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FINITE ELEMENT MODELLING OF THE SPOKE WHEEL TRUING

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Abstract: The present paper is concerned with the mechanical and mathematical modelling of the spoke wheel lateral truing. A finite element model of the wheel which consists of truss and beam elements is created in ADINA software. The warp in the wheel is embedded in the model geometry. This geometric shape imperfection is straightened by altering the spoke tension. A mathematical software is used to find the optimal values of the spoke tension. Results are presented for various truing strategies.
Keywords: spoke wheel, truing, finite element analysis, optimization

INTRODUCTION

Spoke wheels are widely used on bicycles, motorcycles and wheelchairs because of their lightweight and stiff structure. A common failure type of these is the warping of the rim due to excessive load. This causes the wheel wobbling side to side, i.e. in axial direction. This should be avoided, because it leads to additional stresses and power losses in motion. If the warp is of slight degree than it can be repaired by truing the wheels. This process consists of tightening the spoke nipples which alters the spoke tension. Since the spokes are directed in axial and radial directions, the "rim run out" can be straightened.

As regards the mechanical analysis of spoke wheels the first studies appeared in the first half of the last century. Wheels with spokes subjected to compressive load were investigated by Pippard and Baker [10] who calculated the maximal stress. In the articles by Coker [3] and respectively Reynolds and Ehasz [14] have analysed the stress distribution with the use of photoelasticity. Pippard and Duncan have averaged the spokes and smeared it into an equivalent disc to analyse the load transmission between the rim and hub [11].

Pippard and Francis have first dealt with the modelling of tensioned spoke wheels [12] and in a later article Pippard and White compared it with experimental results [13]. Decades later Burgoyne and Dilmaghanian have compared experimental results with results carried out from a model, where a disc was substituted for the spokes [2]. Minguez and Vogwell developed an analytical model in which the rim and the spokes were considered and the effect of pretension was taken into account [7]. In the study by Gavin the stiffness of the wheel in terms of the arrangement of the spokes was determined by analytical, numerical and experimental results [4]. Petrone and Giubilato report on the measurement results about the behaviour of the spoke wheel in radial direction in their article [9].

With fast computers and effective numerical methods investigation of complex problems has become available. Finite element computations are presented by Brandt [1]. In addition to the numerical results, the cited book describes the structure and geometric properties of the spoke wheel and gives instructions how to repair them. Salamon and Oldham investigated the effect of the spoke arrangement using finite element method [15], while Hartz reports on numerical and experimental results [5]. In the study [8] the effect of the number of spokes on the radial stiffness was analysed with the finite element method.

The cited works model the wheel as a two-dimensional structure, therefore they are not able to describe the axial displacements and the response behaviour of the wheel to lateral load. Consequently none of them deal with the modelling of wheel truing.

The present paper deals with the finite element modelling of truing a damaged spoke wheel. The paper outlines the basic assumptions, the finite element model and the mathematical model of wheel truing. Numerical results are also shown, which can be used as a basis for repair instructions.

MECHANICAL MODEL OF THE WHEEL

The object of our examinations is a spoke wheel consisting of a rim, 36 spokes and a hub. The other parts of the wheel are not taken into account. We consider the hub as a stationary and rigid part, consequently it will act as the ground in our investigations.

It is assumed that the displacements are small and the material law is linear, consequently we can use linear theory of elasticity.

The spokes are treated as beams under tension. At the ends of the spokes they connect to the rim and the hub with joint which allow small rotation in every direction. Thus they can be modelled with truss elements. The spokes have a circular cross section. There are many standard values for their diameter. In our investigation we use the common d=2 mm value which corresponds to the cross section area A=3.14 mm². The lay-out of the spoke arrangement follows a 3-cross pattern.

The rim is considered as a Bernoulli beam, with a cross section shown is Figure 1. The relevant second moments of area are calculated by the CAD tool in which the figure was drawn. Their values are I_x=6422.1 mm⁴ and I_y=11217.8 mm⁴ respectively.

