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FERROFLUID SQUEEZE FILM IN CURVED ROUGH POROUS CIRCULAR PLATES WITH SLIP VELOCITY: A COMPARISON OF MAGNETIC FLUID FLOW MODELS

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Abstract: This investigation presents a comparison of all the three magnetic fluid flow models (Neuringer–Rosensweig model, Shliomis model, Jenkins model) concerning the performance of a ferrofluid squeeze film in curved rough porous circular plates considering the slip velocity. The slip model of Beavers and Joseph's has been invoked to study the effect of slip velocity. The stochastic averaging model of Christensen and Tonder has been deployed to evaluate the effect of transverse surface roughness. The pressure distribution is obtained by solving the associated stochastically averaged Reynolds type equation. The expression for load carrying capacity is obtained thereafter. The graphical representations establish that Shliomis model remains more favourable for designing the bearing system. It is also appealing to note that for lower to moderate values of slip, Neuringer–Rosensweig model may be adopted. Besides, Jenkins model may be used when the roughness is at lower level and the slip is at minimum.
Keywords: Circular bearing, Ferrofluid, Roughness, Flow models, Load carrying capacity

INTRODUCTION

It is known that the additives are added to the based lubricants to enhance the bearing characteristics in general. Magnetic fluids are stable colloidal suspensions composed of single–domain magnetic nanoparticles dispersed in a viscous fluid. The main advantage of magnetic fluid as lubricant, over the conventional oil, is that the former can be retained at a desired location by an external magnetic field and still possesses flow ability at the same time. Due to their some important physical and chemical properties the ferrofluids have been attractive in different types of engineering and other fields applications, such as, vacuum sealing, magnetic resonance, imaging, intelligent sensors, buffer solution in chips, drug delivery, grinding, separation, ink–jet printing, damper and so on.

In the last decade, various theoretical models have been put forward to study the continuum description of ferrofluid flow. However, most of the systematic studies about the motion of magnetic fluids are based on the formulation given by Neuringer and Rosensweig [1], Shliomis [2] and Jenkins [3]. Neuringer and Rosensweig [1] proposed a quite simple model where the effect of magnetic body force was measured under the assumption of magnetization vector being parallel to the magnetic field vector. However, Shliomis [2] embarked on a different formulation. He observed that the magnetic particles in the fluid had Brownian motion and their rotation affected the motion of magnetic fluids. Thus, Shliomis developed the equation of motion for ferrofluids by considering internal angular momentum due to the self–rotation of particles. Jenkins [3] modified the idea of Neuringer–Rosensweig to develop a model to describe the flow of a ferrofluid by using Maugin's modification.

A good number of papers (Agrawal [4], Shah and Bhat [5], Shah and Bhat [6], Nada and Osman [7], Deheri and Abhangi [8], Patel el al. [9], Patel and Deheri [10] and Patel el al. [11]) exist in the literature dealing with the theory of different types of bearing using Neuringer and Rosensweig flow model. It was pointed out that Neuringer–Rosensweig model enunciated the pressure and enhanced the performance of bearing system.

Thereafter, many researchers (Kumar et al. [12], Singh and Gupta [13], Lin [14], Patel and Deheri [15]) dealt with the model of Shliomis to examine the performance of different bearing's characteristics. All the above investigations analyze the steady state characteristics of the bearings lubricated with magnetic fluids, resorting to the flow model estimated by Shliomis.

It was concluded that Neuringer–Rosensweig model modified the pressure while Jenkins flow model modified both the pressure and the velocity of the Magnetic fluid. The steady–state characteristics of bearings with Jenkins model based ferrofluids were discussed by Agrawal [4], Ram and Verma [16], Shah and Bhat [17], Ahmad and Singh [18] and Patel and Deheri [19]. It was manifest in all the studies that the load carrying capacity of the bearing system increased with increasing magnetization.

Nowadays, a significant amount of tribology research has been dedicated to the study of the effect of surface roughness or geometric imperfections on hydrodynamic lubrication because the bearings surfaces, in practice, are all rough and the height of the roughness asperities may have the same order as the mean bearing clearance. Under these circumstances, the surface roughness affects the bearing performance noticeably. The deep–rooted stochastic theory of hydrodynamic lubrication of rough surfaces developed by Christensen and Tonder [20–22] formed the basis of this paper. In a series of works (Ting [23], Praksh and Tiwari [24], Guha [25], Turaga et al. [26], Gururajan and Prakash [27], Gadelmawla et al. [28], Sinha and Adamu [29], Adamu and Sinha [30]). In fact, the model was applied to the study of the surface roughness for various geometrical configurations.

The combined effect of slip velocity and surface roughness on the performance of Jenkins model based magnetic squeeze film in curved rough annular plates was examined by Patel and Deheri [31].

It was concluded that the effect of transverse surface roughness remained adverse in general, Jenkins model based ferrofluid lubrication provided some measures in mitigating the adverse effect and this became more apparent when the slip parameter was at reduced level and negatively skewed roughness occurred. Jao et al. [32] studied a lubrication theory that included the coupled effects of surface roughness and anisotropic slips. It was found that the load ratio increased as the dimensionless slip length decreased (except the case of short bearing) or as the slenderness ratio increased. The effect of a magnetic fluid based parallel plate rough slider bearing with the comparison of all the three magnetic fluid flow models (Neuringer–Rosensweig model, Shliomis model, and Jenkins model)was investigated by Patel and Deheri [33].

However, comparison does not exist for the performance of the circular plates bearing system in all the three models. Thus, this paper investigates the combined effect of surface roughness and slip velocity on squeeze film characteristics of circular plates bearing by considering the comparison of three magnetic fluid flow models, namely, Neuringer–Rosensweig model, Shliomis model and Jenkins model.

