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# WEIGHT OPTIMIZATION OF A POST-TENSIONED CONCRETE BEAM

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**Abstract:** This research focuses on the weight optimization of a post-tensioned concrete section under the design guidelines of Eurocode 2. Research shows that optimizing concrete weight can lead to lighter members, smaller sections, and lower costs while maintaining safety and failure requirements. Microsoft Excel was used to develop and optimize the mathematically derived model representing the weight of the post tensioned beam and the constraints related to it. The minimum weight is assumed to coincide with the minimum section dimensions; hence, the design variables consisted of the dimensions of the beam's cross-section. The weight optimization process was demonstrated by comparing the initial and final values of both weights with respect to the beam span and live load on the structure. A weight reduction of 65.06% was observed at a span of 5 m, which then declined to a measly 2.3% when the span increased to 10 m. A similar result also occurred with respect to the applied live load; here a weight reduction of 57.21% was observed when a load of 3 kN/m was placed on the beam; this also reduced to 2.34% when the live load was increased to 8 kN/m.
**Keywords:** weight optimization, post-tensioned concrete section, Eurocode 2, optimization, serviceability constraints

## INTRODUCTION

The main purpose of the design of prestressed concrete is to limit tensile stresses, and hence cracking due to bending moments, in the concrete under working conditions. The design of the prestressed beam in this research is therefore based initially on the requirements of the serviceability limit state.

Prestressing in itself is the process of inducing compressive stress zone of a structural element, which may become tension under external loads. The introduction of compressive stress in the structural element helps to neutralize the tensile stress that may occur so that there is no resultant tension. This means that cracking in the structural part has been fully eradicated under working load, and all of the concrete can be assumed to be effective in carrying load. As a result, lighter sections can carry a given bending moment over much longer spans than reinforced concrete.

Given that the prestressed member is lighter than an identical reinforced concrete member of the same function under the same load, weight optimization is often not considered in the design of prestressed members. In addition, not only is the concrete in the member fully utilised, but also the need for conventional steel reinforcement with bars is unnecessary. The compressive force is usually provided by tensioned steel wires or strands which are anchored against the concrete and, since the stress in this steel is not an important factor in the behaviour of the beam but merely a means of applying the appropriate force, full advantage may be taken of very high strength steels. Hence, the weight of the steel tendons, compared to the concrete is negligible.

Despite the prevalence of Eurocode 2 and its detailed provisions for post-tensioned concrete design, there are still challenges associated with optimizing the weight of post-tensioned concrete beams while maintaining compliance with safety and structural integrity requirements. The problem is multifaceted, involving the need to reduce material usage to promote sustainability and cost-effectiveness, while concurrently ensuring that the structural performance remains reliable.

This research aims to address the aforementioned problem by focusing on the weight optimization of post-tensioned concrete beams while adhering to the guidelines and constraints set forth by Eurocode 2. The optimization process will involve a systematic exploration of design variables, such as concrete strength, beam geometry, and eccentricity of prestressing force with the goal of achieving the most efficient and permissible structural configuration. Advanced computational methods and optimization algorithms will be employed to search for optimal solutions within the defined design space.

The literature surrounding the optimization of post-tensioned concrete structures is rich and diverse. Previous research has predominantly centered on various aspects of post-tensioned concrete design, ranging from detailing and construction techniques to material properties and structural behavior. However, limited research has explicitly focused on the weight optimization of such structures within the

Eurocode 2 framework. Reinforced Concrete Design to Eurocode 2 (Mosley, *et al*., 2012) was studied to investigate the design of post-tensioned concrete beam to Eurocode 2 specifications. It also illustrates the assumptions, design parameters and constraints to be made by the designer when developing an optimization model for the structure within the Eurocode 2 framework.

Chapra and Canale (2015) was thoroughly studied as it discusses the various optimization processes and how they are to be used. It also presented a list of optimization algorithms consisting of: the simplex method, generalized reduced gradient (GRG) search method, genetic algorithms, simulated annealing and Tabu search. In addition, it showed how for the various optimization problems, different algorithms could be applied through the use of Microsoft Excel software by the means of its SOLVER tool which applies either the simplex method, evolutionary algorithm (genetic algorithm) or generalized reduced gradient (GRG) methods.

Oded and Emad (2018), focuses on the optimization of prestressed concrete beams by optimizing the distribution of material in a given design domain. Similar to the aim of this research, it focuses on adjusting the geometry of the beam to achieve the research work's stated targets.

Dissanayake and Jothy (2007), is another research work with similar objectives as this one. This research used Microsoft Excel and SAP2000 structural analysis to organize, manage and direct for solving and optimize a pre-stressed concrete beam section. The cost of manufacture was the objective function, and its optimization was achieved by minimizing the cross-sectional area of both the concrete, steel tendons and compression reinforcement.

