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# COST OPTIMIZATION OF A DOUBLY REINFORCED CONCRETE BEAM

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**Abstract:** In this research, the cost optimization of a simply supported doubly reinforced concrete beam with uniformly distributed load was performed under constraints from Eurocode 2. This research presents a relationship between the components of the reinforced concrete beam, its resultant manufacturing costs and the optimization model developed to minimize such costs. This relationship was constrained geometrically by real life estimates and behaviourally by conventions defined in Eurocode 2. Whilst the cost optimization procedure was carried out using Microsoft Excel, results from further analysis showed a direct relationship between the span, optimized costs and original costs. The relationship observed for the concrete class however, showed that an increase in the grade of concrete led to a decrease in the optimized and original costs.

**Keywords:** cost optimization, doubly reinforced concrete beam, Eurocode 2, optimization, real life estimates

## INTRODUCTION

Reinforced concrete (RC), is one of the most commonly used materials for the construction of the built environment, and with that, reinforced concrete beams are one of the most ubiquitous structural element present in everyday life as every structure experiences flexure due to a combination of permanent (dead) and variable (live) loads. However, despite their common presence, very few of them are designed with the cost of their construction being put among top considerations.

The conventional design process of reinforced concrete members does not take into account the cost of the structural element. Usually a trial and error process is applied given that structural design is an iterative process. The first step in the design process is usually the making of the initial design, after which the designer makes an overall guess about the possible optimum solution consistent with his or her experience, knowledge, constraints, and requirements. The analysis of the structure is then carried out using initial design. Based on the results of the analysis, a re–design of the structure is carried out if any of the constraints is not satisfied. The efficiency of the design process depends heavily on the quality of the initial guess, which, if good, will reduce the number of analysis–design cycles. In the real–life design of the structures, it is inevitable to take into account the cost of the beams at some point when the structure is to be constructed. There is an absence of cost consideration when designing the structural element.

Related concepts to the focus of this research have been explored in previous literature. As seen in Ildiko, *et al.*, (2010), which used the nonlinear programming approach (NLP) to optimize RC beams of rectangular cross–section. The objective function of the beam's construction costs was derived, including material and labor cost items. However, the paper did not provide a comprehensive account of how the optimization was carried out on the objective function with respect to the derived constraints.

Khaled, *et al.*, (2004), which used STAAD III to design safe cross sections and Microsoft Excel to calculate steel and concrete quantities. A sensitivity analysis was also performed on the model.

Bhalchandra and Adsul (2012) conducted a cost optimized design of doubly reinforced beams with uniformly distributed and concentrated load. The results showed that the Genetic Algorithm technique generated a cost less than the GRG and Interior Point optimization techniques.

Galeb (2018) focused on achieving optimization objectives through simulated annealing, which mimics the thermal annealing of heating solids critically. When the temperature is reduced, the atoms tend to be ordered and form crystals with the minimum possible internal energy. This implies that the optimum cost of the beam always takes the minimum bounds of the specified constraints.

Antunes (2017) focused on optimizing shell structures using Building Information Modelling software. The heuristic used evaluates the possible solutions and selects the most suitable solutions. BIM allows for full integration between design and fabrication processes, as well as the ability to store and manipulate multiple layers of object–oriented information.

The research by Salim, *et al.*, (2018) demonstrates the cost minimization of both singly and doubly reinforced concrete rectangular beam sections through the use of the Artificial Neural Networks Application. The derivation of cost coefficients for concrete and steel was not discussed.

With the background of the research and its literature explored, the scope of the research work is limited to the design of the beam with Eurocode 2 and optimizing it with an algorithm applied via an Excel spreadsheet and the thorough testing of the model.

The design of a doubly reinforced concrete beam with Eurocode 2 at ultimate limit state, with constraints placed by the code was the chosen approach. There was a mathematical modelling of the structure as a cost optimization problem. Microsoft Excel was used to run the numerical process to optimize the modelled cost.

The cost model was developed with a consideration of materials alone regardless of labour involved. An evaluation of the cost optimization procedure of the structure was limited with respect to an increasing span and various concrete classes.

