

BUCKLING ANALYSIS OF SIMPLY SUPPORTED SQUARE SYMMETRIC LAMINATED COMPOSITE PLATE

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Abstract: To use the laminated composite plates efficiently, it is necessary to develop appropriate analysis theories to predict accurately their structural and dynamical behavior. The analysis of the behavior of the laminated plates is an active research area because of their complex behavior. The structural instability becomes an important concern in a reliable design of composite plates. The majority of the investigations on laminated plates utilize either the classical lamination plate theory (CLPT), or the first-order shear deformation theory (FSDT). Various geometries of the plates subjected to compressive load are studied. The present work deals about a buckling analysis of simply supported symmetric composite plate with four layers. It is assumed that composite plate is surrounded by external elastic foundation. Composite plate is modeled by the finite element method and subjected to biaxial compression load. Governing equations are derived based on Classical Laminated Plate Theory (CLPT) and computed critical buckling loads were compared with numerical results.

Keywords: buckling; symmetric composite plate; orientation of layers; compression load

INTRODUCTION

A composite laminate is composed of reinforcement (fibres, particles, flakes, and/or fillers) embedded in a matrix (polymers, metals, or ceramics). A laminate is called symmetric if the material angle, and the thickness of plies are the same above and below the midplane. The matrix holds the reinforcement to form the desired shape while the reinforcement improves the overall mechanical properties of the matrix.

To use the laminated composite plates efficiently, it is necessary to develop appropriate analysis theories to predict accurately their structural and dynamical behavior. The analysis of the behavior of the laminated plates is an active research area because of their complex behavior. The structural instability becomes an important concern in a reliable design of composite plates. Several studies on laminated plates stability were concentrated on rectangular plates [1-3]. It is known that buckling strength of the rectangular plates depends on the boundary conditions, plies orientation and geometrical ratio [2-4]. The thin composites structures which are largely used become unstable when they are subjected to mechanical or thermal loadings which leads to buckling. The buckling of the composite plates is a very complicated subject and more details can be seen in references [1-4]. To predict buckling load and deformation mode of a structure, the linear analysis can be used as an evaluation technique [5]. The buckling analysis of rectangular laminated composite plate with and without cutouts for the effects of fiber angle orientation and cutout shapes on critical buckling load are determined [12]. The effect of aspect ratio, orthotropic ratio and fiber orientation for antisymmetric laminated composite plates subjected to in plane loading are discussed to obtain critical buckling load [13].

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geometries of the plates subjected to compressive load are studied. In [10] buckling analysis of the laminated composites is performed by using finite element analysis software ANSYS. Buckling analysis of a simply supported rectangular plate subjected to various types of non-uniform compressive loads has been studied [15]. The effect of fibre orientation on buckling behavior in a rectangular composite laminate with central circular hole under uniform in-plane loading has been studied by using finite element method [16].

In this paper, buckling behavior of symmetric laminated composite plates under biaxial compression load using ANSYS software is studied. The main contribution of this work is to perform a composite laminated plates analysis by using the Classical Laminated Plate Theory (CLPT) and ANSYS ACP. The ANSYS results are validated with the results predicted by classical laminated plate theory. The composite plate is modeled as shell model and then it is loaded by compression load. Then the obtained critical loads are compared for two different orientations of layers calculated using CLPT.

The laminated composite plates are thin shell elements composed of fibre lamination and epoxy resin is used to bond the lamina. The strength of these composite plates depended on type and properties of fibre material used along with epoxy resin. In structural designing field ANSYS, COSMOS, ABAQUS and so on are the most used finite element analysis software's. ANSYS is most trusted finite element software as it provides ease of work to analyse the laminated composite plates under buckling in biaxial loading. Where as in experimental study the biaxial loading is complicate to perform and requires energy and resources.

THEORETICAL FORMULATION

The buckling analysis of the symmetric square composite plate with 4 plies made of two types of materials and two different layouts of plies is performed (Figure 1).

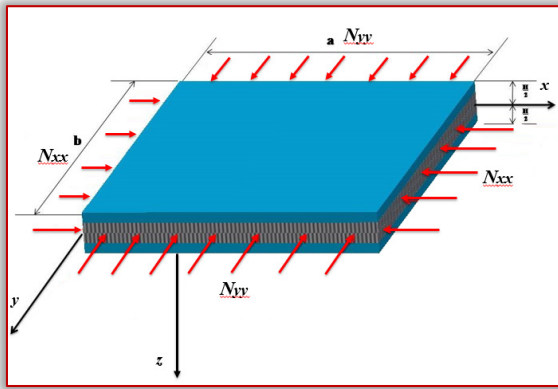


Figure 1. Symmetric square composite plate with 4 plies

The material properties of composite plate are kevlar 49/CE 3305 as material 1 (M1): $E_1 = 82$ GPa, $E_2 = 4$ GPa, $G_{12} = 2.8$ GPa, $\nu_{12} = 0.25$ and graphite-epoxy AS-1/3501-5A as material 2 (M2): $E_1 = 127.6$ GPa, $E_2 = 11$ GPa, $G_{12} = 4.5$ GPa, $\nu_{12} = 0.25$

The thickness of one ply is $h = 0.25$ mm. The symmetric orientations of plies α in composite are defined as M10/M230/M230/M10 and M20/M130/M130/M20.

