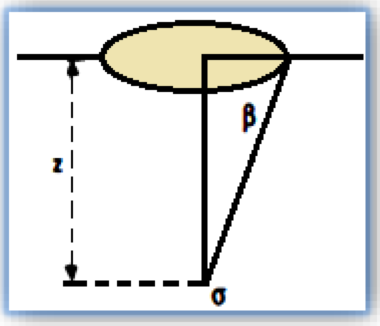

Figure 11. Vertical stress under a circular plate [23]

CONCLUSIONS

The soil doesn't have a heterogeneous structure, for which reason the modeling of artificial compaction is not easy to determine, in this regard being developed numerous models that considered various factors influencing this process:

- *soil bulk density and three empirical parameters (Bailey's model);*
- *soil bulk density, three empirical parameters and two connection parameters (Assouline's model);*
- *change of soil volume (Gupta and Larson's model);*
- *state of cylindrical stress in the soil (Bailey and Johnson's model);*
- *estimation of soil density after tire passage depending on contact pressure (Schwanghart's model);*
- *calculus of soil compaction state;*
- *estimation of soil sinkage after multiple passes of a tractor (Abebe's model);*
- *estimation of soil bulk density on the longitudinal central axis of the wheel, from the ruts formed on the soil (COMPSOIL and O'Sullivan models);*
- *determination of the contact area between agricultural tires and soil (Komandi's empirical model);*
- *estimation of footprint (FRIDA model);*
- *determination of footprint area (Grecenko's empirical models);*
- *estimation of stress distribution at soil-tire interface (Söhne's model);*
- *estimation of stresses in the footprint (Johnson and Burt's model);*
- *distribution of stress in the soil (Keller's model);*
- *estimation of stress distribution under the agricultural tire (Keller's model);*
- *calculus of contact pressure between tire and rigid surface (Karafiath and Nowatsky's model);*
- *calculus of pressure in the footprint (Koolen's model);*
- *estimation of stress distribution in the soil (Helenelund's model).*

Each of these models verify experimental data for the considered parameters, but each individual model cannot be extrapolated for any type of soil artificially compacted, because soil composition, the parameters influencing the compaction process and the atmospheric conditions vary from one plot to another.

REFERENCES

[1.] Assouline S. (2002). Modeling soil compaction under uniaxial compression. Soil Sci. Soc. Am. J. 66: 1784 – 1787;

[2.] Bailey A. C., Raper R. L., Johnson C. E., Burt E. C. (1995). An integrated approach to soil compaction prediction. J. Agric. Engng. Res. 61: 73-80;

[3.] Biriş S. Şt. (2005). Metoda elementelor finite. Concepte fundamentale, Editura Printech, Bucureşti;

[4.] Biriş S.Şt., Maican E., Bayer M., Vlăduţ V., Bungescu S., Prunău M. (2007). Advanced modelling of stress distribution in agricultural soil under different traction devices. Research People and Actual Tasks on Multidisciplinary Sciences. Lozenec, Bulgaria;

[5.] Biriş S. Şt. (2010). Modelarea matematică a compactării solului agricol, Editura Printech, Bucureşti;

[6.] Biriş S. Şt., Ungureanu N., Maican E., Paraschiv G., Voicu Gh., Manea M. (2011). FEM model for the study of interaction between the driving wheel and the rolling track for agricultural vehicles. 39th International Symposium "Actual Tasks on Agricultural Engineering", Croaţia, Opatija, pp. 95-105;

[7.] Biriş S.Şt., Vlăduţ V., Ungureanu N., Matache M., Voicea I. (2012). Researches on the development of an equation for the contact area calculus for agricultural tires. Proceedings of the 40 International Symposium "Actual Tasks on Agricultural Engineering", pag. 181-194, ISSN 1333-2651. Opatija, Croaţia;

[8.] Cui K., Defossez P., Richard G. (2007). A new approach for modeling vertical stress distribution at the soil/tyre interface to predict the compaction of cultivated soils by using the PLAXIS code. Soil & Tillage Research 95: 277-287;