ANALYSIS

Figure 1 presents the formation of the squeeze film circular bearing considering the laminar flow of an incompressible fluid and transversely rough bearing surfaces between two circular plates, each of radius$a$. The upper curved plate approaches the lower curved plate with normal uniform velocity $\dot{h_0}$, where h_0 is the central film thickness.

In view of the theory of Christensen and Tonder [20–22], the expression for the film thickness is assumed to be made up of two parts

$$h = \bar{h} + h_s \qquad (1)$$

where $\bar{h}$ denotes the mean film thickness and h_s represents the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces.$h_s$is taken to be stochastic in nature and governed by a probability density function

$$f(h_s) = \begin{cases} \dfrac{35}{32c}\left(1 - \dfrac{h_s^2}{c^2}\right)^3, & -c \le h_s \le c \\ 0 & , \text{elsewhere} \end{cases}$$

c being the maximum deviation from the mean film thickness. The details of the meanα, the standard deviation σ and the parameter ε, which is the measure of symmetry of the random variable h_s are explained in Christensen and Tonder [20–22].

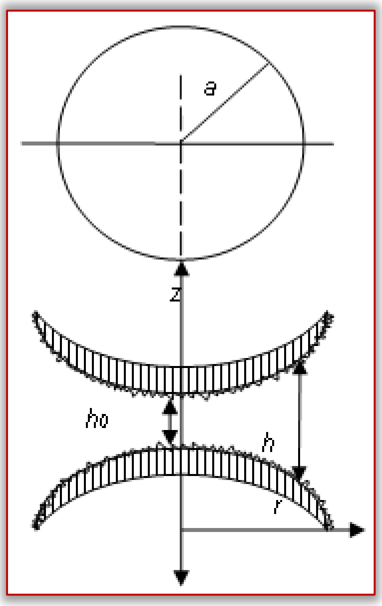

Figure 1: Circular bearing configuration

The deliberations of Bhat [34] and Patel and Deheri [35], discuss that the upper plate lying along the surface determined by the relation

$$z_u = h_0 \exp(-\beta r^2);\ 0 \le r \le a$$

goes towards the lower plate lying along the surface given by

$$z_l = h_0 \left[\frac{1}{1+\gamma r} - 1\right];\ 0 \le r \le a$$

with normal velocity $\dot{h_0}$. Where β and γ are the curvature parameters of the respective plates. The film thickness then, is given by (Bhat [34], Patel and Deheri [36])

$$h(r) = h_0 \left[\exp(-\beta r^2) - \frac{1}{1+\gamma r} + 1\right];\quad 0 \le r \le a \qquad (2)$$

A simple flow model proposed by Neuringer and Rosensweig [1] examined the steady flow of magnetic fluids in the presence of slowly changing external magnetic fields. The model was developed by the following expressions

$$\rho(\bar{q}\nabla)\bar{q} = -\nabla p + \eta\nabla^2\bar{q} + \mu_0(\bar{M}\nabla)\bar{H} \qquad (3)$$

$$\nabla\bar{q} = 0 \qquad (4)$$

$$\nabla \times \bar{H} = 0 \qquad (5)$$

$$\bar{M} = \bar{\mu}\bar{H} \qquad (6)$$

$$\nabla(\bar{H} + \bar{M}) = 0 \qquad (7)$$

where ρ is the fluid density, $\bar{q}$ being the fluid velocity in the film region, $\bar{H}$ represents external magnetic field, $\bar{\mu}$ isthe magnetic susceptibility of the magnetic field, $p$represents the film pressure, η denotes the fluid viscosity and μ_0 representsthe permeability of the free space. The details can be seen from Bhat [34] and Prajapati [37].

Using equations (4)–(6), equation (3) becomes

$$\rho(\bar{q}\nabla)\bar{q} = -\nabla\left(p - \frac{\mu_0\bar{\mu}}{2}M^2\right) + \eta\nabla^2\bar{q}$$

The Reynolds equation with modification governing the film pressure for Neuringer and Rosensweig model then, is given by

$$\frac{1}{r}\frac{d}{dr}\left[h^3 r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}M^2\right)\right]=12\eta\dot{h_0} \quad (8)$$

Shliomis [2] pointed out that magnetic particles of a magnetic fluid could relax in two ways, by the rotation of magnetic particles in the fluid and by rotation of the magnetic moment with in the particles, when the applied magnetic field changed. According to Bhat [34] and Patel and Deheri [35], the modified Reynolds type equation for Shliomis model comes out to be

$$\frac{1}{r}\frac{d}{dr}\left(h^3 r\frac{dp}{dr}\right)=12\eta_a\dot{h_0}$$
$$=12\eta(1+\tau)\dot{h_0} \quad (9)$$

The details of the derivation of the expression (9) are already discussed in Bhat [34] and Patel and Deheri [35, 31].

Jenkins [3] modified the theory of Neuringer–Rosensweig model and investigated a model to depict the flow of a ferrofluid. Using the Maugin's modification, equation for the model for steady flow becomes (Jenkins [3], Ram and Verma [16], Patel and Deheri [10]).

$$\rho(\bar{q}.\nabla)\bar{q}=-\nabla p+\eta\nabla^2\bar{q}+\mu_0(\bar{M}.\nabla)\bar{H}+\frac{\rho A^2}{2}\nabla$$
$$\times\left[\frac{\bar{M}}{M}\times\{(\nabla\times\bar{q})\times\bar{M}\}\right] \quad (10)$$

together with equations (4)–(7), Adenotes a material constant. From equations (3) and (10) it is found that Jenkins model is a generalization of Neuringer– Rosensweig model with an additional term

$$\frac{\rho J^2}{2}\nabla\times\left[\frac{\bar{M}}{M}\times\{(\nabla\times\bar{q})\times\bar{M}\}\right]$$
$$=\frac{\rho A^2\bar{\mu}}{2}\nabla\times\left[\frac{\bar{H}}{H}\times\{(\nabla\times\bar{q})\times\bar{H}\}\right] \quad (11)$$

which improves the velocity of the fluid.