Krauser (2009) examines a parametric study of a post-tensioned flat plate floor system. To construct the parametric analysis of a hotel/condominium grid plan, the load balanced by post-tensioning, slab depth, and concrete strength were modified. To carry out the parametric study, research on the development of post-tensioning, methods of analysis for two-way slab design, and posttensioning techniques of analysis were carried out. The design was done by hand using a series of Excel spreadsheets.

Samartin and Utrilla (1990) provide a review of the current strategies for optimizing prestressed concrete bridge decks. The sizes of prestressing cables with a given fixed geometry are determined using linear optimization. This simple procedure of linear optimization is also used to obtain the 'best' cable profile, by combining a series of feasible cable profiles.

Shengping and Tiong (2004) evaluated the cost effectiveness of post-tensioned concrete structural systems, taking into account various grid systems and loads to determine the best cost efficient solution for a building. The work investigated the costs of materials, labor, transportation, and necessary equipment. The paper focuses on construction in Singapore and provides an example of how to do a cost analysis. The variables of material cost and labor cost will be used in the cost analysis for this project.

The scope of this research encompasses a comprehensive investigation into the weight optimization of post-tensioned concrete beams, with a primary focus on Eurocode 2 compliance. The study will involve weight optimization on a selected beam with defined parameters, a demonstration of the weight optimization procedure with respect to various applied load values, and a performance of the weight optimization procedure with respect to various span lengths of the beam.. The justification for this work lies in the increasing demand for sustainable construction practices, where reducing material usage and enhancing structural efficiency are paramount.

Given that the weight of steel in the post-tensioned beam is negligible (at serviceability limit state), the weight of the concrete can be optimized, leading to lighter members, smaller sections and all at a cheaper cost while still fulfilling safety and failure requirements. By optimizing the weight of post-tensioned concrete beams, this research contributes to the broader objectives of sustainability, safety, and cost-effectiveness in civil engineering.

Since the weight optimization is achieved by minimizing the cross-section properties, the model could also serve as a means of topological (shape) optimization. It could also be further developed to handle larger and less-idealized post-tensioned members like bridge-girders, with the consideration of the member at ultimate limit state or compression reinforcements.

A main goal of the research was to have a thorough grasp of Eurocode 2 and its requirements for post-tensioned concrete design. This was then followed by an investigation into the important design variables and limitations influencing the weight optimization of post-tensioned concrete beams. An accurate depiction of the behavior of post-tensioned concrete beams could be achieved using created computational models, as well as recommendations for practical implementation and future study in the field of post-tensioned concrete optimization. In pursuit of these goals, our research intends to give significant insights and practical solutions to the optimization of post-tensioned concrete beams, in line with the construction industry's increasing needs.

## MATERIALS AND METHODS

### Materials

Eurocode 2 (2004) was used to develop a mathematical representation of a concrete structure. An Excel spreadsheet was used to set up the model and the optimization process was executed using Excel's Solver Tool.

### Development of the Objective Function

Consider the transverse section of a post-tensioned concrete beam as shown in Figure 1.

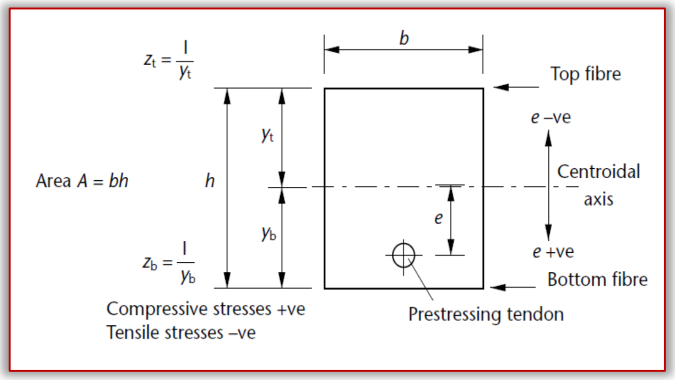


Figure 1: Cross-section of a post-tensioned concrete beam.

The weight of the post-tensioned beam was taken as a function of the density of concrete, cross-sectional area of the beam and its span.

$$\text{Weight}, W = \rho \times b \times h \times L \quad (1)$$

where:

$\rho$, is the density of the concrete grade.
b, is the width of the beam.
h, is the height of the beam.
L, is the span across the beam.

### Required Inputs

These are the terms and parameters needed to calculate and derive the objective function, constraints and other relevant parameters.