For laminates of total thickness of 1mm with four sheets of individual thickness of 0.25mm, bending stiffness matrix D has the following form [8]:

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})^k (h_k^3 - h_{k-1}^3)$$

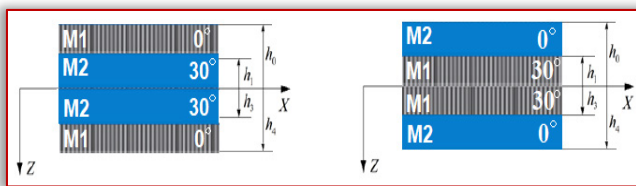


Figure 2. Symmetric orientation of plies and bending stiffness matrix

Based on the above material properties and using the MATLAB software package, bending stiffness matrix for selected laminate schemes $\theta = 0^\circ, 30^\circ$ are obtained.

$$[D] = \begin{bmatrix} 6,7954 & 0,3167 & 0,3990 \\ 0,3167 & 0,4800 & 0,1298 \\ 0,3990 & 0,1298 & 0,4841 \end{bmatrix} \quad [D] = \begin{bmatrix} 9,8649 & 0,3548 & 0,2588 \\ 0,3548 & 0,9093 & 0,0941 \\ 0,2588 & 0,0941 & 0,5000 \end{bmatrix}$$

— Governing equations of biaxially compressed composite plate

The governing equation for biaxially compressed orthotropic composite plate [14], which is based on Classical Laminated Plate Theory (CLPT), have following form

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \quad (1)$$

We assume that composite plate is biaxially compressed in the directions of x and y axes, $N_x = N_y$. Now we can define compression ratio which equals the ratio between the forces acting in y and x directions

$$\delta = \frac{N_{yy}}{N_{xx}} \rightarrow N_{yy} = \delta N_{xx} \quad (2)$$

Substitution of equation (2) in equation (1) we derive the general form of governing equation

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + N_x \left(\frac{\partial^2 w}{\partial x^2} + \delta \frac{\partial^2 w}{\partial y^2} \right) = 0 \quad (3)$$

It is assumed that all edges on composite plate are simply supported. This means that both the displacements and moments at the composite plate edges are zero. This can be expressed by following equations

$$w_i(0, y, t) = 0, \quad w_i(a, y, t) = 0, \\ w_i(x, 0, t) = 0, \quad w_i(x, b, t) = 0 \quad i = 1, 2 \quad (4)$$

$$M_i(0, y, t) = 0, \quad M_i(a, y, t) = 0, \quad M_i(x, 0, t) = 0, \quad M_i(x, b, t) = 0 \quad (5)$$

We assume that the buckling mode of the composites system as

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin(\alpha x) \sin(\beta y) \quad (6)$$

In the upper equation:

$$\alpha = \frac{m\pi}{a}, \\ \beta = \frac{n\pi}{b} \quad (7)$$

where m and n are the half wave numbers.

Substituting equation (6) into equation (3), we get critical buckling load

$$N_{cr} = \frac{D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4}{(\alpha^2 + \delta\beta^2)} \quad (8)$$

Each composite plate had the length, a and width b . We assume that composite plates are biaxially compressed by forces N_{xx} and N_{yy} in the directions of x and y axes (Figure 3).

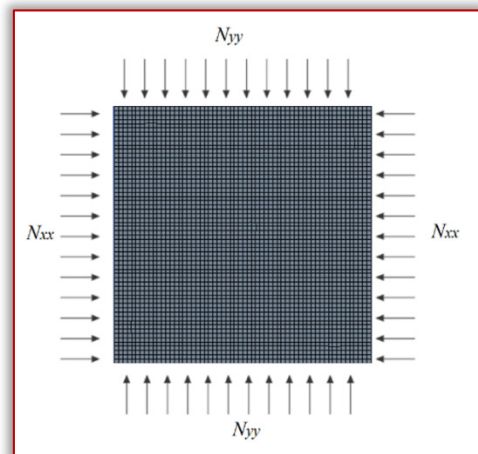


Figure 3. Composite plate loaded by biaxial compression load

RESULTS AND DISCUSSION

Non-dimensional buckling load was calculated for the number of half waves $m=1, n=1$ and $m=2, n=2$, while the compression ratio was $\delta=1$. The thickness of one composite plate is $h = 0,25$ mm, while the length and width take values $a=0,3m$ and $b=0,3$ (square plate). We investigated buckling behavior of square symmetric composite plate under biaxial

compression load using Classical Laminated Plate Theory (CLPT) and computed critical buckling loads were compared with results obtained in ANSYS 19.2 ACP (Ansys Composites PrepPost). The present ANSYS model validation has been obtained for two different laminated composite plates and are presented in Table 1.

