1.Mustefa JIBRIL, 2.Tesfabirhan SHOGA

COMPARISON OF H∞ AND μ-SYNTHESIS CONTROL DESIGN FOR QUARTER CAR ACTIVE SUSPENSION SYSTEM USING SIMULINK

1.Dire Dawa Institute of Technology, DireDawa, ETHIOPIA
2.Jimma Institute of Technology, Jimma, ETHIOPIA

Abstract: In order to improve the road handling and passenger comfort of a vehicle, suspension system is provided. An active suspension system is considered to be better than passive suspension system. In this paper, a linear quarter car active suspension system is designed, which subjected to different road disturbances. Since parametric uncertainty in the spring, damper and actuator has been considered, therefore robust control is used. H∞ and μ-synthesis controllers are used to improve the ride comfort and road handling ability of the car as well as to check the robust stability and performance of the system. In H∞ design, we design a controller for passenger comfort purpose and to keep the suspension deflection small and to reduce the road disturbance to suspension deflection. For the μ-synthesis design, we design a controller with hydraulic actuator and uncertainty model. We design a MATLAB/SIMULINK model for the active suspension system with the H∞ and μ-synthesis controllers and we made test using four road disturbance inputs (bump, random, sine pavement and slope) for suspension deflection, body acceleration and body travel for passive, active suspension with controller and active suspension without controller. Finally we compare the H∞ and μ-synthesis controllers with a Simulink model for suspension deflection, body acceleration and body travel simulation and the result shows that both designs give good performance but H∞ controller has superior performance as compared to μ-synthesis controller.

Keywords: Quarter Car Active Suspension System, H∞ Controller, μ-Synthesis Controller, Robust Performance, Robust Stability

INTRODUCTION

At present, the world's leading automotive companies and research institutions have invested considerable human and material resources to develop a cost-effective vehicle suspension system, in order to be widely used in the vehicle. The main aim of suspension system is to isolate a vehicle body from road irregularities in order to maximize passenger ride comfort and retain continuous road wheel contact in order to provide road holding. Many studies have shown that the vibrations caused by irregular road surfaces have an energy draining effect on drivers, affecting their physical and mental health [1]. Demands for better ride comfort and controllability of road vehicles like passenger cars has motivated to develop new type of suspension systems like active and semi active suspension systems. These electronically controlled suspension systems can potentially improve the ride comfort as well as the road handling of the vehicle. An active suspension system has the capability to adjust itself continuously to changing road conditions. By changing its character to respond to varying road conditions, active suspension offers superior handling, road feel, responsiveness and safety.

An active suspension system has the ability to continuously adjust to changing road conditions. By changing its character to respond to different road conditions, the active suspension offers superior handling, road feel, responsiveness and safety.

Active suspension systems dynamically respond to changes in the road profile because of their ability to supply energy that can be used to produce relative motion between the body and wheel. Typically, the active suspension systems include sensors to measure suspension variables such as body velocity, suspension displacement, and wheel velocity and wheel and body acceleration. An active suspension is one in which the passive components are augmented by actuators that supply additional forces. These additional forces are determined by a feedback control law using data from sensors attached to the vehicle.

The existing active suspension system is inefficient if there are changes in parameter of the system or of actuator, then controlling the suspension system becomes a big problem. Therefore H∞ and μ-synthesis control technique are used. H∞ and μ-synthesis control effectively suppresses the vehicle vibrations in the sensitive frequency range of the human body. The desired robust performance and robust stability are achieved in the closed loop system for a quarter vehicle model in the presence of structured uncertainties.

REVIEW ON DIFFERENT CONTROL METHODS OF CAR ACTIVE SUSPENSION SYSTEM

Vivek Kumar Maurya and Narinder Singh Bhangal [2] presented optimal control of vehicle active suspension system. A linear model of active suspension system has been developed since it contains all the basic parameters of its performance body deflection, suspension deflection and body acceleration. Two control techniques PID and LQR are used to suppress the vibrations of the system. A comparison between passive and active suspension system using PID and LQR control technique with road disturbance as input has been made.

Michiel Haemers et al. [3] presented a proportional-integral state-feedback controller optimization for a full-car active suspension setup using a genetic algorithm. In his work, an optimal control for a full-car electromechanical active suspension was presented. Therefore, a scaled-down lab setup model of this full-car active suspension was established, capable of emulating a car driving over a road surface with a much simpler approach in comparison with a classical full-car setup. A kinematic analysis is performed to assure system behavior which matches typical full-car dynamics. A state-space model was deducted, in order to accurately simulate the behavior of a car driving over an actual road profile, in agreement with the ISO 8608 norm. The active suspension control makes use of a Multiple-Input-Multiple- Output (MIMO) state-feedback controller with proportional and integral actions. The optimal controller tuning parameters are determined using a Genetic Algorithm, with respect to actuator constraints and without the need of any further manual re-tuning.

Ahmed A. Abougarair and Muawia M. A. Mahmoud [6] have introduced the design and simulation of optimal controller for quarter car active suspension system. In their work, the Proportional Derivative (PD), Linear Quadratic Control (LQR) and Fuzzy logic tune PD controllers' techniques implemented to the active suspensions system for a quarter car model. The comparison between these controllers by means of the reduction of body displacement trajectory under influence two road profiles and variation of body mass.

J. Marzbanrad and N. Zahabi [7] have presented H ∞ active control of a vehicle suspension system exited by harmonic and random roads. In their work they proposes a controller based upon H ∞ control approach in order to improve the vehicle performance under two different road profiles. In their study, there are two control targets that they are car body travel and suspension deflection. In fact, H ∞ controller is responsible for minimizing the infinity norm of two subsystems. The first one is from car body travel to road disturbance and second is from suspension deflection to road disturbance. These two control targets must be improved by a logical control input that is determined by H ∞ control approach. In order to improve the performance of the quarter–car, weighting functions are also defined. Disturbance that is the system input is considered as two types of road profiles, harmonic and random. The results show that the H ∞ controller is able to improve the quarter–car performance for both roads. In addition, the sensitivity analysis is done to show that the active suspension system is able to work when sprung mass changes as may be occurred when passengers added.

Shital M. Pawar and A.A. Panchwadkar [8] presents estimation of state variables of active suspension system using kalman filter. In their work, active suspension systems use an actuator which is governed by the control strategy using an ECU. Most of the literature focuses on feedback control of active suspension systems. Their study aims to make use of an algorithm which predicts the states of the suspension systems in response to the road input using Kalman filter and control the suspension travel between the sprung and unsprung masses of the suspension. The Kalman filter is used as the observer which will observe the system states and predict the next states of the plant model. A linear Quadratic Regulator (LQR) and a Linear Quadratic Gaussian (LQG) control strategy was used to control the required force to minimize the suspension travel. The suspension system model was prepared in Matlab™/Simulink and simulated. Comparison of the estimation errors in open loop (passive suspension), the LQR control and LQG control using Kalman filter was made to study the effectiveness of the new control strategy.