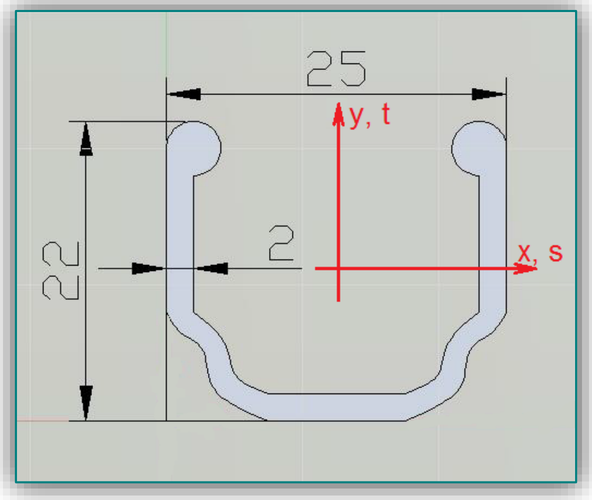

Figure 1. Cross section of the rim

The Saint-Venant torsional constant I_T is calculated by the formula valid for thin-walled open cross section [6]:

$$I_T = \frac{1}{3}\int_S t^3 ds \qquad (1)$$

where the S curve denotes the centreline of the section and t the thickness. The calculated value for the rim is I_T=144 mm⁴. The section area of the rim is A=122.7 mm².

The rim and spokes is made of steel, which is regarded as homogeneous isotropic material with E=210 Gpa modulus of elasticity and ν=0.3 Poisson's ratio.

The finite element analysis is made with ADINA software. The model is built of truss and beam elements. The pretension of the spokes was considered as initial strains with the value ε_0=0.00076 which correspond to an initial stress of σ=159.6 Mpa. Using only the initial strains as load, the mesh on the rim was refined until the obtained results converged to a certain value. The finite element model of the rim is shown in Figure 2. It consists of 649 nodes.

The lateral warp of the rim is of 1 mm. This damage is considered as an initial geometric property. Consequently no residual stresses are taken into account. The form of the rim is originally a circle. This circle is drawn in the *xz* plane. In the deformed rim some points have a lateral displacement, i.e. they are taken off the plane in *y*-direction In the present model 1 mm warp is placed at one of the spoke connections to the rim, and 0.5 mm warp is at the neighbouring spoke connections. This is demonstrated in Figure 3 where the belonging node numbers are also marked.

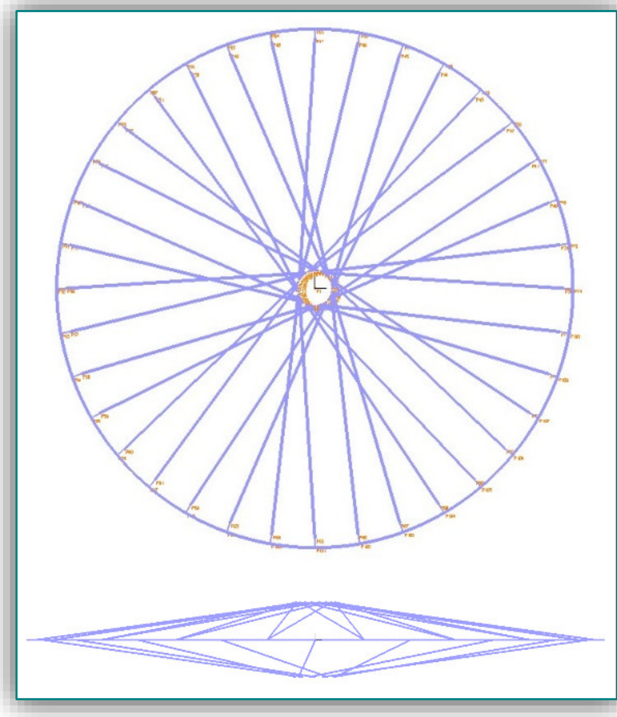

Figure 2. FEM Model

Together with the neighbouring spoke-rim connections, which have initially zero *y*-coordinates (they are not deformed), these points constitute the domain of the rim, which is regarded as the damaged domain of the wheel. In the following this domain will be examined. The node numbers of these points together with their initial *y*-coordinate is listed in Table 1. The placement of these nodes are shown in Figure 4.