As described earlier, cost considerations are often lacking when designing structural elements such as in this case, a doubly reinforced beam. In addition, most optimization methods tend to be only applied in concrete design and mixture proportions, with most of these methods not considering the costs of plain concrete, reinforcement and formwork costs as functions. These actions often have consequences later on in the life cycle of the project, as unforeseen changes can wield considerable influence over the cost of constructing the structural element. However, a great influence on cost can be achieved at the initial phases of the life cycle of the project.

In this paper, the cost optimization of a doubly reinforced concrete beam is carried out using the moment constraint of Eurocode 2 (2004).

## MATERIALS AND METHODS

### ▦ Materials

Eurocode 2 (2004) was used to develop a mathematical representation of a concrete structure. An Excel spreadsheet was used to set up the model and the optimization process was executed using Excel's Solver Tool.

### ▦ Methods

#### — Development of the Objective Function

The total cost of constructing the doubly RC beam, is the sum of material costs required for the fabrication of certain constituents and construction of the entire member (Ildiko, *et al.*, 2010). Figure 1 shows the cross–section of a doubly reinforced concrete beam.

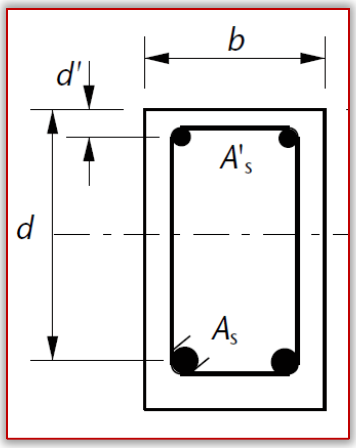


Figure 1: Section of a doubly reinforced concrete beam.

The objective function was derived as follows:

$$\text{Total Cost } (C_T) = c_c[(bh - (A_{sc} + A_{st})) \times L \times u_c] + c_s[A_{sc} + A_{st}] + c_f[b + 2h] \qquad (1)$$

where:

$C_T$ = Total cost of manufacturing the singly reinforced concrete beam.

$C_c$ = Cost coefficient of concrete in cost per mass (naira per kg).

$C_s$ = Cost coefficient of reinforcement steel, in cost per cross–sectional area (naira per mm²).

$C_f$ = Cost coefficient of formwork, in cost per length (naira per m).

b = Width of the beam (mm).

h = Height of the beam (mm).

d = Effective depth of tension reinforcement bar (mm).

$d^l$ = Effective depth of compression reinforcement bar (mm).

$A_{st}$ = Total area of tension reinforcement steel (mm²).

$A_{SC}$ = Total area of compression reinforcement steel (mm²).

L = Length of the beam (m).

$U_C$ = Unit weight of concrete (kg/m³).

#### — Design Constraints

The stress–strain diagram for the doubly reinforced concrete section is shown in Figure 2. The constraints developed included the behavioural constraints and geometric constraints. Given the nature of the structure, flexural constraints were the focus of the design constraints. The geometric constraints were developed from realistic values of doubly reinforced beams in use.

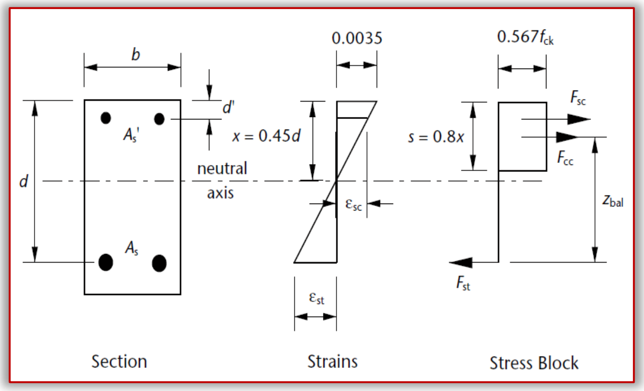


Figure 2: A doubly reinforced concrete section with strain diagram and stress block.