The gap of this study is there is no design of MATLAB/Simulink model for the active suspension system with H ∞ and μ - synthesis controllers in the presence of uncertainty. No comparison has been made between H ∞ and μ - synthesis in the tests of suspension deflection, body acceleration and body travel using random, sine pavement and slope road disturbances for passive, active suspension with H ∞ and μ - synthesis controller and active suspension without controller and no comparison has been made based on impulse, step, Nichols, NY Quist and Bode magnitude response of the active suspension system with H ∞ and μ - synthesis controllers in the presence of uncertainty.

MATHEMATICAL MODELS

— Passive Suspension System Mathematical Model

Vibration control system design should start with the establishment of mathematical model of the system, and then determine the design requirements, and formal description of it. And then select one or several design methods to design the control system, and further attached to simulation or model experiments to identify the control system is designed to meet the performance requirements. Therefore, the establishment of the mathematical models of the system is a prerequisite for the entire control designs and control system design is closely related to the control system quality evaluation model.

As with other engineering control system, a mathematical model of the suspension control system refers to the formal model, for short the mathematical model. Such models typically rely on the dynamic principle to be derived or through some of the system dynamics test. Then it experiences mathematical simulation and optimization, or statistical approach. The key to create a system of mathematical models is to provide a description of the model form and determine its parameters.

In vibration control area, there are three kinds of models most popular to describe the form, the state space description, transfer function description and weight function description. In accordance with the implementation of continuous control and discrete control of different characteristics, they are each divided into a time continuous and time discrete mathematical description.

The automotive is a complex vibration system, should simplify based on the analysis of the problem. Simplification of motor vehicles there are several ways, but according to the convenience of the study, in this paper we simplify it into a system model as shown in Figure 1 below:

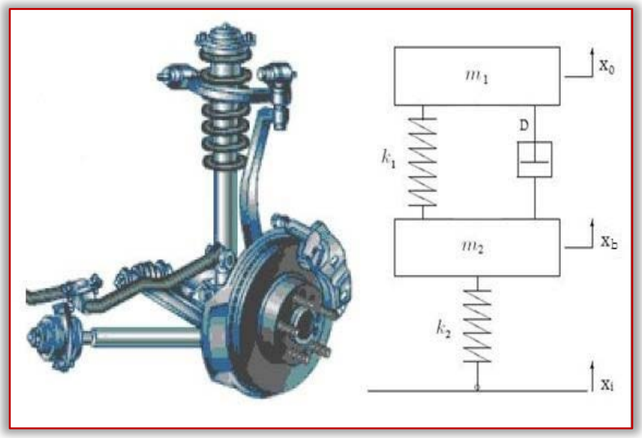

Figure 1. Quarter Model of Passive suspension system

Figure 1 shows a quarter vehicle model of the passive suspension system. The sprung mass m_1 represents the vehicle body, and the unsprung mass m_2 is an assembly of the axle and wheel. The tire is assured to contact the surface of the road when the vehicle is traveling, and is modeled as a linear spring with stiffness k_2. The linear damper, whose average damping coefficient is D, and the linear spring, whose average stiffness coefficient is k_1, consist of the passive component of the suspension system. The state variables x_0 (t) and x_b (t) are the vertical displacements of the sprung and unsprung masses, respectively, and $x_i(t)$ is the vertical road profile.

This is a vehicle body and wheel dual-mass vibration system model. From this model, we can analyze the vehicle suspension system dynamics and establish two degrees of freedom motion differential equations. Their equilibrium position is the origin of coordinates; we can get the equations as follow (1) and (2).

$$m_1\ddot{x}_0(t)+D\left[\dot{x}_0(t)-\dot{x}_b(t)\right]+k_1\left[x_0(t)-x_b(t)\right]=0 \qquad (1)$$

$$m_2\ddot{x}_b(t)-D\left[\dot{x}_0(t)-\dot{x}_b(t)\right]+k_1\left[x_b(t)-x_0(t)\right]+k_2\left[x_b(t)-x_i(t)\right]=0 \qquad (2)$$

Letting

$$W = x_0 - x_b$$

where: W-Suspension Deflection; X_i-Road Disturbance

If we make a Laplace transformation to the above equation, we can get equation (3):

$$\frac{W}{X_i}=\frac{M_1K_2}{M_1M_2s^2+(M_2+K_2)Ds+(M_2K_1+M_1K_1+M_2K_2+K_1K_2)} \qquad (3)$$

Table 1: Parameters of quarter vehicle model

Model parameters	Symbol	Symbol Values
Vehicle body mass	m 1	300 Kg
Wheel assembly mass	m 2	40 Kg
Suspension stiffness	k 1	15,000 N/m
Tire stiffness	k 2	150,000 N/m
Suspension damping	D	1000 N-s/m

The passive suspension system P_{C1} (s) transfer function is

$$P_{C1}(s)=\frac{4.5\times10^{9}}{12000s^{2}+1.5\times10^{8}s+2.261\times10^{9}}$$

— **Active Suspension System Mathematical Model**

The mathematical model and the simulation made by the following sections are only discussing the amount of force created by the active suspension. Active suspensions allow the designer to balance these objectives using a feedback-controller hydraulic actuator which is driven by a motor between the chassis and wheel assembly. The force U applied between the body and wheel assembly is controlled by feedback and represents the active component of the suspension system.

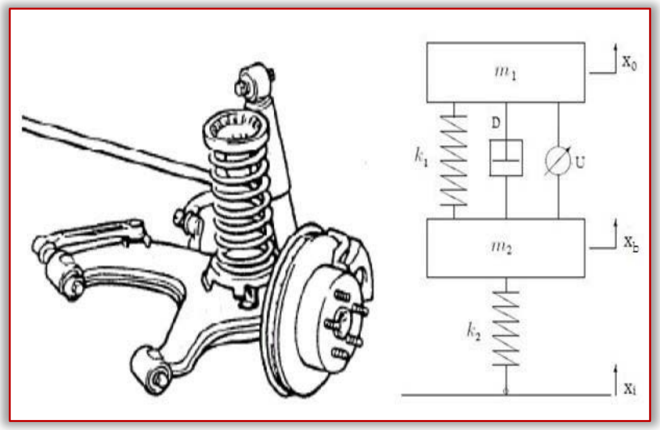

Figure 2. Quarter Model of active suspension system with actuating force (u) between sprung and unsprung mass.