The geometric properties of the beam considered include:

- The width of the beam (b).
- The height of the beam (h).
- The span of the beam (L).
- The eccentricity of prestressing force (e).
- The material properties of the concrete considered were its density ($\rho$). This was taken as 2400 kg/m³.

The prestress loss was considered by the loss factor (K). The loading on the beam is considered by the live load acting on it ($q_k$). This was used to determine the moment variation as illustrated in the next section.

### Design Constraints

Given that the weight optimization is achieved by selecting the minimum section properties, the main design constraint relates the section modulus ($z_t$ and $z_b$) to the moment acting on the member due to the imposed live load alone. Mathematically:

$$z_t \geq \frac{M_v}{(f_{max} - K f^I_{min})} \quad (2)$$

$$z_b \geq \frac{M_v}{(K f^I_{max} - f_{min})} \quad (3)$$

where:

$$M_v = \frac{q_k L^2}{8} \quad (4)$$

Whilst equations 2 and 3 provide the lowest permissible value of the section modulus, the initial values of the section modulus for the initial beam provided is to be derived from the formula:

$$z_t = z_b = z = \frac{bh^2}{6} \quad (5)$$

To derive the section properties, the larger section modulus between equations 2 and 3 is used and related to equation 5 to find the smallest possible sections and hence, the minimum weight. For a single-span, simply supported post-tensioned beam it is usually the stresses that govern the behaviour of the beam are constrained by (Eurocode, 2004):

$$\frac{P}{A} - \frac{Pe}{z_t} + \frac{M_{min}}{z_t} = f_t^I \geq f_{min}^I \quad (6)$$

$$\frac{P}{A} + \frac{Pe}{z_b} - \frac{M_{max}}{z_b} = f_b^I \leq f_{max}^I \quad (7)$$

$$\frac{KP}{A} - \frac{KPe}{z_t} + \frac{M_{max}}{z_t} = f_t \leq f_{max} \quad (8)$$

$$\frac{KP}{A} + \frac{KPe}{z_b} - \frac{M_{min}}{z_b} = f_b \geq f_{min} \quad (9)$$

where:

$$M_{min} = \frac{q_k L^2}{8} \quad (10)$$

$$M_{min} = \frac{(q_k + W) L^2}{8} \quad (11)$$

$$\frac{P}{A} - \frac{Pe}{z_t} + \frac{M_{min}}{z_t} = f_{min}^I \quad (12)$$

$$\frac{P}{A} + \frac{Pe}{z_b} - \frac{M_{max}}{z_b} = f_{max}^I \quad (13)$$

$$\frac{P}{A} - \frac{Pe}{z_t} + \frac{M_{max}}{z_t} = f_{max} \quad (14)$$

$$\frac{P}{A} + \frac{Pe}{z_b} - \frac{M_{min}}{z_b} = f_{min} \quad (15)$$

The geometric constraints used were:

$$150 \leq b \leq 300 \quad (16)$$

$$200 \leq h \leq 400 \quad (17)$$

### Optimization Model and Required Outputs

Using the previously specified equations, the model was summarized as:

$$\text{Minimize Weight, } W = \rho bhL \quad (18)$$

Subject to:

$$z_t \geq \frac{M_v}{(f_{max} - Kf^{I}{}_{min})} \quad (19)$$

$$z_b \geq \frac{M_v}{(Kf^{I}{}_{max} - f_{min})} \quad (20)$$

$$\frac{P}{A} - \frac{Pe}{z_t} + \frac{M_{min}}{z_t} = f_t{}^{I} \geq f_{min}{}^{I} \quad (21)$$

$$\frac{P}{A} + \frac{Pe}{z_b} - \frac{M_{max}}{z_b} = f_b{}^{I} \leq f_{max}{}^{I} \quad (22)$$

$$\frac{KP}{A} - \frac{KPe}{z_t} + \frac{M_{max}}{z_t} = f_t \leq f_{max} \quad (23)$$

$$\frac{KP}{A} + \frac{KPe}{z_b} - \frac{M_{min}}{z_b} = f_b \geq f_{min} \quad (24)$$

$$150 \leq b \leq 300 \quad (25)$$

$$200 \leq h \leq 400 \quad (26)$$

To find $X = [X_1 X_2]^T$ which minimizes the objective function while satisfying the constraints stated above. Let:

$$b = X_1$$
$$h = X_2$$

### Optimization Process

The weight optimization was carried out by replicating the mathematical model in a Microsoft Excel spreadsheet and using the Excel Solver to generate the new optimal solutions.

### Development of the Excel Spreadsheet

The objective function, input parameters, design parameters, computed values, constraints and their aforementioned formulas were appropriately placed in the Excel spreadsheet shown in Plate 3.2.