In view of the discussions of Bhat [34] and Patel and Deheri [35], the modified Reynolds equation for Jenkins model can be found in the form of,

$$\frac{1}{r}\frac{d}{dr}\left(\frac{h^3}{\left(1-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right)\right)$$
$$=12\eta\dot{h_0} \quad (12)$$

Taking into accountthe usual assumptions of hydrodynamic lubrication (Bhat [34], Prajapati [37], Deheri et al. [38]) and the stochastic modelling of Christensen and Tonder [20–22], the modified Reynolds' equation leading to the pressure distribution takes the form for Neuringer–Rosensweig model, Shliomis model and Jenkins model, respectively as,

$$\frac{1}{r}\frac{d}{dr}\left[g(h)r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}M^2\right)\right]=12\eta\dot{h_0} \quad (13)$$

$$\frac{1}{r}\frac{d}{dr}\left(g(h)r\frac{dp}{dr}\right)=12\eta(1+\tau)\dot{h_0} \quad (14)$$

and

$$\frac{1}{r}\frac{d}{dr}\left(\frac{g(h)}{\left(1-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right)\right)$$
$$=12\eta\dot{h_0} \quad (15)$$

where

$$g(h)=(h^3+3h^2\alpha+3(\sigma^2+\alpha^2)h+3\sigma^2\alpha+\alpha^3+\varepsilon$$
$$+12\phi H_0)\left(\frac{4+sh}{2+sh}\right),$$

ϕ being the permeability of the porous facing and H_0denotes the thickness of the porous facing.

The following dimensionless quantities are considered

$$\bar{h}=\frac{h}{h_0},R=\frac{r}{a},P=-\frac{h_0^3 p}{\eta a^2\dot{h_0}},B=\beta a^2,$$
$$C=\gamma a,\bar{\sigma}=\frac{\sigma}{h_0},\bar{\alpha}=\frac{\alpha}{h_0},\bar{\varepsilon}=\frac{\varepsilon}{h_0^3},$$
$$M^2=H^2=kr^2\frac{(a-r)}{a},\mu^*=-\frac{k\mu_0\bar{\mu}h_0^3}{\eta\dot{h_0}},$$
$$\bar{A}^2=\frac{\rho A^2\bar{\mu}\sqrt{k}a}{2\eta},\bar{s}=sh_0,\bar{\psi}=\frac{\phi H}{h_0^3} \quad (16)$$

The associated boundary conditions are

$$P(1)=0,\left(\frac{dP}{dR}\right)_{R=0}=0 \quad (17)$$

Using the dimensionless quantities (16), the equations (13–15) convert respectively into,

$$\frac{1}{R}\frac{d}{dR}\left[g(\bar{h})R\frac{d}{dR}\left(P-\frac{\mu^*}{2}R^2(1-R)\right)\right]=-12 \quad (18)$$

$$\frac{1}{R}\frac{d}{dR}\left(g(\bar{h})R\frac{dP}{dR}\right)=-12(1+\tau) \quad (19)$$

and

$$\frac{1}{R}\frac{d}{dR}\left(\frac{g(\bar{h})}{(1-\bar{A}^2R\sqrt{1-R})}R\frac{d}{dR}\left(P-\frac{\mu^*}{2}R^2(1-R)\right)\right)$$
$$=-12 \quad (20)$$

where

$$g(\bar{h})=(\bar{h}^3+3\bar{h}^2\bar{\alpha}+3(\bar{\sigma}^2+\bar{\alpha}^2)\bar{h}+3\bar{\sigma}^2\bar{\alpha}+\bar{\alpha}^3+\bar{\varepsilon}$$
$$+12\bar{\psi})\left(\frac{4+\bar{s}\bar{h}}{2+\bar{s}\bar{h}}\right)$$

With the help of the boundary conditions (16), solving equations (18–20), the non–dimensional pressure for Neuringer–Rosensweig model, Shliomis model and Jenkins model, found respectively as,

$$P=\frac{\mu^*}{2}R^2(1-R)-6\int_1^R\frac{R}{g(\bar{h})}dR \quad (21)$$

$$P=-6(1+\tau)\int_1^R\frac{R}{g(\bar{h})}dR \quad (22)$$

and

$$P=\frac{\mu^*}{2}R^2(1-R)$$
$$-6\int_1^R\frac{R}{g(\bar{h})}(1-\bar{A}^2R\sqrt{1-R})dR \quad (23)$$

One can obtain the dimensionless load carrying capacity for all three cases, respectively as,

$$W = -\frac{h_0^3}{2\pi\eta a^4 \dot{h}_0} w = \int_0^1 RPdR = \frac{\mu^*}{40} + 3\int_0^1 \frac{R^3}{g(\bar{h})} dR \tag{24}$$

$$W = -\frac{h_0^3}{2\pi\eta a^4 \dot{h}_0} w = \int_0^1 RPdR = 3(1+\tau)\int_0^1 \frac{R^3}{g(\bar{h})} dR \tag{25}$$

and

$$W = -\frac{h_0^3}{2\pi\eta a^4 \dot{h}_0} w = \int_0^1 RPdR = \frac{\mu^*}{40} + 3\int_0^1 \frac{R^3}{g(\bar{h})}\left(1-\bar{A}^2 R\sqrt{1-R}\right)dR \tag{26}$$

RESULTS AND DISCUSSIONS

It is noticed that equations (24–26) determine the non–dimensional$W$. It is observed that the Wenhances by $\frac{\mu^*}{40}$ in the case of Neuringer–Rosensweig model and Jenkins model while the increase in load carrying capacity for the case of Shliomis model is found to be $3\tau\int_0^1 \frac{R^3}{g(\bar{h})}dR$ as compared to the case of traditional lubricant based bearing system. This is perhaps due to the fact that the viscosity of the lubricant gets increased owing to magnetization, there by leading to increased pressure and hence the load carrying capacity. A glance at the expression of the load suggests that the expressions are linear with respect to the magnetization parameter. This means an increase in the magnetization parameter would always result in enhanced W.