Figure 2 shows a vehicle quarter model of active suspension system. The mass m 1 (in kilograms) represents the car chassis (body) and the mass m 2 (in kilograms) represents the wheel assembly.

The spring K 1 and damper D represent the passive spring and shock absorber placed between the car body and the wheel assembly. The spring K 2 models the compressibility of the pneumatic tire. The variables x 0 , x b and x i (all in meters) are the body travel, wheel travel, and road disturbance, respectively. The actuator force f s (in KiloNewtons) applied between the body and wheel assembly is controlled by feedback and represents the active component of the suspension system.

From this model, we can analyze the vehicle suspension system dynamics as a linear system model and establish two degrees of freedom motion differential equations will be as follow:

$$m_1\ddot{x}_0(t)+D\left[\dot{x}_0(t)-\dot{x}_2(t)\right]+k_1\left[x_0(t)-x_2(t)\right]=u$$

$$m_2\ddot{x}_2(t)-D\left[\dot{x}_0(t)-\dot{x}_2(t)\right]+k_1\left[x_2(t)-x_0(t)\right]+k_2\left[x_2(t)-x_1(t)\right]=-u$$

We can set:

$$x_1=x_2(t),\ x_2=x_0(t),\ x_3=\dot{x}_2(t),\ x_4=\dot{x}_0(t)$$

The system state space equation can be express as:

$$\frac{dX}{dt}=AX+BU$$

In this equation, state variable matrixes are:

$$X=\begin{pmatrix}x_1 & x_2 & x_3 & x_4\end{pmatrix}^T$$

Constant matrixes A and B are shown as below:

$$A=\begin{pmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\frac{k_1+k_2}{m_2} & \frac{k_1}{m_2} & -\frac{D}{m_2} & \frac{D}{m_2}\\ \frac{k_1}{m_1} & -\frac{k_1}{m_1} & \frac{D}{m_1} & -\frac{D}{m_1}\end{pmatrix}$$

$$B=\begin{pmatrix}0 & 0\\ 0 & 0\\ \frac{k_2}{m_2} & \frac{1}{m_2}\\ 0 & -\frac{1}{m_1}\end{pmatrix}$$

The system input variable matrix will be:

$$U=\begin{pmatrix}x_1(t) & u\end{pmatrix}^T$$

The vehicle suspension system output matrix equation will be:

$$Y=CX+DU$$

In above equation, the output variable matrix Y will be:

$$Y=\left(k_2\left[x_1(t)-x_2(t)\right] \quad \ddot{x}_0(t) \quad x_0(t)\right)$$

Y will also express as the following equation:

$$Y=\left(k_2\left[x_1(t)-x_2(t)\right] \quad \ddot{x}_0(t) \quad x_0(t)\right)$$

Constant matrixes C and D will be shown as below:

$$C=\begin{pmatrix}-k_2 & 0 & 0 & 0\\ \frac{k_1}{m_1} & -\frac{k_1}{m_1} & \frac{D}{m_1} & -\frac{D}{m_1}\\ 0 & 1 & 0 & 0\end{pmatrix}$$

$$D=\begin{pmatrix}k_2 & 0\\ 0 & -\frac{1}{m_1}\\ 0 & 0\end{pmatrix}$$

ROAD PROFILES

Four types of road disturbance input signal will be used to simulate different kinds of road condition. They are bump input signal, sine pavement input signal, random input signal and slope road input signal. These inputs are the prerequisite to simulate the vehicle suspension system, and they should be accurately reflecting the real road condition when a vehicle drives on the road. Precise signal is crucial to the result of the simulation. We assume the vehicle is a linear system.

— Bump Road Disturbance:

Bump input signal is a basic input to research the suspension system. It simulated a very intense force for a very short time, such as a vehicle drive through a speed hump. This road disturbance has a maximum height of 10 cm as shown in Figure 3.

Figure 3. Bump road disturbance

— Random Road Disturbance:

Numerous researches show that it is necessary to test a car to a random road disturbance to check the spring and damper respond quickly and correctly.

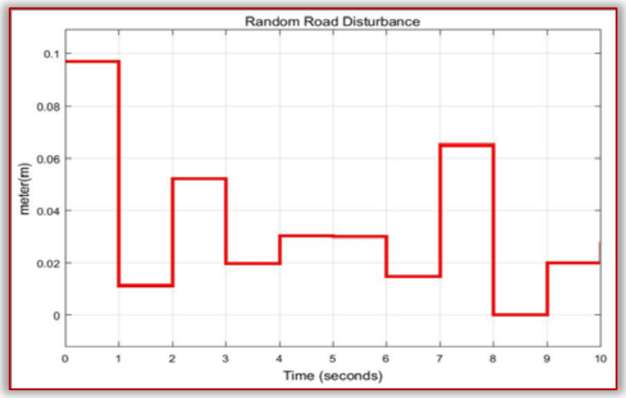

Figure 4. Random road disturbance

The random road disturbance has a maximum height of 10 cm and minimum height of 0 cm as shown in Figure 4.

— **Sine Pavement Road Disturbance:**

Sine wave input signal can be used to simulate periodic pavement fluctuations. It can test the vehicle suspension system elastic resilience ability while the car experiences a periodic wave pavement. Sine input pavement test is made by every automotive industries before a new vehicle drives on road. The sine pavement road disturbance has a height of -10 cm to 10 cm as shown in Figure 5.

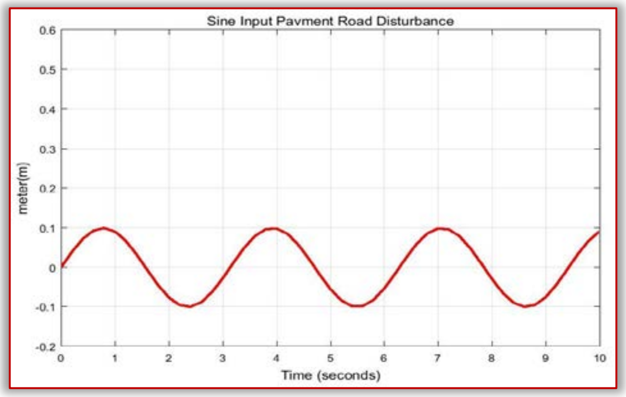

Figure 5. Sine Input pavement road disturbance

— **Slope Road Disturbance:**

The suspension performance is tested using slope road disturbance by checking the degree of elevation of the road that the suspension handle. The proposed slope road disturbance is 45^0 degree elevated as shown in Figure 6.