Behavioural constraints:

$$M_{bal} \leq M \leq M_R \quad (2)$$

$$K > 0.167 \quad (3)$$

$$\frac{x}{d} < 0.45 \quad (4)$$

Geometric constraints:

$$\frac{d^I}{d} \leq 0.171 \quad (5)$$

$$150 \leq b \leq 350 \quad (6)$$

$$400 \leq h \leq 700 \quad (7)$$

$$1500 \leq A_{st} \leq 2500 \quad (8)$$

$$300 \leq A_{sc} \leq 800 \quad (9)$$

## Optimization Model Formulation

With the objective function, constraints and the required input parameters defined, the model was summarized in the following mathematical format:

$$\text{Minimize: Total Cost}, C_T = c_c\left[\left(bh - (A_{sc} + A_{st})\right) \times L \times u_c\right] + c_s[A_{sc} + A_{st}] + c_f[b + 2h] \quad (10)$$

Subject to:

$$M_{bal} \leq M \leq M_R \quad (11)$$

$$K > 0.167 \quad (12)$$

$$\frac{x}{d} < 0.45 \quad (13)$$

$$\frac{d^I}{d} \leq 0.171 \quad (14)$$

$$150 \leq b \leq 350 \quad (15)$$

$$400 \leq h \leq 700 \quad (16)$$

$$1500 \leq A_{st} \leq 2500 \quad (17)$$

$$300 \leq A_{sc} \leq 800 \quad (18)$$

To find:

$$X = [X_1 X_2 X_3 X_4]^T$$

where:

$b = X_1$

$h = X_2$

$A_{st} = X_3$

$A_{sc} = X_4$

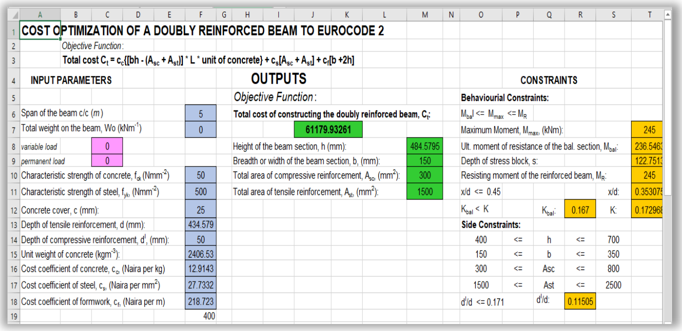
The optimized values of the aforementioned design variables are obtained by optimization.

## Optimization Process

The mathematical expressions described above for the model was replicated in Microsoft Excel using a spreadsheet. This was then optimized with Excel's Solver Tool, after which the required results were recorded.

## Development of the Excel Spreadsheet

The objective function, input parameters, design parameters, computed values, constraints and their aforementioned formulas were appropriately placed in the Excel spreadsheet shown in Figure 3, with the corresponding Solver dialogue box in Figure 4.



**COST OPTIMIZATION OF A DOUBLY REINFORCED BEAM TO EUROCODE 2**

*Objective Function:*

**Total cost $C_t = c_c\{[bh - (A_{sc} + A_{st})] * L * \text{unit of concrete}\} + c_s[A_{sc} + A_{st}] + c_f[b + 2h]$**

| INPUT PARAMETERS | | |
|---|---|---|
| Span of the beam c/c (*m*) | | 5 |
| Total weight on the beam, Wo (kNm$^{-1}$) | | 0 |
| *variable load* | 0 | |
| *permanent load* | 0 | |
| Characteristic strength of concrete, $f_{ck}$ (Nmm$^{-2}$) | | 50 |
| Characteristic strength of steel, $f_{yk}$, (Nmm$^{-2}$) | | 500 |
| Concrete cover, c (mm): | | 25 |
| Depth of tensile reinforcement, d (mm): | | 434.579 |
| Depth of compressive reinforcement, $d^I$, (mm): | | 50 |
| Unit weight of concrete (kgm$^{-3}$): | | 2406.53 |
| Cost coefficient of concrete, $c_c$, (Naira per kg) | | 12.9143 |
| Cost coefficient of steel, $c_s$, (Naira per mm$^2$) | | 27.7332 |
| Cost coefficient of formwork, $c_f$, (Naira per m) | | 218.723 |
| | | 400 |