The variation of $W$with respect to magnetization presented in Figures 2–7 ensures that an increase in the magnetic strength leads to enhance W, the most increase being in the case of Shliomis model. Although a nominal increase in W is noticed for Neuringer–Rosensweig model and Jenkins model, the effect of μ^* is more sharp in the case of Shliomis model.

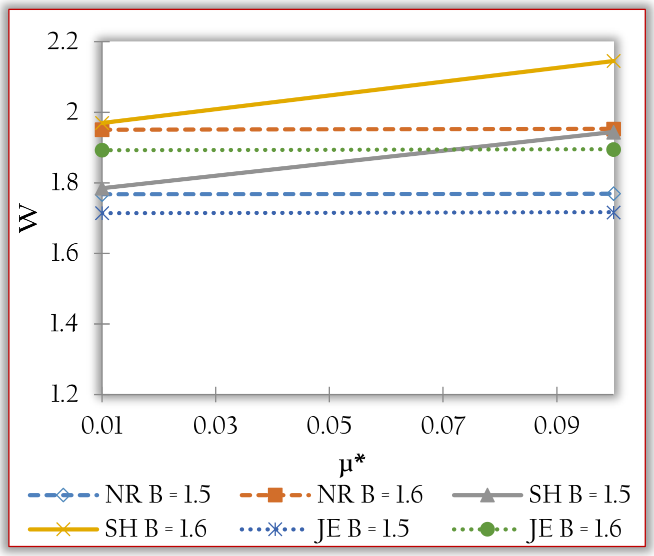

Figure 2: Variation of W with respect to $\mu^*$and$B$.

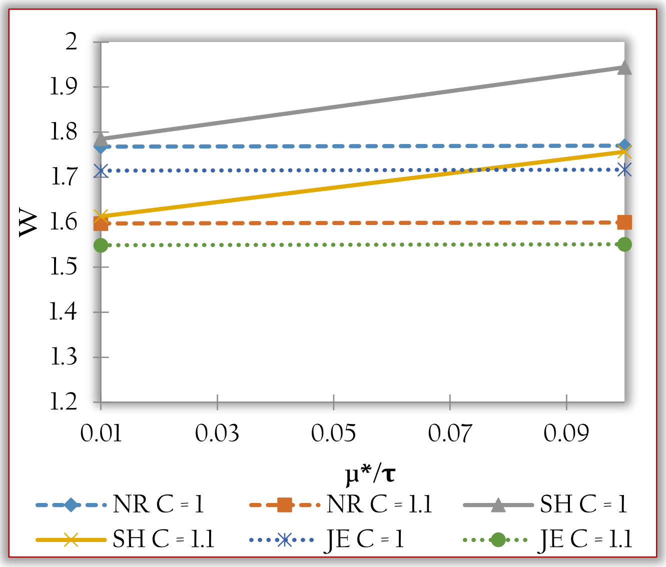

Figure 3: Variation of W with respect to $\mu^*$and$C$.

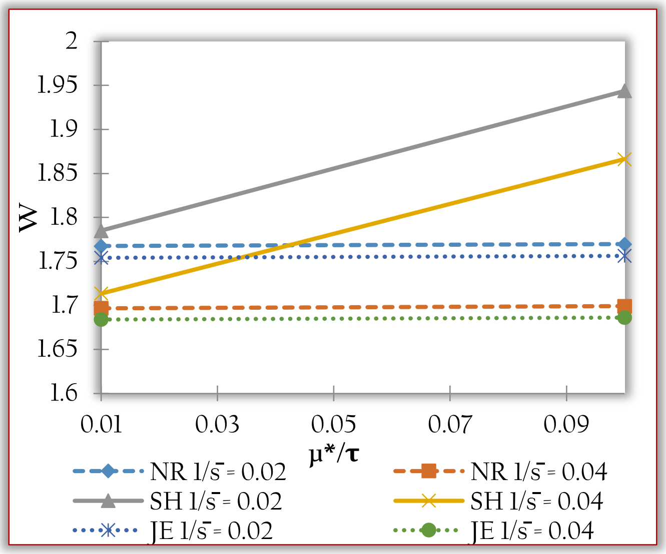

Figure 4: Variation of W with respect to $\mu^*/\tau$and$1/\bar{s}$.

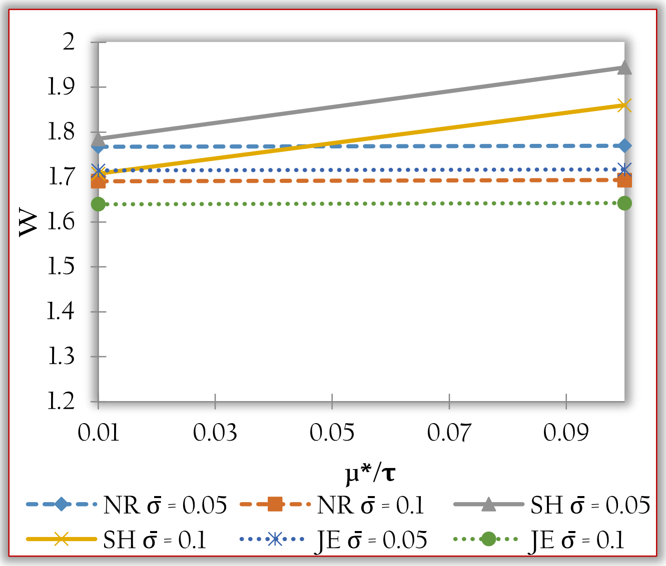

Figure 5: Variation of W with respect to μ^*/τ and $\bar{\sigma}$.