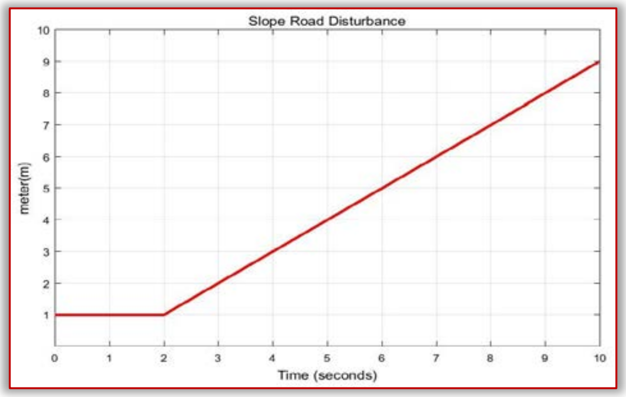

Figure 6. Slope road disturbance

THE PROPOSED H ∞ CONTROL DESIGN

The design of active suspension system to provide passenger comfort and road handling is developed using H ∞ controller design. The main aim of the controller design is to minimize suspension deflection, body acceleration and body travel of the system. H ∞ synthesis is the method used to design the proposed controller by achieving the performance objective via minimizing the weighted transfer function norm. The H infinity interconnected design for active suspension system is shown in Figure 7.

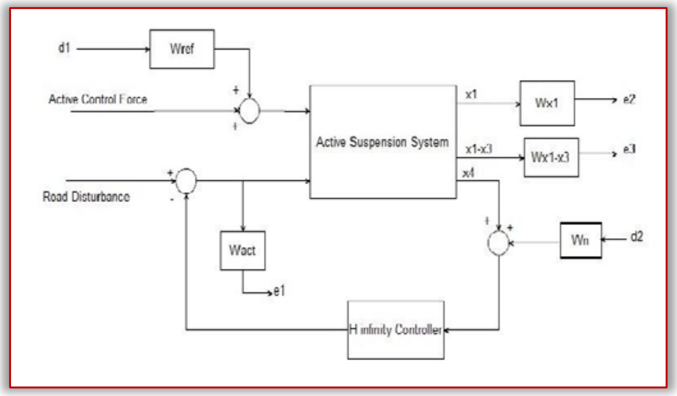

Figure 7. H ∞ system interconnected block diagram

There are two purposes for the weighted functions norm: for a given norm, there will be a direct comparison for different performance objectives and they are used for knowing the frequency information incorporated into the analysis. The output or feedback signal y is

$$y = (x\,4 + d2 * W_n) * H\infty \text{ Controller}$$

The controller's acts on the y signal to produce the road disturbance signal. The W_n block modelled the sensor noise in the channel. W_n is given a sensor noise of 0.05 m.

$$W_n = 0.05$$

W_n is used to model the noise of the displacement sensor. The magnitude of the active control force is scaled using the weight W_{ref}. Let us assume the maximum control force is 0.1Newton which means

$$W_{ref} = 0.1$$

The weighting function W_{act} is used to limit the magnitude and frequency content of the input road disturbance signal. Choosing

$$W_{act} = \frac{80}{11}\frac{s+60}{s+600}$$

— **H infinity Controller Design G_{c1} (s):**

W_{x1} and W_{x1-x3} are used to keep the car deflection and the suspension deflection small over the desired range. The car body deflection W_{x1} is given as

$$W_{x1} = \frac{508.1}{s+56.55}$$

The suspension deflection is used via weighting function

W_{x1-x3}. The weighting function is given as

$$W_{x1-x3} = \frac{15}{0.2s+1}$$

— **The Proposed μ-Synthesis Control Design**

In the active suspension system, μ-synthesis design included the hydraulic actuator dynamics. In order to account for the difference between the actuator model and the actual actuator dynamics, we used a first order model of the actuator dynamics as well as an uncertainty model. The μ-synthesis interconnection block diagram for active suspension system is shown in Figure 8.

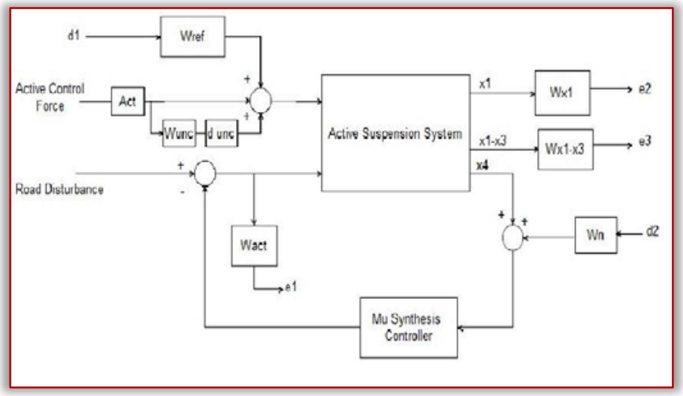

Figure 8. μ-synthesis interconnection block diagram

— **µ-Synthesis Controller Design S_{c1} (s):**

The nominal model for the hydraulic actuator is

$$HYD_{act} = \frac{1}{\frac{1}{50}s+1}$$

We describe the actuator model error as a set of possible models using a weighting function because the actuator model itself is uncertain. The model uncertainty is represented by weight W_{unc} which corresponds to the frequency variation of the model uncertainty and the uncertain LTI dynamics object 4_{unc} which is Unc=Uncertain LTI dynamics "*unc*" with 1 outputs, 1 inputs, and gain less than 1.

$$W_{unc} = \frac{0.03s+0.15}{0.001667s+1}$$

The uncertain actuator model represents the model of the hydraulic actuator used for control. A μ-synthesis controller is synthesized using D-K iteration. The D-K iteration method is an approximation to synthesis that attempts to synthesize the controller. There is two control input the road disturbance signal and the active control force. There are three measurement output signals, the suspension deflection, car body acceleration and car body travel.

RESULT AND DISCUSSION

— **Simulation of the Proposed Controllers**

In this subsection we simulate passive suspension system, active suspension system with G_{C1} (*s*) controller, active suspension system with S_{C1} (*s*) controller and active suspension system without controller for suspension deflection, body acceleration and body travel using bump, random, sine pavement and slope road disturbances.

» **Simulation of a Bump Road Disturbance:**

The Simulink model for a bump input road disturbance and active control force input is shown in Figure 9. In this Simulink model, we simulate passive suspension system, active suspension system with G_{C1} (*s*) controller, active suspension system with S_{C1} (*s*) controller and active suspension system without controller for suspension deflection, body acceleration and body travel. Here in this Simulink we assign d1 and d2 as a random signal with amplitude of 0.001 and period of 10 seconds and there are three error signals named e1, e2 and e3 and there are three active control force for the active suspension with controller and without controller with a step function input.