**OUTPUTS**

*Objective Function:*

**Total cost of constructing the doubly reinforced beam, $C_t$:** 61179.93261

| | |
|---|---|
| Height of the beam section, h (mm): | 484.5795 |
| Breadth or width of the beam section, b, (mm): | 150 |
| Total area of compressive reinforcement, $A_{sc}$, (mm$^2$): | 300 |
| Total area of tensile reinforcement, $A_{st}$, (mm$^2$): | 1500 |

**CONSTRAINTS**

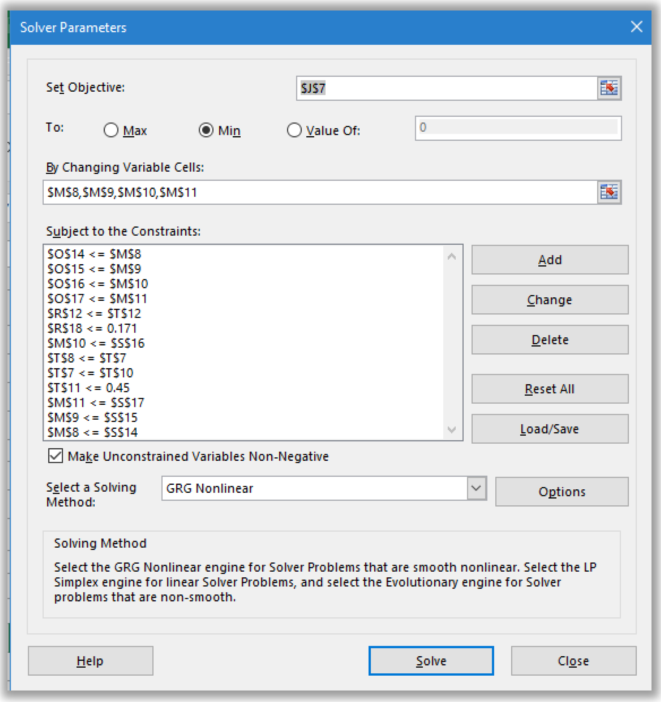
**Behaviourial Constraints:**

| | | | | |
|---|---|---|---|---|
| $M_{bal} <= M_{max} <= M_R$ | | | | |
| Maximum Moment, $M_{max}$, (kNm): | | | | 245 |
| Ult. moment of resistance of the bal. section, $M_{bal}$: | | | | 236.5463 |
| Depth of stress block, s: | | | | 122.7513 |
| Resisting moment of the reinforced beam, $M_R$: | | | | 245 |
| x/d <= 0.45 | | | x/d: | 0.353075 |
| $K_{bal}$ < K | $K_{bal}$: | 0.167 | K: | 0.172968 |

**Side Constraints:**

| | | | | |
|---|---|---|---|---|
| 400 | <= | h | <= | 700 |
| 150 | <= | b | <= | 350 |
| 300 | <= | Asc | <= | 800 |
| 1500 | <= | Ast | <= | 2500 |
| $d^I/d$ <= 0.171 | $d^I/d$: | 0.11505 | | |

Figure 3: Excel spreadsheet developed to perform cost optimization on a doubly reinforced concrete beam to Eurocode 2.



Set Objective: $J$7

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells: $M$8,$M$9,$M$10,$M$11

Subject to the Constraints:
$O$14 <= $M$8
$O$15 <= $M$9
$O$16 <= $M$10
$O$17 <= $M$11
$R$12 <= $T$12
$R$18 <= 0.171
$M$10 <= $S$16
$T$8 <= $T$7
$T$7 <= $T$10
$T$11 <= 0.45
$M$11 <= $S$17
$M$9 <= $S$15
$M$8 <= $S$14

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 4: Excel Solver dialogue box with the model's objective function, design variables and constraints in place.

## Generation of Results

After using the Solver dialogue box to solve the optimization model, the values in the cells containing the design variables were changed due to the success of the operation. This also led to a corresponding change in the cost of the beam. The previous values of both the objective function and design variables are recorded, as well as the corresponding values after the optimization process has been completed.

## RESULTS AND DISCUSSION

### Design Example

A selected beam of known dimensions was setup in the model for cost optimization. The relevant input parameters utilized for this are shown in the table below.