Table 1. Validate the ANSYS results with reference [9] for 4-layer symmetric square plate

Biaxial compression		Ncr		
Composite plate	Half wave numbers	CLPT	ANSYS	% error
M10/M230/M230/M10	m=1 n=1	7,1496	5,288	26
	m=2 n=2	28,5982	21,673	24
M20/M130/M130/M20	m=1 n=1	6,7451	7,0868	4,82
	m=2 n=2	26,980	28,738	6,11

The present ANSYS ACP model (Figure 4 and Figure 5) is being validate by comparing the results with references [9] by taking the same material properties, geometrical parameters and boundary conditions.

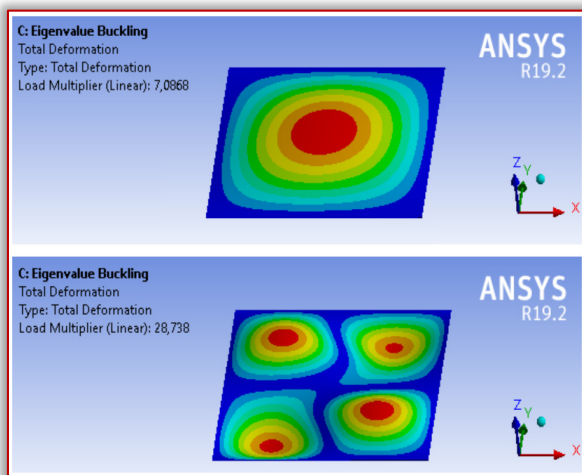


Figure 4. ANSYS results for laminate M10/M230/M230/M10

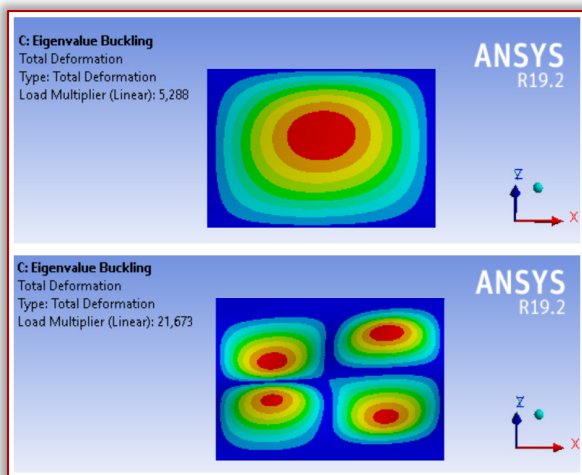


Figure 5. ANSYS results for laminate M20/M130/M130/M20

In Table 1 the present ANSYS results are showing good agreement for laminate M20/M130/M130/M20 and the values are max 6,11% error compared to the reference [9]. The values for laminate M10/M230/M230/M10 are higher with max 26% error compared to the reference [9]. It is because of the fact that the present model is developed in the finite element analysis software ANSYS whereas in the reference the model is developed based on analytical solution using CLPT. For biaxial compressive loading the values are obtained for all edges simply supported (SSSS) boundary condition. The critical buckling load of the composite plate is almost the same for all two laminates.

CONCLUSION

The paper studied the buckling behavior of the square symmetric composite plates with two different layer orientations and four layers. The all composite plates were modeled using finite element method in ANSYS 19.2 ACP. The finite element model of composite plate consisted from shell elements, which had defined the material of composite plate, the thickness of composite plate and layout of layers. The composite plates were loaded by biaxial compression load. The boundary conditions on the parallel edges with x and y were applied. The computed critical buckling loads for all configurations showed that:

- ≡ the orientations of composite layers effect the value of critical buckling load, for example the composite plate with orientation of layers M20/M130/M130/ M20 is more sensitive to compression load than the composite plate with orientation of layers M10/M230/M230/ M10
- ≡ the buckling shapes are slightly effected by position of layer in composite plate.
- ≡ for value of half wave m=2 and n=2 we get higher value of non-dimensional buckling load.

It has been shown that with the change of layer position and angle of fiber orientation the value of the non-dimensioning critical load is changed. Laminate have different minimum and maximum values of non-dimensional critical force.

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