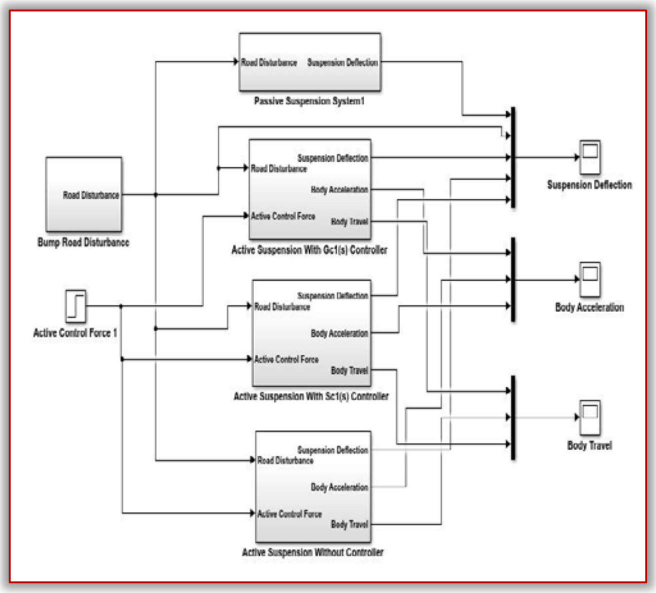

Figure 9. Simulink model of a Bump road disturbance

The suspension deflection, body acceleration and body travel simulation output is shown in Figure 10, Figure 11 and Figure 12 respectively for a bump road disturbance and active control force inputs.

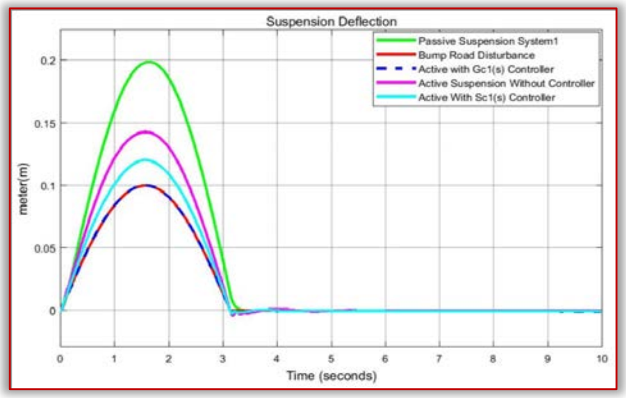

Figure 10. Suspension deflection for Bump road disturbance

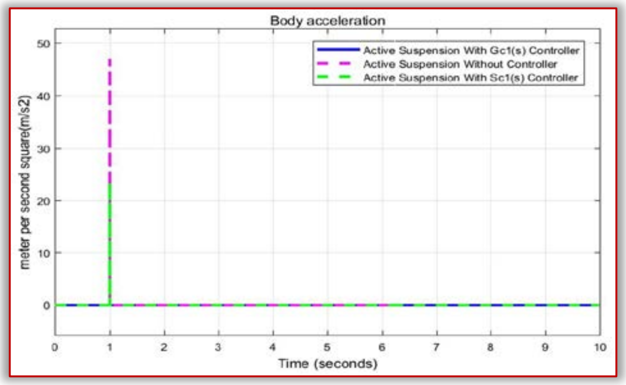

Figure 11. Body acceleration for Bump road disturbance

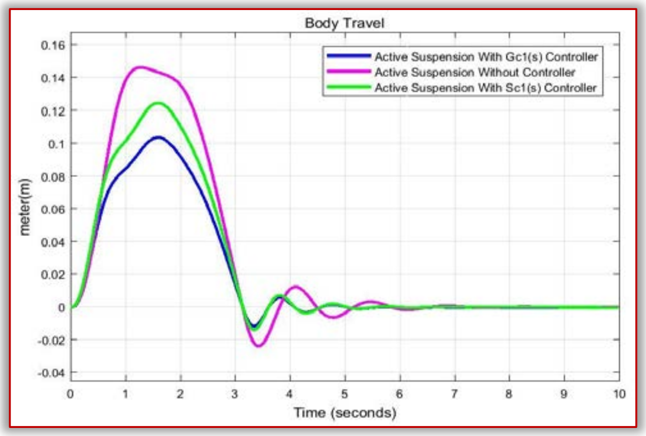

Figure 12. Body travel for Bump road disturbance

» **Simulation of a Random Road Disturbance:**

The Simulink model for a random road disturbance and active control force inputs is shown in Figure 13.The suspension deflection, body acceleration and body travel simulation is shown in Figure 14, Figure 15 and Figure 16 respectively.