Table 1: Input Parameters for the case study used for the design example.

| Input Parameters | Values |
|---|---|
| Span of the beam c/c (*m*) | 5 |
| Maximum Moment, $M_{max}$, (kNm): | 443 |
| Characteristic strength of concrete, $f_{ck}$ ($Nmm^{-2}$) | 25 |
| Characteristic strength of steel, $f_{yk}$, ($Nmm^{-2}$) | 500 |
| Concrete cover, c (mm): | 25 |
| Depth of tensile reinforcement, d (mm): | 510 |
| Depth of compressive reinforcement, $d^I$, (mm): | 50 |
| Unit weight of concrete ($kgm^{-3}$): | 2406.53 |
| Cost coefficient of concrete, $c_c$, (Naira per kg) | 12.914 |
| Cost coefficient of steel, $c_s$, (Naira per $mm^2$) | 27.733 |
| Cost coefficient of formwork, $c_f$, (Naira per m) | 218.723 |

Upon the execution of the optimization procedure, the new values for the design variables and cost objective function were observed and recorded as is shown in Table 2.

Table 2: Results from the cost optimization of the design example.

| Design Variables | Original Values | Optimal Values | |
|---|---|---|---|
| h | 560 | 673.031 | |
| b | 280 | 264.965 | |
| $A_{sc}$ | 694.368 | 434.844 | |
| $A_{st}$ | 2365.773 | 2075.482 | |
| Objective Function | Original Cost | Optimal Cost | Gain (%) |
| $C_T$ | 109063.915 | 97292.96 | 10.793 |

The original cost is decreased by 10.793 % of its value. This is the cost savings due to the cost optimization of the selected beam under the preassigned parameters. A reduction all the values of the design variables except for the overall beam depth which was increased from 560 mm to 673.031 mm was also observed.

### Effect of Span Length on Cost Optimization of the Model

An evaluation of the relationship between the span length and the resulting original cost and optimal costs under cost optimization was performed on the model. Keeping loading conditions constant and increasing the span of the beam, the resulting effects of the cost optimization was observed and recorded.

The span increase was directly proportional to the increase in the original and optimized costs as seen in Figure 5, even though the optimized costs closely matched the direction and slope of the original costs.

The gains from cost optimization with respect to increasing span lengths however were very minute, as seen in the trend line generated in Figure 6. The gains ranged from 0.264% at 1 m span to 0.210% at 10 m, with the highest gains being at 3 m with 0.229% and the lowest at 5 m with 0.176% gain.

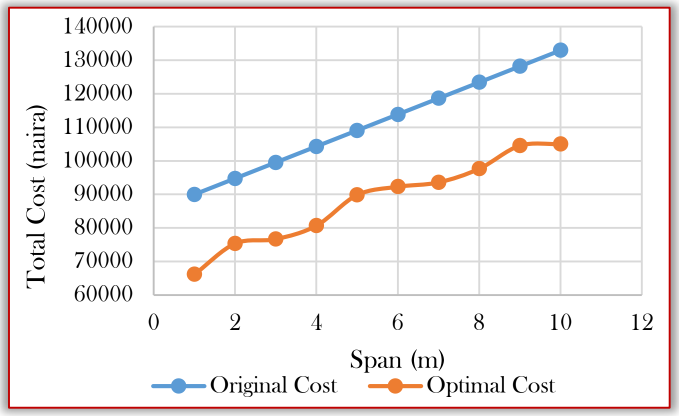


Figure 5: Graph of costs against span under cost optimization.

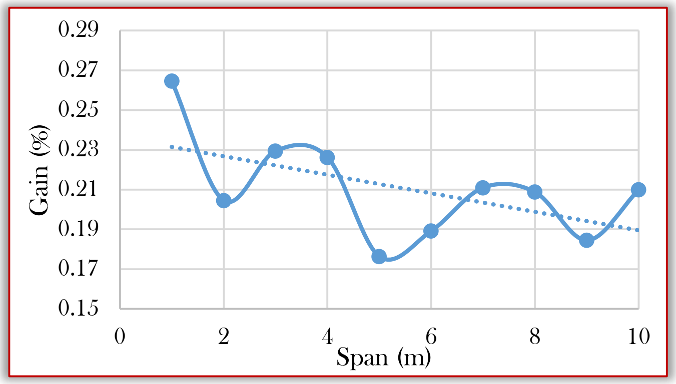


Figure 6: Graph of gain (%) against span under cost optimization.