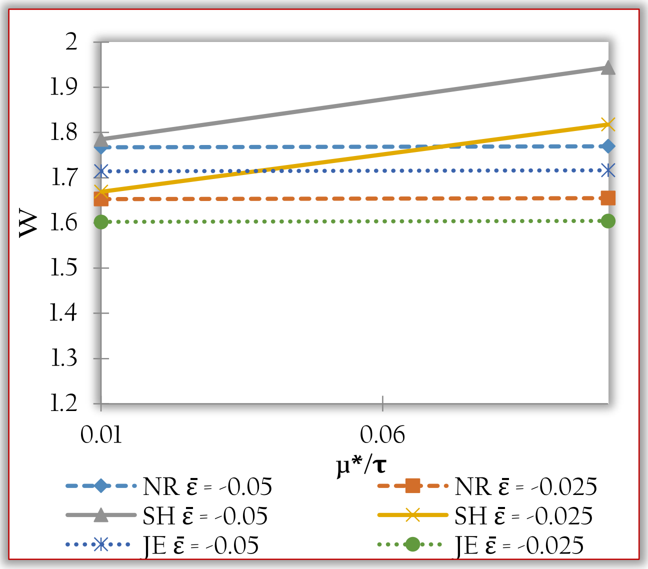

Figure 6: Variation of W with respect to μ^*/τ and $\bar{\varepsilon}$.

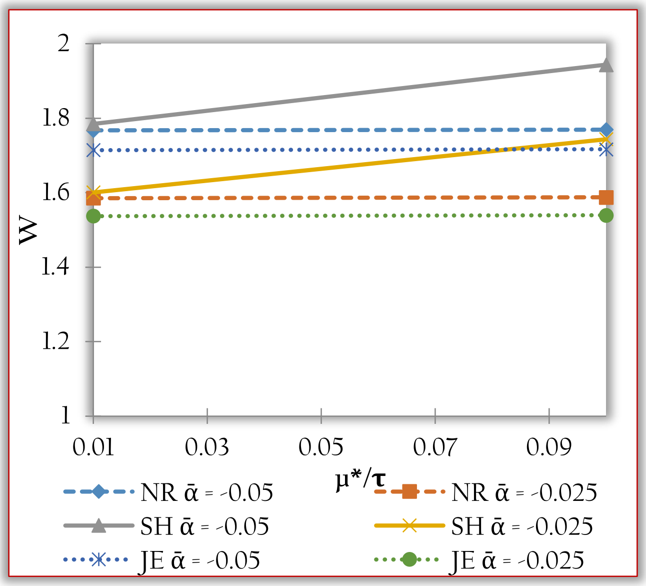

Figure 7: Variation of W with respect to μ^*/τ and $\bar{\alpha}$.

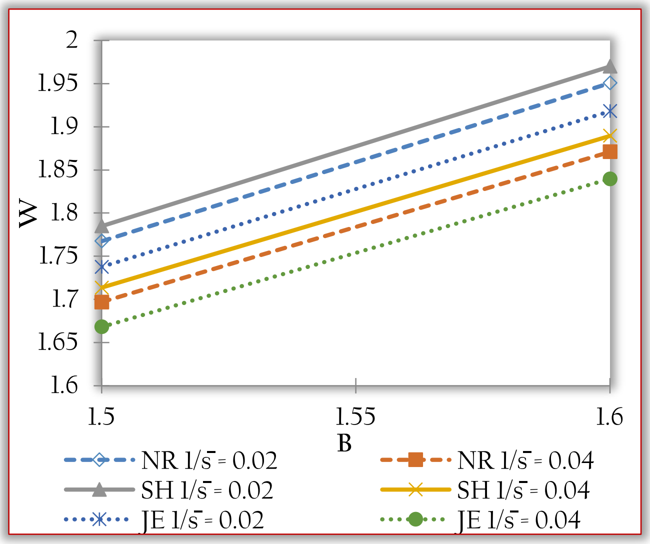

Figure 8: Variation of W with respect to B and $1/\bar{s}$.

The combined effect of curvature parameters given in Figures 8–11 indicates that the lower plates curvature parameters affects the most in the case of Jenkins model.

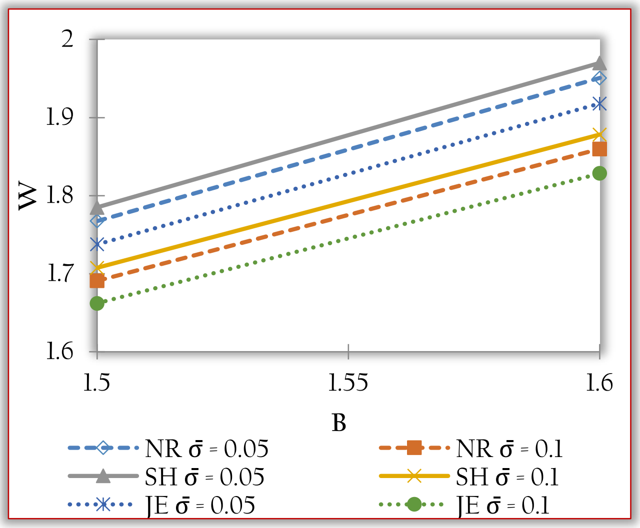

Figure 9: Variation of W with respect to B and $\bar{\sigma}$.