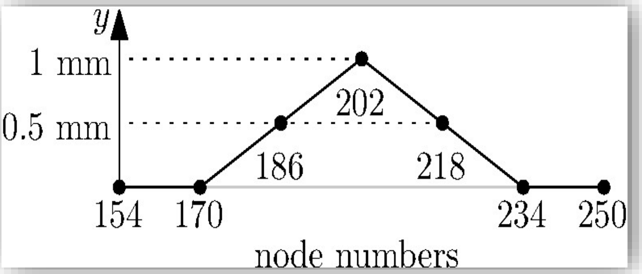

Figure 3. Initial y positions

Table 1. Initial *y*-position of the examined points

Node 170	0.00000E+00
Node 186	5.00000E-01
Node 202	1.00000E+00
Node 218	5.00000E-01
Node 234	0.00000E+00

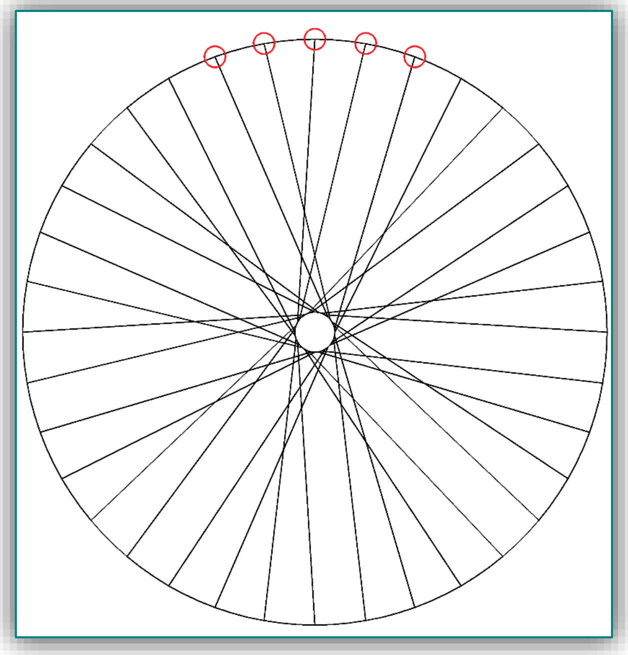

Figure 4. Placement of the examined nodes

MATHEMATICAL MODEL OF TRUING

The main goal of truing is straightening the described lateral warp. The original ideal form cannot be achieved, but the failure can be reduced in a satisfying measure which is within a given margin of error. The best result is achieved, if the warp of the rim is minimal. Therefore we have to solve an optimisation problem.

In order to solve the problem we define an objective function which we have to minimize. The input values are the initial strains of the spokes, consequently we have 36 input values. The function calls ADINA which does the finite element calculation with the given initial strains. Then the *y*-positions of the examined points are taken from the calculated results. The square sum of these *y*-positions is a good index of the magnitude of the warp. Consequently this number is taken as the value of the function that has to be minimized.

As a result of the truing process tightness of the spokes alter. Extremely loose and tight spokes should be avoided. Therefore we prescribe the inequality constrains

$$0.00038 \le \varepsilon_{0,i} \le 0.00114 \quad i = 1,\dots 36 \qquad (2)$$

which mean that the initial strains should not exceed the original value by ±50%. The inequality constrains are handled by a penalty function.

The minimization is done by a computer program written in the GNU Octave mathematical software. We use a built-in function which uses the Nelder & Mead Simplex algorithm for minimization.

RESULTS

If the initial strains of all spokes are taken into account for minimization, i.e. every value is altered, then a minimum is found where the *y*-positions of the nodes in Table 1 become the values listed in Table 2.

Table 2. Results for all spokes altered

Node 170	-3.24500E-01
Node 186	5.05318E-02
Node 202	5.03589E-01
Node 218	4.33440E-02
Node 234	-3.35638E-01

The warp reduces overall, but the rim deforms largely at other locations far from the examined points. This circumstance is demonstrated in Figure 5 with scaled displacements. This is an undesired behaviour. In order to solve this problem, we take more points into account, when we evaluate square sum of the *y*-positions. In addition we select three more points on the rim whose placement is shown in Figure 6.

The results show that the magnitude of the warp is of the same order as in the previous calculations, but the rim is not affected by large deformations at other locations of the rim. Therefore this solution is a better as the previous one. The *y*-positions are listed in Table 3.

