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CALCULATION OF DIELECTRIC PARAMETERS BASING ON MEASURED ELECTRICAL PARAMETERS

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Abstract: Determination of dielectric properties can provide the electrical or magnetic characteristics of the materials, which useful in many research and development fields improving design, processing, quality and control of product. Many different measurement methods have been developed, but most of them are limited in used frequency and properties of the examined material. At University of Szeged measuring equipment was developed. Measuring method we used is based on reflection method. There are four diodes detecting electrical signs. From these electrical parameters dielectric properties can be determined.

Keywords: dielectric parameters, reflection method, standing wave ratio

DIELECTRIC PARAMETERS

The material making up atoms, molecules are generally electrically neutral, uncharged (except eg. ionic crystals) but their position is not necessarily symmetrical. The centre of gravity of positive and negative charges does not coincide in several molecule, these are polar molecules having permanent electric dipole moment. Without an electric field these moments are placed completely irregularly extinguishing the effect of each other. Symmetrical molecules are non-polar molecules their electric dipole moment is zero. Asymmetric charge distribution is formed, electric dipole moment is induced in the initially symmetrical, non-polar molecules and direction of dipole moment of initially polar molecules is changed by electric field. Dipole moment induced in non-polar molecules can be occurred by two reasons, on the one hand the electron cloud moves relative to the nucleus (electron polarization), on the other hand, the nucleus moves in relation to one another (atomic polarization). In both cases induced dipole moment of the molecule is proportional to the actual local electric field strength. In case of polar molecules a new effect orientation polarization is added to atomic and electron. For studying the dynamics of polarization put the material in harmonically time varying electric field. Using low frequency polarization emerging in the material is in phase with the electric field but in case of quite high frequency polarization isn't able to follow the

changes of electric field, behinds in phase and the value of it is reduced as well. Formally it can be described that permittivity is a complex number. [1]

$$\varepsilon = \varepsilon' - \varepsilon'' \cdot j \quad (1)$$

ε' is the real, ε'' is the imaginary part of the complex permittivity. The real part of permittivity is a measure of how much energy from an external electric field is stored in a material. The imaginary part corresponds to the polarization lagging 90° to the electric field. Changing over time it causes displacement current which is in phase with electric field. It means that the material takes up energy from the electric space. Because of it ε'' is called dissipation or loss factor. It is a measure of how dissipative or lossy a material is to an external electric field. The imaginary part of permittivity is always greater than zero and is usually much smaller than the real part. The loss factor includes the effects of both dielectric loss and conductivity.

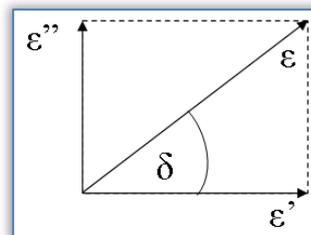


Figure 1. Loss tangent vector diagram
Drawing complex permittivity in a simple vector diagram (Figure 1) the real and imaginary part are 90° out of phase.

The resultant vector forms an angle δ with the real axis (ϵ'). The relative loss of the material can be defined as the ratio of the energy lost to the energy stored. It is called tangent loss.

$$\operatorname{tg}\delta = \frac{\epsilon''}{\epsilon'} \quad (2)$$

TRANSMISSION LINES

Transmission lines are specially designed electrically conducting wires transmitting high-frequency alternating current with low power loss. On a transmission line the voltage and current vary along the structure in time and distance as indicated in Figure 2.

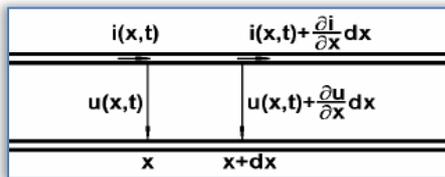


Figure 2. A very short piece of a transmission line. The following distributed parameters characterize the circuit properties of a transmission line.

R = resistance per unit length, (Ω/m) (due to losses in conductors)

L = inductance per unit length, (H/m) (due to current in conductors, and magnetic flux linking current path)

G = conductance per unit length, (S/m) (due to losses in dielectric, between conductors)

C = capacitance per unit length, (F/m) (due to time varying electric field, between the two conductors)

x = increment of length, (m)

Developing the equations for $I(x,t)$ and $U(x,t)$ we need to represent transmission line as a series connection of many small cells containing series inductors and resistors, and parallel capacitors and resistors, as in Figure 3.