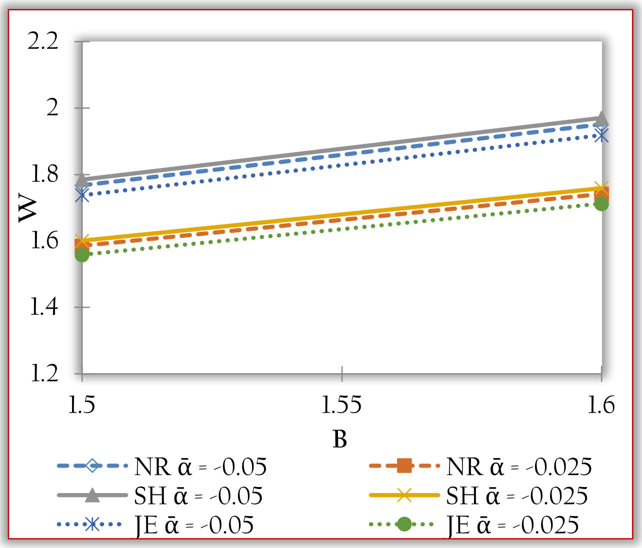

Figure 10: Variation of W with respect to B and $\bar{\alpha}$.

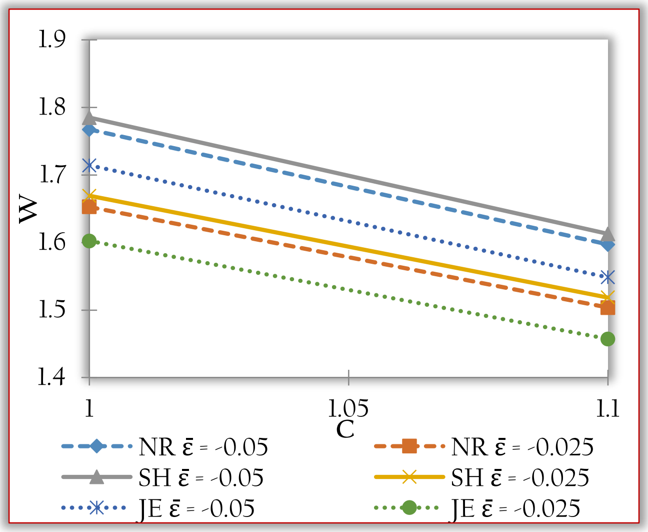

Figure 11: Variation of W with respect to C and $\bar{\varepsilon}$.

The effect of slip velocity encountered in Figures 12–14 suggests that the slip effect is comparatively more in the case of Jenkins model.

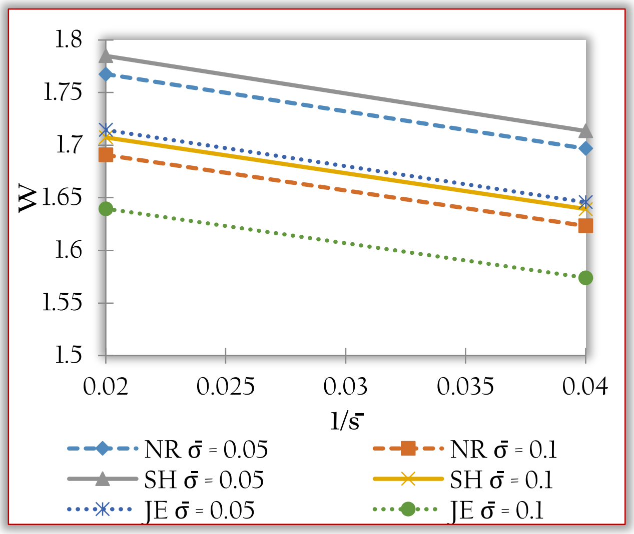

Figure 12: Variation of W with respect to $1/\bar{s}$ and $\bar{\sigma}$.

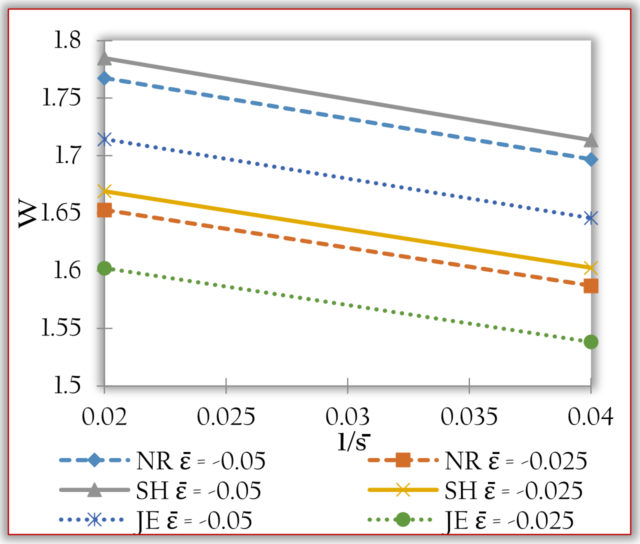

Figure 13: Variation of W with respect to $1/\bar{s}$ and $\bar{\varepsilon}$.