Table 3. Results with additional examined points

Node 170	-3.46355E-01
Node 186	6.74146E-02
Node 202	5.37462E-01
Node 218	7.04624E-02
Node 234	-3.41745E-01

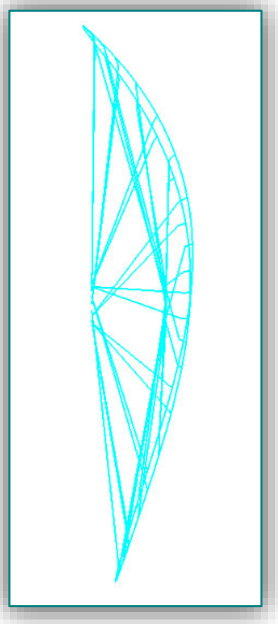

Figure 5. Deformed wheel

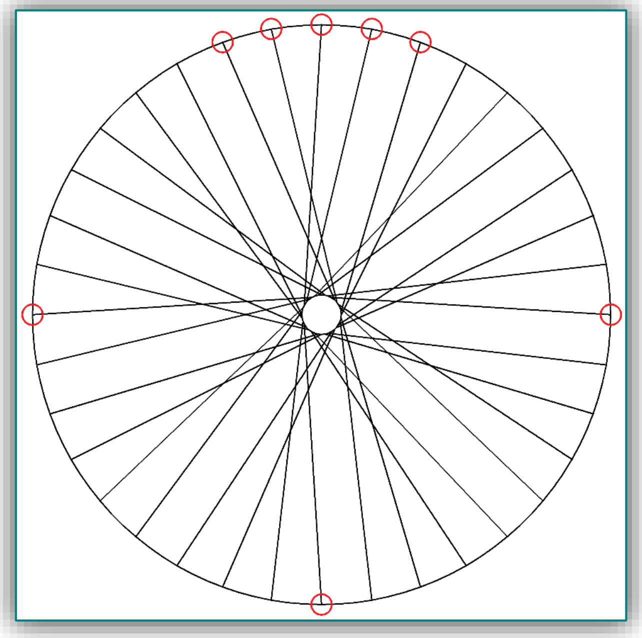

Figure 6. Additional examined points

Another approach comes from practical experience. Professional wheel builder only alter the spokes near the damaged domain in the truing process. Reducing the number of spokes whose initial strain is altered corresponds with this. Since the number of input variables reduces, this also causes less computation time.

The spokes with altered initial strains are marked in the Figures 7 and 8. The number of the altered spokes are 9 and 7 respectively. The results of the computations are listed in Table 4 and 5. According to the results it is enough to alter the initial strain of seven spokes in the damaged area.

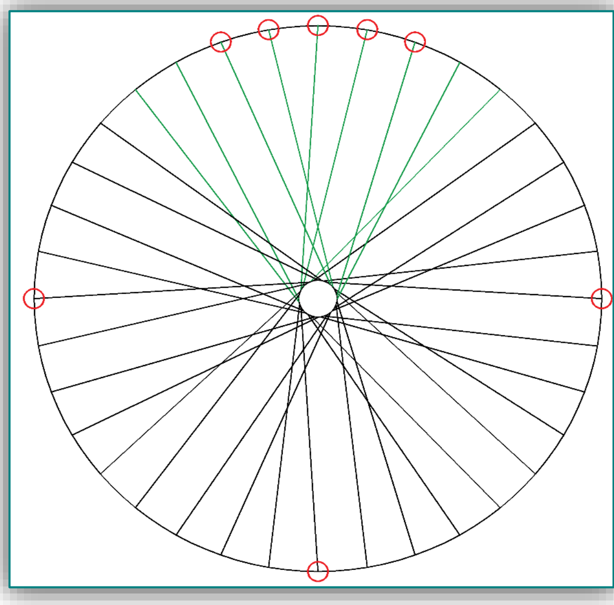

Figure 7. Nine altered spokes

Table 4. Results for 9 altered spokes

Node 170	-3.12930E-01
Node 186	1.16757E-01
Node 202	5.71028E-01
Node 218	5.82344E-02
Node 234	-4.19403E-01