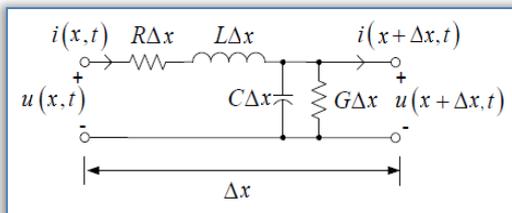


Figure 3. Circuit-theory approximation of a transmission line with losses

Using Kirchoff's laws for this section of transmission line:

$$u(x,t) - R \cdot \Delta x \cdot i(x,t) - L \cdot \Delta x \frac{\partial i(x,t)}{\partial t} - u(x+\Delta x,t) = 0 \quad (3)$$

$$i(x,t) - G \cdot \Delta x \cdot u(x+\Delta x,t) - C \cdot \Delta x \frac{\partial u(x+\Delta x,t)}{\partial t} - i(x+\Delta x,t) = 0 \quad (4)$$

If $\Delta x \rightarrow 0$, these lead to following equations:

$$-\frac{\partial u(x,t)}{\partial x} = R i(x,t) + L \frac{\partial i(x,t)}{\partial t} \quad (5)$$

$$-\frac{\partial i(x,t)}{\partial x} = G u(x,t) + C \frac{\partial u(x,t)}{\partial t} \quad (6)$$

In case of sinusoidal varying voltages and currents:

$$-\frac{dU(x)}{dx} = R \cdot I(x) + j \cdot \omega \cdot L \cdot I(x) = (R + j \cdot \omega \cdot L) \cdot I(x) \quad (7)$$

$$-\frac{dI(x)}{dx} = G \cdot U(x) + j \cdot \omega \cdot C \cdot U(x) = (G + j \cdot \omega \cdot C) \cdot U(x) \quad (8)$$

After decoupling we get the transmission line equations:

$$\frac{dU^2(x)}{dx^2} = \gamma^2 \cdot U(z) \quad (9)$$

$$\frac{dI^2(x)}{dx^2} = \gamma^2 \cdot I(z) \quad (10)$$

where γ is the complex propagation constant given by

$$\gamma = \alpha + \beta \cdot j = \sqrt{(R + j \cdot \omega \cdot L)(G + j \cdot \omega \cdot C)} \quad (11)$$

Its real part α is the attenuation constant (Np/m) and its imaginary part β is the phase constant (rad/m). Generally, these quantities are functions of ω .

When a transmission line is terminated by an impedance that does not match the characteristic impedance of the transmission line, part of the incident signal is reflected back down the transmission line. The forward signal mixes with the reverse signal to cause a voltage standing wave pattern on the transmission line. The ratio of the maximum to minimum voltage is known as Voltage Standing Wave Ratio (VSWR). [2]

$$r = \frac{U_{\max}}{U_{\min}} \quad (12)$$

If the signals from the diode detectors are quadratic, as is typical, the standing wave ratio will be:

$$r = \sqrt{\frac{U_{\max}}{U_{\min}}} \quad (13)$$

The solutions to transmission lines equations are superposition of forward and reverse waves:

$$U(x) = A \cdot e^{-\gamma x} + B \cdot e^{\gamma x} \quad (14)$$

$$U_f = A \cdot e^{-\gamma x} \text{ (forwarded travelling wave)} \quad (15)$$

$$U_r = B \cdot e^{\gamma x} \text{ (reflected travelling wave)} \quad (16)$$

A and B are complex vectors. The ratio of the reflected and the forwarded travelling wave is the voltage reflection coefficient.

$$\Gamma = \frac{U_r}{U_f} = \frac{B \cdot e^{-\gamma l}}{A \cdot e^{\gamma l}} = \frac{B}{A} e^{-2\gamma l} = \Gamma_0 e^{-2\gamma l} = |\Gamma| e^{j\phi} \quad (17)$$

The maximum of the amplitude of the standing wave is formed in the transmission lines where the two waves are added and the minimum of the amplitude is where the two waves are deducted. [1]

$$r = \frac{U_{\max}}{U_{\min}} = \frac{|A| + |B|}{|A| - |B|} \quad (18)$$

The relationship between VSWR and reflection coefficient is the following:

$$|\Gamma| = \frac{r-1}{r+1} \quad (19)$$

MEASURING DEVICE

A water cooled magnetron is located at the left side of the measurement unit. The operating frequency of it is 2450MHz ± 30MHz. Output power from the generator (magnetron) enters into the transmission line as form outrivaling wave. The transmission line is a length of about 3λ_g (where λ_g is the wavelength in the waveguide) rectangular cross-section waveguide with diode detectors placed in specific locations. Sample holder is located in a sufficient distance from the detectors.