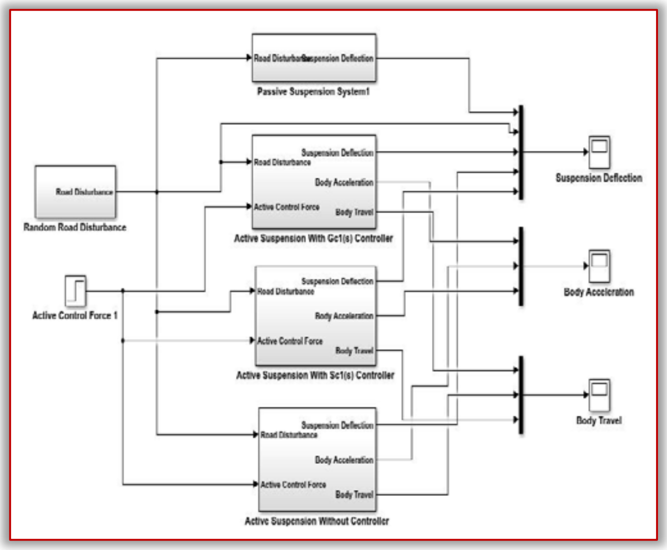

Figure 13. Simulink model for a Random road disturbance

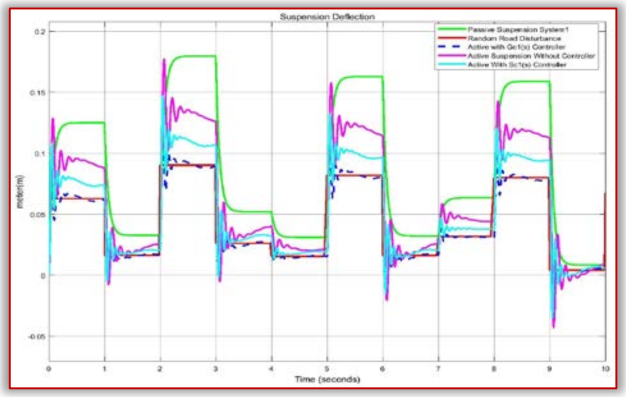

Figure14.Suspension deflection for Random road disturbance

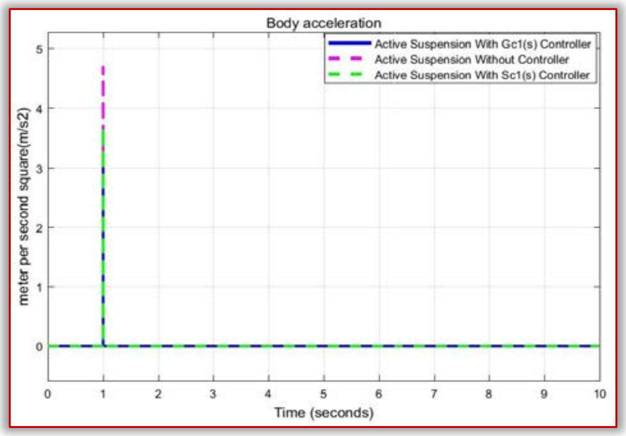

Figure 15. Body acceleration for Random road disturbance

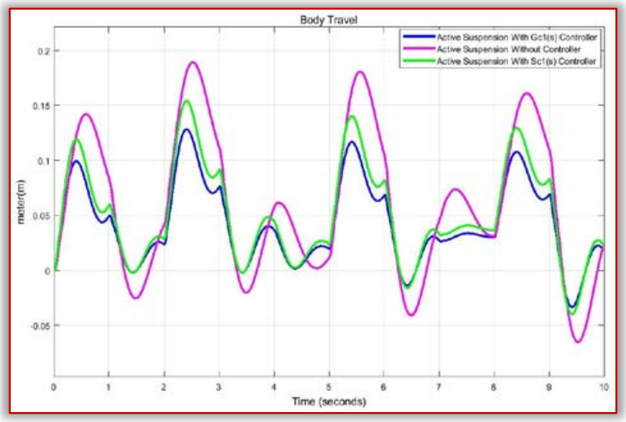

Figure 16. Body travel for Random road disturbance

» **Simulation of a Sine Input Pavement Road Disturbance:**

The Simulink model for a sine input pavement road disturbance and active control force inputs is shown in Figure 17. The suspension deflection, body acceleration and body travel simulation is shown in Figure 18, Figure 19 and Figure 20 respectively.

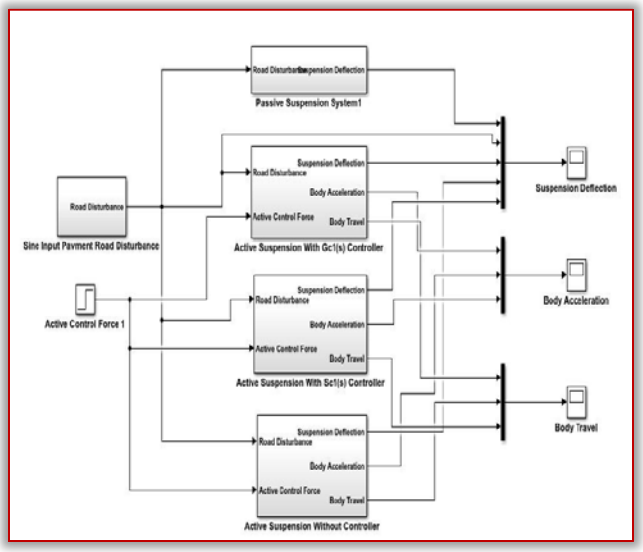

Figure 17. Simulink model for a Sine input pavement road disturbance

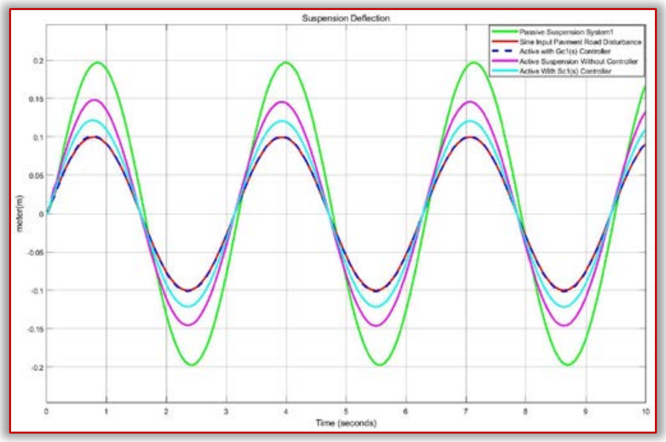

Figure 18. Suspension deflection for Sine input pavement road disturbance

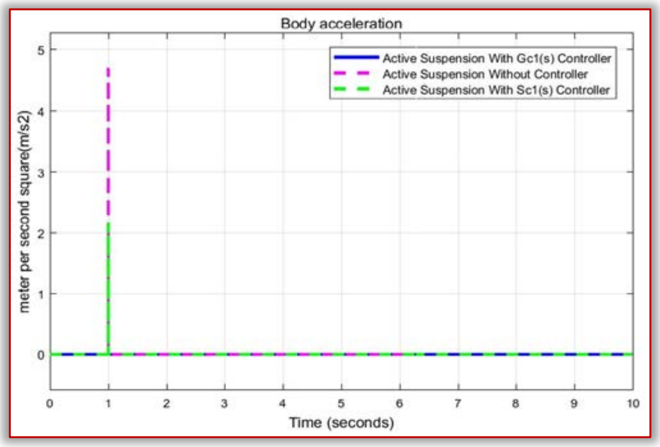

Figure 19. Body acceleration for Sine input pavement road disturbance

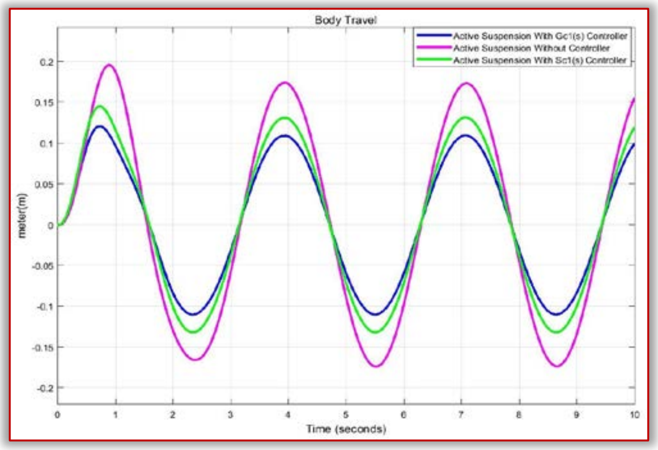

Figure 20. Body travel for Sine input pavement road disturbance

» **Simulation of a Slope Road Disturbance:**

The Simulink model for a slope road disturbance and active control force inputs is shown in Figure 21.The suspension deflection, body acceleration and body travel simulation is shown in Figure 22, Figure 23 and Figure 24 respectively.