### Effect of Concrete Grade on Cost Optimization of the Model

The identical procedure carried out in the previous section was repeated for various concrete classes. The characteristic strength values of these classes ($f_{ck}$) is a parameter that influences the costs of the structure and is unique and representative of each class.

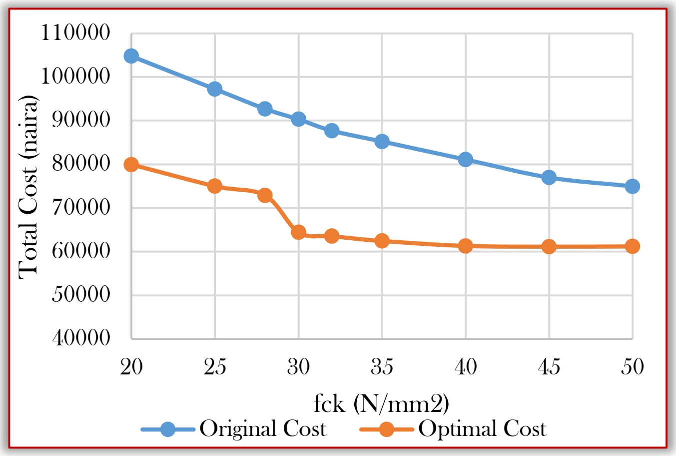


Figure 7: Graph of costs against characteristic concrete strengths under cost optimization

Increase in the characteristic strength of concrete (which implies the use of a different grade or class of concrete for design or manufacture) led to a steady decline in the both the original cost and optimized cost, as seen in Figure 7. The gain (%) however, shown in Figure 8, due to increase in the characteristic concrete strength of the concrete classes decreased initially before increasing to 28.68%, after which it steadily declined to 18.345% for the concrete class of 50/60.

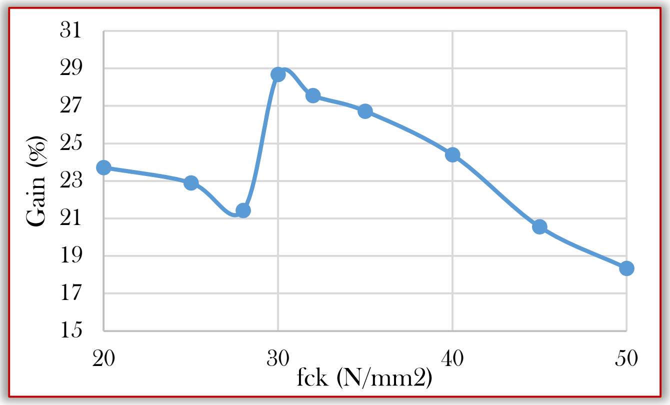


Figure 8: Graph of gain (%) against characteristic concrete strengths under cost optimization.

## CONCLUSION

Based on the results obtained from this study, the following conclusions can be drawn:

- The height of the beam was increased in the case study whilst all other design variables were decreased on the execution of the cost optimization process. The height increased from 560 mm to approximately 673.0311 mm.
- Increase in the span of the member led to an increase in the original costs, as well as the optimized costs, even though the latter were smaller than the former.
- The gains from cost optimization with respect to increasing span lengths however were very minute. The gains ranged from 0.264% at 1 m span to 0.210% at 10 m, with the highest gains being at 3 m with 0.229% and the lowest at 5 m with 0.176% gain.
- The minute values of gain indicated that there were no additional gains derived by increasing the span of the beam whilst under cost optimization.
- Increase in the concrete class led to a decrease in the original and optimized costs of the beam. The decline in the values of the optimized costs however was observed to be slower and there could possibly be a scenario where a high enough concrete class would receive no benefit from the developed model's cost optimization.
- The gain (%) due to increase in the characteristic concrete strength of the concrete classes decreased initially before increasing to 28.68%, after which it steadily declined to 18.345% for the concrete class of 50/60.

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