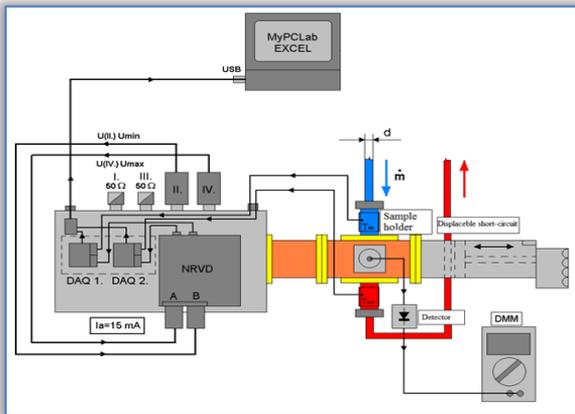


Figure 4. Measuring device

A specially formed reflecting surface can be found at the end of the waveguide from which the incident waves are reflected. The position of short-circuit can be changed. Electromagnetic waves pass through the sample, it modifies wave front emerged in the waveguide and absorbs electromagnetic energy. Reflected waves interfere with forwarded waves and so a special waveform is created in the waveguide by two important factors. One of them is the sample which changes the electromagnetic field; it is polarized and dissipates energy from electromagnetic field. The other is the position of the reflecting surface which affects the phase of the reflected wave. These diodes detect the resultant field strength of the forwarded and reflected waves. Electric signal of the dielectrometer, the inlet and outlet temperature of the material are received by the measurement data collector and recorded on-line by software and displayed in the computer screen.

EVALUATING METHODS

Method 1.

From Maxwell's equations that in the case of a plane wave complex amplitudes of vectors **E** and **H** are linked characteristic impedance of the medium:

$$Z_L = \frac{\omega\mu_a}{\gamma} = \sqrt{\frac{\mu_a}{\epsilon_a}} \quad (20)$$

so that the characteristic impedance for the non-magnetic environments ($\mu_a = \mu_0$)

$$Z_L = \frac{\dot{E}}{\dot{H}} = \frac{\sqrt{\epsilon\epsilon_0}}{\sqrt{1-jtg\delta_\epsilon}} = \frac{120\pi}{\sqrt{\epsilon}} e^{j\frac{\delta_\epsilon}{2}}, [\Omega] \quad (21)$$

Given that

$$\epsilon_a = \epsilon\epsilon_0(1-jtg\delta_\epsilon) \quad (22)$$

expression of

$$\dot{\Gamma} = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (23)$$

where

$$Z_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi = 376,99[\Omega] \quad (24)$$

and $\dot{\Gamma}$ -is a complex reflection coefficient, we obtain:

$$\dot{\Gamma} = \frac{1 - \sqrt{\epsilon}(1-jtg\delta_\epsilon)^{\frac{1}{2}}}{1 + \sqrt{\epsilon}(1-jtg\delta_\epsilon)^{\frac{1}{2}}} \quad (25)$$

from

$$\frac{1 - \dot{\Gamma}}{1 + \dot{\Gamma}} = \sqrt{\epsilon}(\sqrt{1+tg^2\delta_\epsilon})e^{-j\frac{\pi}{2}} \quad (26)$$

Substituting this expression in $\dot{\Gamma} = -|\Gamma|e^{-j\phi}$, equating phases and modules of both sides, we get:

$$\delta_\epsilon = 2 \left\{ \arctg \left[\frac{|\Gamma| \sin \phi}{1 - |\Gamma| \cos \phi} \right] - \arctg \left[\frac{|\Gamma| \sin \phi}{1 + |\Gamma| \cos \phi} \right] \right\} \quad (27)$$

$$\epsilon' = \frac{1}{\sqrt{1+tg^2\delta_\epsilon}} \left(\frac{1 + |\Gamma|^2 + 2|\Gamma| \cos \phi}{1 + |\Gamma|^2 - 2|\Gamma| \cos \phi} \right) \quad (28)$$

$$\epsilon'' = \epsilon' tg\delta_\epsilon \quad (29)$$

Method 2.

The transmission-line method (TLM) belongs to a large group of nonresonant methods of measuring complex dielectric permittivity of different materials in a microwave range [3,4]. Several modifications to this method exist, including the free-space technique [5], the open-circuit network method (see previous section), and the short-circuited network method.