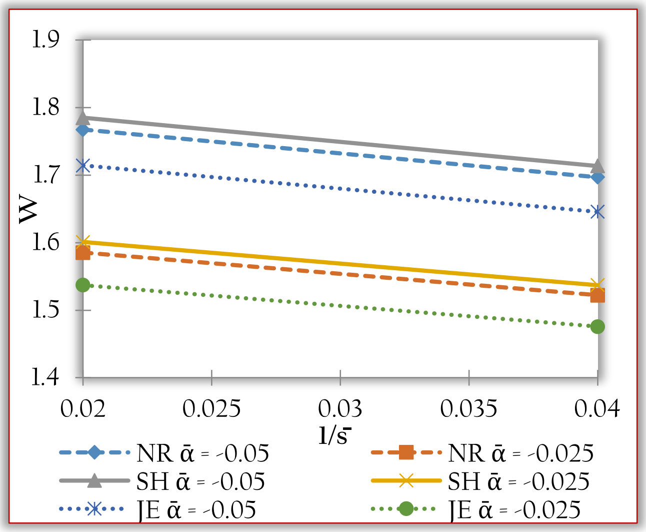

Figure 14: Variation of W with respect to $1/\bar{s}$ and $\bar{\alpha}$.

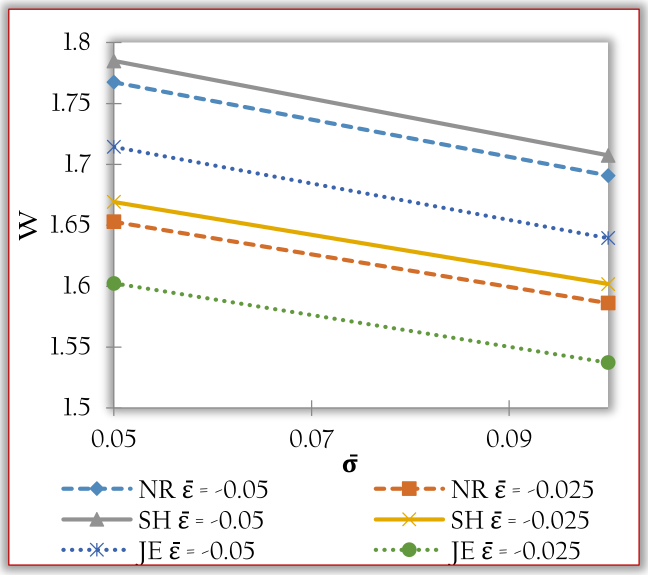

Figure 15: Variation of W with respect to $\bar{\sigma}$ and $\bar{\varepsilon}$.

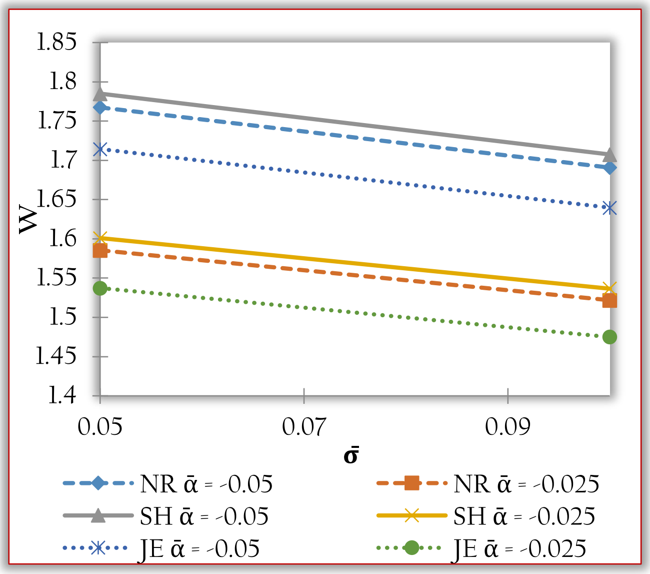

Figure 16: Variation of W with respect to $\bar{\sigma}$ and $\bar{\alpha}$.

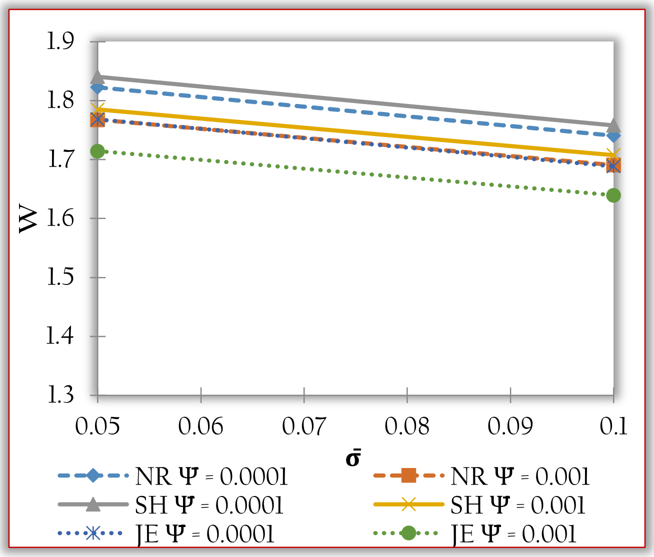

Figure 17: Variation of W with respect to $\bar{\sigma}$ and $\bar{\psi}$.

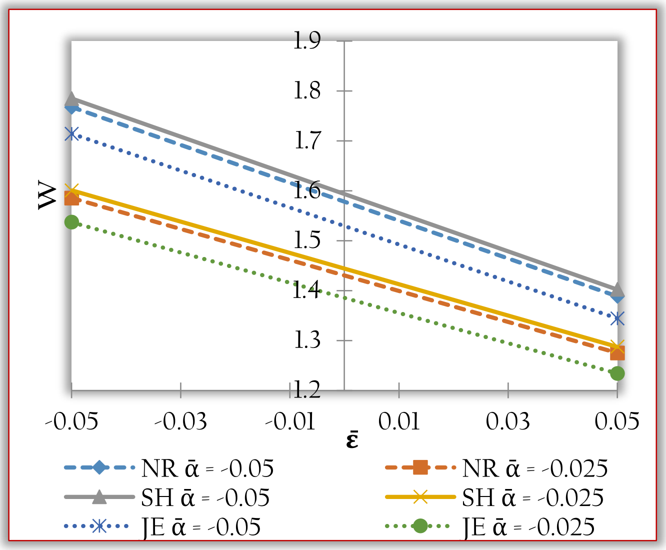

Figure 18: Variation of W with respect to $\bar{\varepsilon}$ and $\bar{\alpha}$.