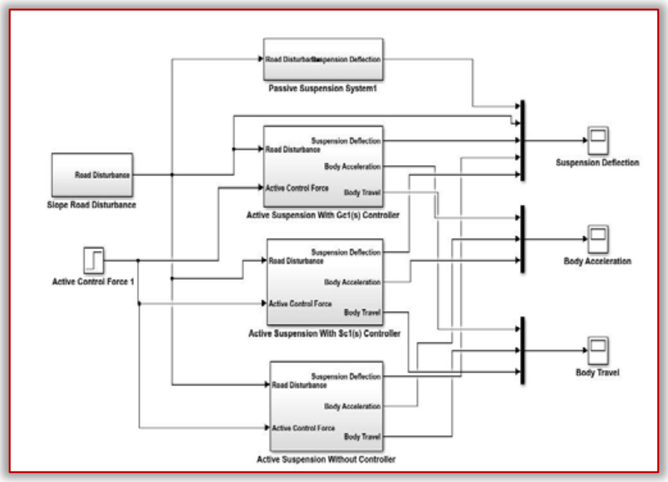

Figure 21. Simulink model for a Slope road disturbance

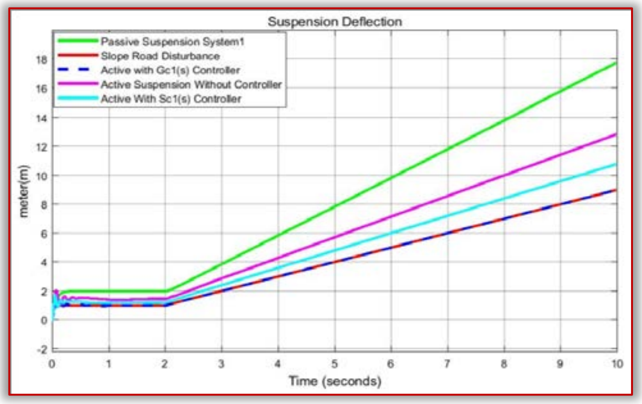

Figure 22. Suspension deflection for Slope road disturbance

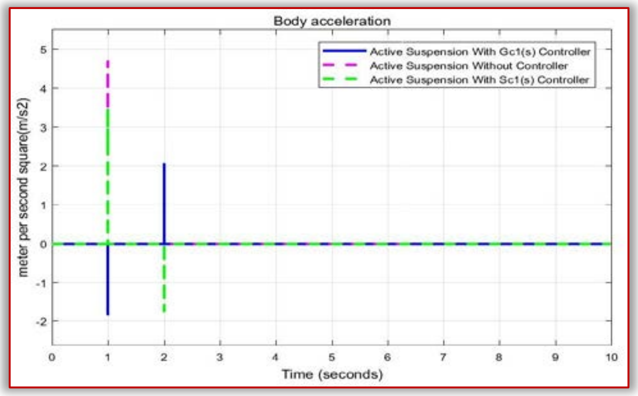

Figure 23. Body acceleration for Slope road disturbance

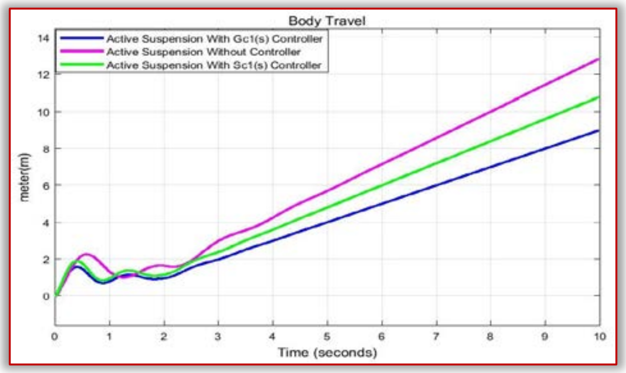

Figure 24. Body travel for Slope road disturbance

— **Comparison of Active Suspension System With H∞ G_{C1} (s) and μ–Synthesis S_{C1} (s) Controllers**

Here in this section, we compare active suspension system with *H∞* controller (*G_{C1} (s)*) and *μ*–synthesis controller (*S_{C1} (s)*) for suspension deflection, body acceleration and body travel with bump, random, sine and slope road disturbances.

» **Comparison for Bump Road Disturbance:**

In the suspension deflection simulation as shown in Figure 10, the active suspension system with *S_{C1} (s)* controller strokes are larger than the road surface wave amplitude while the active suspension system with *G_{C1} (s)* controller strokes fits the road surface wave amplitude. In the body acceleration as shown in Figure 11, the acceleration is effectively reduced in the active suspension system with *G_{C1} (s)* controller. In the body travel as shown in Figure 12, the vertical distance that the body travels is effectively reduced in the active suspension system with *G_{C1} (s)* controller. The reduction in overshoot value is shown in Table 2.

Table 2 Reduction in overshoot value for bump road disturbance

Parameters	S_C1 (s)	G_C1 (s)	% in Reduction
Suspension Deflection	0.13 m	0.1 m	23.08 %
Body Acceleration	$24\frac{m}{s^2}$	$5\frac{m}{s^2}$	79.2 %
Body Travel	0.13m	0.11 m	15.38 %

» **Comparison for Random Road Disturbance:**

In the suspension deflection simulation as shown in Figure 14, the active suspension system with *S_{C1} (s)* controller strokes have a larger amplitude than the active suspension system with *G_{C1} (s)* controller. In the body acceleration as shown in Figure 15, the acceleration is effective reduced in the active suspension system with *G_{C1} (s)* controller. In the body travel as shown in Figure 16, the vertical distance that the body travels is effectively reduced in the active suspension system with *G_{C1} (s)* controller. The reduction in overshoot value is shown in Table 3.