Usually three main types of transmission lines are used as the measurement cell in TLM: rectangular waveguide, coaxial line, and microstrip line. Analyzed sample is placed near the short-circuited end of transmission line. The dielectric properties of the sample are determined using the following expressions:

$$\epsilon' = \left(\frac{\lambda}{2\pi d} \right)^2 (x^2 - y^2) + \left(\frac{\lambda}{\lambda_{qc}} \right)^2 \quad (30)$$

$$\epsilon'' = \left(\frac{\lambda}{2\pi d} \right)^2 2xy \quad (31)$$

where λ is the free-space wavelength, λ_{qc} is the quasicutoff wavelength, d is the sample thickness

$x=\text{Re}(Z_{in})$, and $y=\text{Im}(Z_{in})$, Γ , and Z_{in} is the input impedance of the short-circuited line:

$$Z_{in} = \frac{K_t^2 + \text{tg}^2\left(\frac{2\pi}{\lambda_g}l\right)}{K_t \left[1 + \text{tg}^2\left(\frac{2\pi}{\lambda_g}l\right) + j(1 - K_t^2)\text{tg}\left(\frac{2\pi}{\lambda_g}l\right) \right]} \quad (32)$$

where l is the distance between the dielectric surface and the first minimum of the standing wave, λ_g is the wavelength in unloaded part of transmission line, and K_t is the travelling-wave coefficient that is calculated when $K_t \geq 0.4$ as

$$K_t = \sqrt{\frac{E_{\min}}{E_{\max}}} \quad (33)$$

where E_{\min} and E_{\max} are the minimum and maximum values of electric field amplitude.

Method 3

Many aspects of wave propagation are dependent on the permittivity and permeability of a material. Let's use the "optical view" of dielectric behaviour. [6] Consider a flat slab of material (MUT) in space, there will be incident, reflected and transmitted waves. Consider a flat slab of material (MUT) in space, with a wave incident on its surface (Figure 5).

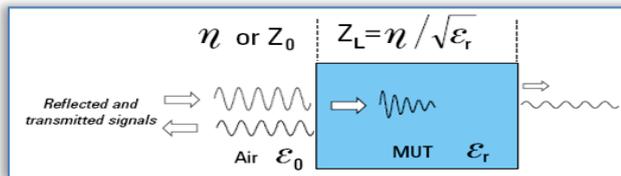


Figure 5. Reflected and transmitted signals

Since the impedance of the wave in the material Z is different (lower) from the free space impedance η (or Z_0) there will be impedance mismatch and this will create the reflected wave:

$$Z_L = \frac{\eta}{\sqrt{\epsilon_r}} \quad (34)$$

impedance lower, where

$$\eta = Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \quad (35)$$

Part of the energy will penetrate the sample. Once in the slab, the wave velocity v , is slower than the speed of light c :

$$v = \frac{c}{\sqrt{\epsilon_r}} \quad (36)$$

The wavelength λ_{MUT} is shorter than the wavelength λ_g in the waveguide according to the equations below:

$$\lambda_{MUT} = \frac{\lambda_g}{\sqrt{\epsilon_r}} \quad (37)$$

Since the material will always have some loss, there will be attenuation or insertion loss. From these, it follows that

$$\sqrt{\epsilon_r} = \frac{Z_0}{Z_L} \quad (38)$$

$$\epsilon_r = \left(\frac{Z_0}{Z_L}\right)^2 \quad (39)$$

and since

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (40)$$

this may transform the last expression as follows:

$$\epsilon_r = \left(\frac{Z_0}{Z_L}\right)^2 = \left(\frac{1+\Gamma}{1-\Gamma}\right)^2 \quad (41)$$

where $\Gamma = |\Gamma|e^{j\phi} = |\Gamma|(\cos \phi + j \sin \phi)$, and $|\Gamma| = \frac{r-1}{r+1}$

SUMMARY

Same electrical parameters are needed to measure in all three evaluation methods, the differences are in the way of evaluation. In case of Method1 from electrical parameters Voltage Standing Wave Ratio, phase shift, and voltage reflection coefficient are counted. Dielectric parameters can be determined on their basis. In case of Method2 travelling-wave coefficient is calculated first, then impedance of the short-circuited line and its real and imaginary part can be determined. According to these and the geometry of the waveguide dielectric parameters are determinable. In case of Method3 Voltage Standing Wave Ratio, phase shift, and voltage reflection coefficient are counted as in Method1, but on their basis complex permittivity can be determined. For comparing the three methods, experiments with materials having well-known dielectric parameters are needed to perform.

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