[9.] Cui K. (2008). Vers une approche hydro-mecanique du comportement des sols cultives: Experimentations et simulations. Agro Paris Tech;

[10.] Iler G. S., Stevenson C. K. (1991). The effects of soil compaction on the production of processing vegetables and field crops. A review. Ridgetown College of Agricultural Technology, Ontario;

[11.] Johnson C. E., Burt E. C. (1990). A method of predicting soil stress state under tires. American Society of Agricultural Engineers. Vol. 33(3): 713-717;

[12.] Keller T. (2004). Soil compaction and soil tillage – studies in agricultural soil mechanics. Doctoral thesis, Agraria 489. Swedish University of Agricultural Sciences, Uppsala, Sweden;

[13.] Keller T., Lamandé M. (2010). Challenges in the development of analytical soil compaction models. Soil & Tillage Research, 111, pp. 54-64;

[14.] Keller T., Lamandé M., Peth S., Berli M., Delenne J.-Y., Baumgarten W., Rabbel W., Radja F., Rajchenbach J., Selvadurai A.P.S., Or D. (2013). An interdisciplinary approach towards improved understanding of soil deformation during compaction. Soil & Tillage Research, 128, pp. 61-80;

[15.] Komandi, G. (1976). The determination of the deflection, contact area, dimensions, and load carrying capacity for driven

pneumatic tires operating on concrete pavement. Journal of Terramechanics. Vol. 13(1), pg. 14-20;

[16.] Koolen A.J., Kuipers H. (1983). Agricultural Soil Mechanics. Advanced Series in Agricultural Sciences, Vol. 13. Springer, Heidelberg, pp. 241;

[17.] Lefebrve M. P. (2010). Spatialisation de modèles de fonctionnement hydromécanique des sols appliquée à la prévision des risques de tassement à l'échelle de la France, Université d'Orléans;

[18.] Lozano N., Rolim M. M., Oliveira V.S., Tavares U. E., Pedrosa E.M.R. (2013). Evaluation of soil compaction by modelling field vehicle traffic with SoilFlex during sugarcane harvest. Soil & Tillage Research 129, pp. 61-68;

[19.] Model cu distribuţie spaţială pentru prognoza stării agrofizice a solului, sub influenţa lucrărilor agricole – MOSTA, Raport 2. Institutul Naţional de Cercetare – Dezvoltare pentru Pedologie, Agrochimie şi Protecţia Mediului–I.C.P.A, Bucureşti;

[20.] Saarilahti M. (2002). Soil interaction model. Evaluation of the WES-method in assessing the trafficability of terrain and the mobility of forest tractors. University of Helsinki, Department of Forest Resource Management;

[21.] Saarilahti M. (2002). Soil interaction model. Modelling of the wheel and soil. University of Helsinki, Department of Forest Resource Management;

[22.] Saarilahti M. (2002). Soil interaction model. Modelling of the wheel and tyre. Tire and soil contact. University of Helsinki, Department of Forest Resource Management;

[23.] Saarilahti M. (2002). Soil interaction model. Modelling of the wheel and tire. Tire/soil models for predicting rut formations and soil compaction. University of Helsinki, Department of Forest Resource Management;

[24.] Saffih-Hdadi K., Défossez P., Richard G., Cui Y.-J., Tang A.-M., Chaplain V. (2009). A method to predict the soil susceptibility to compaction of surface layers as a function of water content and bulk density. Soil Tillage Research 105, pp. 96-103;

[25.] Schjønning P., Lamandé M., Tøgersen F. A., Arvidsson J., Keller T. (2008). Modelling effects of tyre inflation pressure on the stress distribution near the soil-tyre interface. Biosystems Engineering 99: 119-133, Elsevier.

ACTA Technica CORVINIENSIS
BULLETIN OF ENGINEERING
ISSN:2067-3809