Table 3 Reduction in overshoot value for random road disturbance

Parameters	S_C1 (s)	G_C1 (s)	% in Reduction
Suspension Deflection	0.18 m	0.13 m	27.78 %
Body Acceleration	$3.7\frac{m}{s^2}$	$3\frac{m}{s^2}$	19 %
Body Travel	0.16 m	0.13 m	18.75

» **Comparison for Sine Pavement Road Disturbance:**

In the suspension deflection simulation as shown in Figure 18, the active suspension system with *S_{C1} (s)* controller strokes are larger than the road surface wave amplitude while the active suspension system with *G_{C1} (s)* controller strokes fits the road surface wave amplitude. In the body acceleration as shown in Figure 19, the acceleration is effectively reduced in the active suspension system with *G_{C1} (s)* controller. In the body travel as shown in Figure 20, the vertical distance that the body travels has a large amplitude in the active suspension system with *S_{C1} (s)* controller and is effectively reduced in the active suspension system with *G_{C1} (s)* controller. The reduction in overshoot value is shown in Table 4.

Table 4 Reduction in overshoot value for sine pavement road disturbance

Parameters	S_C1 (s)	G_C1 (s)	% in Reduction
Suspension Deflection	0.13 m	0.1 m	23.08 %
Body Acceleration	$2.\frac{m}{s^2}$	$2.\frac{m}{s^2}$	4.56 %
Body Travel	0.14 m	0.12 m	14.3 %

» **Comparison for Slope Road Disturbance:**

In the suspension deflection simulation as shown in Figure 22, the active suspension system with *S_{C1} (s)* controller slopes are larger than the road surface wave amplitude while the active suspension system with *G_{C1} (s)* controller slope fits the road surface wave amplitude. In the body acceleration as shown in Figure 23, the acceleration is effectively reduced in the active suspension system with *G_{C1} (s)* controller. In the body travel as shown in Figure 24, the body travels has a large slope and vibration in the active suspension system with *S_{C1} (s)* controller and is effectively aligned with small vibration in the active suspension system with *G_{C1} (s)* controller. The reduction in overshoot value is shown in Table 5.

Table 5 Reduction in overshoot value for slope road disturbance

Parameters	S_C1 (s)	G_C1 (s)	% in Reduction
Suspension Deflection	51.3°	45°	12.3 %
Body Acceleration	3.5 $\frac{m}{s^2}$	$2\frac{m}{s^2}$	43 %
Body Travel	51.3°	45°	12.3 %

CONCLUSION

In this paper, H ∞ controller and μ - synthesis controllers are successfully designed using MATLAB/SIMULINK for quarter car active suspension system. We design a Simulink model that represents the active suspension system with H ∞ controller, μ - synthesis controller, without controller and passive suspension system and tasted with bump, sine input pavement, random and slope road disturbances for suspension deflection, body acceleration and body travel. We compared the active suspension system with H ∞ controller and μ - synthesis controller for the three parameters and we analyze the percentage reduction in overshoot of the two controllers.

The simulation results shows that the active suspension system with H ∞ controller is capable of stabilizing the suspension system very effectively than the active suspension system with μ - synthesis controller for suspension deflection, body acceleration and body travel parameters with the four road input disturbances. The system with H ∞ controller has a percentage reduction in overshoot than a system with μ - synthesis controller.

We conclude that an active suspension system with H ∞ controller has the best performance with the different tests we made on the system and it achieves the passenger comfort and road handling criteria that it needed to make the active suspension system is the best suspension system.

Acknowledgments

First and formost, I would like to express my deepest thanks and gratitude to Dr.Parashante and Mr.Tesfabirhan for their invaluable advices, encouragement, continuous guidance and caring support during my journal preparation.

Last but not least, I am always indebted to my brother, Taha Jibril, my sister, Nejat Jibril and my family members for their endless support and love throughout these years. They gave me additional motivation and determination during my journal preparation.

References

[1] Nasir, Ahmed and Al-awad "Genetic Algorithm Control of Model Reduction Passive Quarter Car Suspension System" I.J. Modern Education and Computer Science, 2019.

[2] Vivek Kumar Maurya and Narinder Singh Bhangal "Optimal Control of Vehicle Active Suspension System" Journal of Automation and Control Engineering Vol. 6, No. 1, 2018.

[3] Michiel Haemers "Proportional-Integral State-Feedback Controller Optimization for a Full-Car Active Suspension Setup using a Genetic Algorithm" The 3rd IFAC Conference on Advances in Proportional Integral-Derivative Control, Ghent, Belgium, 2018.

[4] Vinayak S. Dixit and Sachin C. Borse "SemiactiveSsuspensionSsystem Design for QuarterCcar Model and itsAanalysis with Passive Suspension Model" International Journal of Engineering Sciences & Research Technology, 2017

[5] Yakubu G.1, Adisa A. B., "Simulation and Analysis of Active Damping System for Vibration Control", American Journal of Engineering Research, Vol. 6, Issue 11, 2017.

[6] Ahmed A. Abougarair and Muawia M. A. Mahmoud "Design and Simulation Optimal Controller for Quarter Car Active Suspension System" 1st Conference of Industrial Technology (CIT2017), 2017.

[7] J. Marzbanrad and N. Zahabi "H∞ Active Control of a Vehicle Suspension System Exited by Harmonic and Random Roads" Journal of Mechanics and Mechanical Engineering Vol. 21, No. 1 pp 171–180, 2017.

[8] Shital M. Pawar and A.A. Panchwadkar "Estimation of State Variables of Active Suspension System using Kalman Filter" International Journal of Current Engineering and Technology E-ISSN 2277 – 4106 , P-ISSN 2347 – 5161,Vol.7, No.2, 2017.

[9] Libin Li and Qiang Li "Vibration Analysis Based on Full Multi Body Model for a Commercial Vehicle Syspension System"ISPRA'07 Proceeding of the 6th WSEAS International, 2016.

[10] Narinder Singh"Robust Control of Vehicle Active Suspension System" International Journal of Control and Automation Vol. 9, No. 4 (2016), pp. 149-160, 2016.

[11] Panshuo Li, James Lam, Kie Chung Cheung "Experimental Investigation of Active Disturbance Rejection Control for Vehicle Suspension Design"International Journal of Theoretical and Applied Mechanics Volume 1, 2016.

[12] Ahmed S Ali, Gamal A.Jaber, Nouby M Ghazaly, "H∞ Control of Active Suspension System for a Quarter Car Model", International Journal of Vehicle Structures and Systems Vol.8, No.1, 2016.

[13] M. P. Nagarkar and G. J. Vikhe Patil "Multi-Objective Optimization of LQR Control Quarter Car Suspension System using Genetic Algorithm" FME Transactions, 44, 187-196, 2016.

ACTA TECHNICA CORVINIENSIS – Bulletin of Engineering
ISSN: 2067-3809