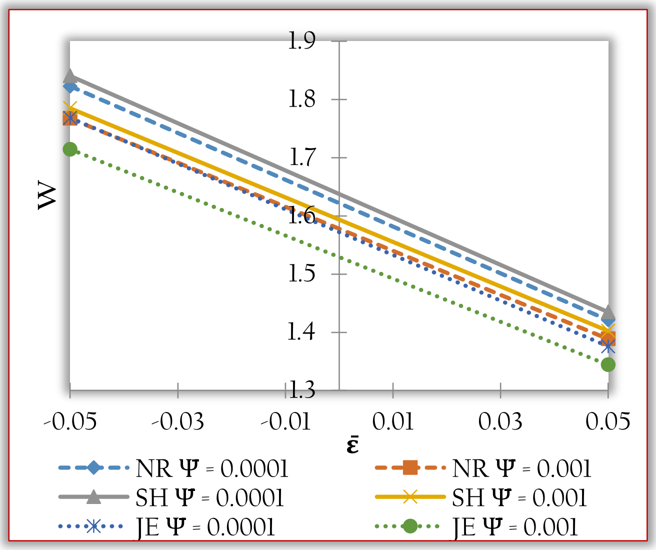

Figure 19: Variation of W with respect to $\bar{\varepsilon}$ and $\bar{\psi}$.

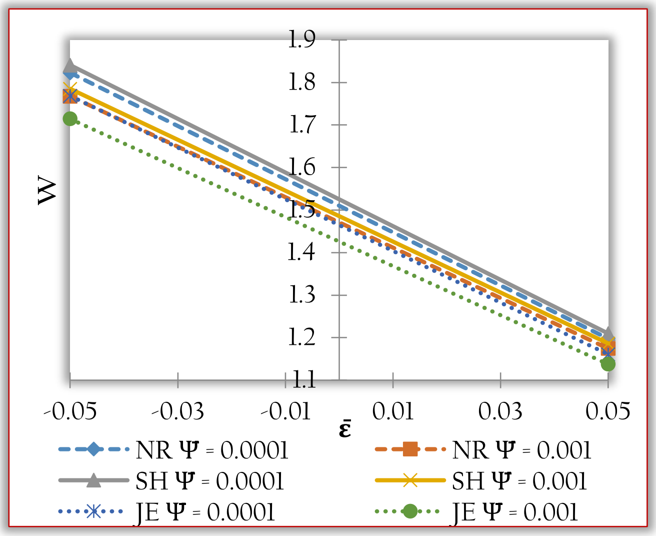

Figure 20: Variation of W with respect to $\bar{\varepsilon}$ and $\bar{\psi}$.

The effect of transverse surface roughness on the Wfound in Figures 15–20 establishes that the adverse effect of transverse surface roughness is registered to be more in the case of Jenkins model. It is interesting here to note that for moderate to higher values of roughness the Shliomis model goes past the Neuringer–Rosensweig model in reducing the effect of surface roughness.

As it happens mostly, the porosity leads to decreased Wand the situation turns out to be worse when the higher values of slip are involved.

In addition, the comparison of graphical representations reveals the following:

— All the three models improve the bearing performance. So far as magnetization is concerned Neuringer–Rosensweig model performs a little better as compared to Jenkins model while the Shliomis model remains the best.
— The Shliomis model comes out to be more effective in comparison with other two models exclusively, from surface roughness point of view. Further, Neuringer–Rosensweig model and Jenkins model differ a little when the combined effect of negative skewness and variance (–ve) is considered.
— A key point to be noted is that the standard deviation reduces the **W** significantly which fails to happen in the case of parallel plate slider bearing in the absence of slip.
— When the slip is at minimum the effect of negatively skewed roughness may provide some amount of help to improve the bearing performance in the case of all the three models when the variance (–ve) occurs.
— If one considers the combined effect of roughness and slip the Shliomis model stays ahead of the other two models for all the values of porosity.
— Up to certain extent the effect of standard deviation remains more manifest in the Neuringer–Rosensweig model as compared to Jenkins model.
— Besides, the Shliomis model scores over the other two models in lowering the adverse effect of porosity and slip.

CONCLUSION

This study concludes that the load carrying capacity gets increased approximately by 2 to 3 percent as compared to the case of conventional fluid based curved rough porous circular squeeze films with slip velocity. Some of the graphical representations indicate that the Neuringer–Rosensweig model may be deployed to compensate the effect of surface roughness when the slip and porosity are at reduced level. However, for a bearing design point of view Shliomis model may be preferred for moderate to higher loads irrespective of the slip effect.

For nominal roughness and moderate slip velocity Neuringer–Rosensweig model and Jenkins model perform alike. The bearing system always supports certain amount of load even when there is no flow which never happens in the case of traditional lubricant based bearing system. However, the load supported by the bearing system in the absence of flow remains significantly higher in the case ofShliomis model. At the same time this discussion underlines that the roughness aspect is required to be carefully evaluated while designing the bearing system even if Shliomis model is in force. It is needless to say that a suitable choice of the ratio of curvature parameters may provide augmented performance of the squeeze film bearing system.

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