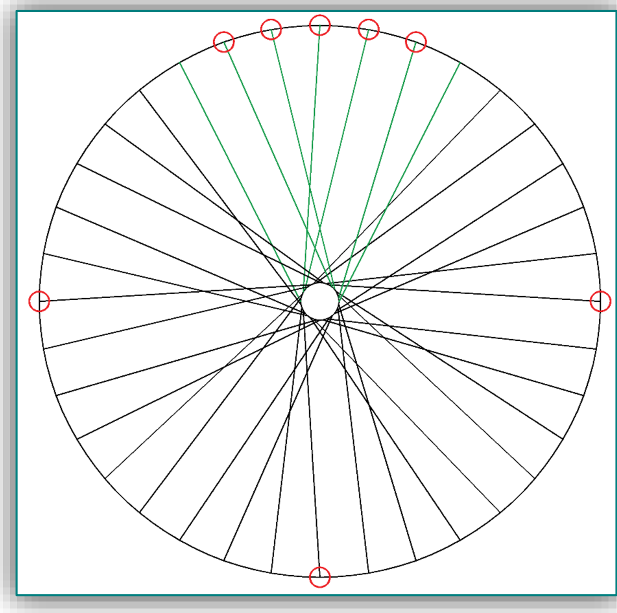

Figure 8. Seven altered spokes

Table 5. Results for 7 altered spokes

Node 170	-3.73946E-01
Node 186	7.39701E-02
Node 202	5.49029E-01
Node 218	5.53532E-02
Node 234	-4.07693E-01

Since the initial strains are only altered in a smaller range, it is not needed to consider the additional points when the square sum of the y-position is evaluated. Figure 9 presents the case if seven spoke strains are altered and only the five points in the damaged area are taken into account. The results for the y-positions of the selected points are presented in Table 6.

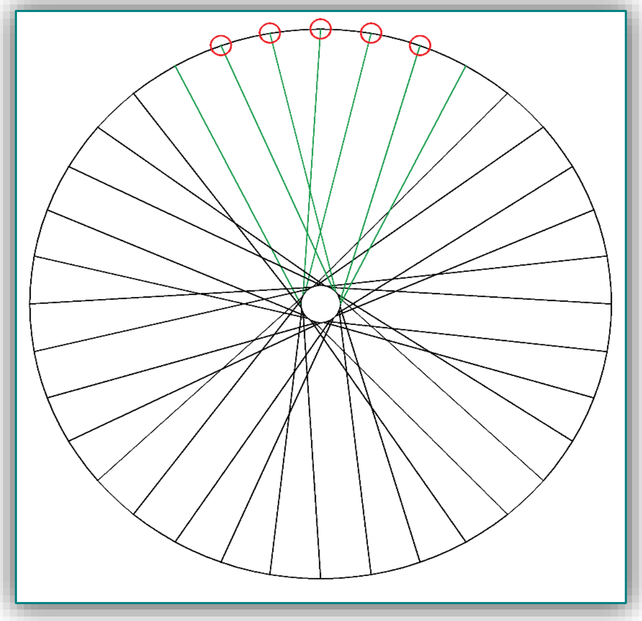

Figure 9. Seven altered spokes without additional examined points

Table 6. Results for 7 altered spokes without additional examined points

Node 170	-3.74249E-01
Node 186	7.17306E-02
Node 202	5.45161E-01
Node 218	5.05212E-02
Node 234	-4.12753E-01

Since the difference of the results between these and the previous ones is minimal this solution is considered satisfactory and it correspond to practical experience. The computational costs are also smaller in this case, therefore this case is considered as the preferred choice. Regarding to the results the warp is approximately reduced to half.

The values of initial strains where the minimum lies are also computed. If we have knowledge about these values together with the spoke length and pitch of the spoke nipple, repair instructions can be given about the necessary turns on the nipples.

CONCLUSION

The present paper has presented the finite element model of a spoke wheel which consists of a rim, 36 spokes and a rigid hub. A program was implemented in order to model the truing of a damaged wheel. According to the results altering the tension in the range around the failure leads to satisfactory melding, which corresponds to practical experience. The results for the optimal initial strains of the spokes is a good basis for repair instructions of the wheel.

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