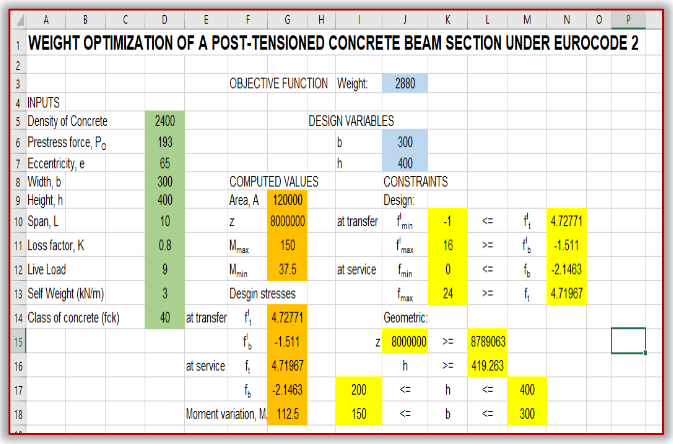


Figure 2: Excel Spreadsheet to perform weight optimization of a post-tensioned beam in development.

### Generation of Results

After using the Solver dialogue box to solve the optimization model, the values in the cells containing the design variables were changed due to the success of the operation. This also led to a corresponding change in the weight of the beam. The previous values of both the objective function and design variables are recorded, as well as the corresponding values after the optimization process has been completed.

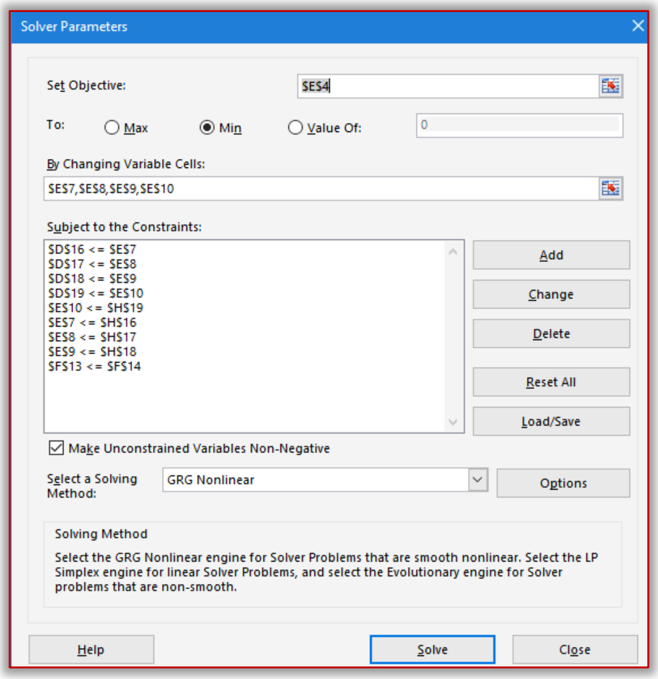


Figure 3: Solver dialogue box with relevant cells filled with information from the spreadsheet.

## RESULTS AND DISCUSSION

The model created was used to optimize the weight of a preselected prestressed beam with a span of 10 m, live load of 3 kN/m and prestess loss of 0.8. The initial and final optimal values are given in the Table 1.

Table 1: Results of the weight optimization of the case study.

| Design Variable | Original Values | Optimized Values | Gain (%) |
|---|---|---|---|
| h (mm) | 250 | 150 | 40 |
| b (mm) | 350 | 342.327 | 2.192 |
| Objective Function | Original Values | Optimized Values | Weight Reduction (%) |
| Weight (kg) | 2100 | 1232.38 | 41.3152 |

The weight reduction and gain were derived from the formulas:

$$\text{Weight Reduction (\%)} = \frac{\text{Original Weight} - \text{Optimal Weight}}{\text{Original Weight}} \times 100\% \quad (27)$$

$$\text{Gain (\%)} = \frac{\text{initial value} - \text{optimal value}}{\text{initial value}} \times 100\% \quad (28)$$

The results of the weight optimization are shown in Table 1. The original weight of the beam was 2100 kg, which was reduced to 1232.38 kg after the resulting weight optimization process was executed. This resulted in a weight reduction of 41.315%.

The dimensions of the beam having been changed in the optimization process, showed both a decrease in height from 350 mm to 342.327 mm and width from 250 mm to 150 mm. The resulting gains from these changes were 2.19% for the height and 40% for the width.

### Weight Optimization with Respect to Span Length

Weight optimization was also executed at various span lengths of the beam. Its span was incrementally increased from 5 to 10 meters and weight optimization was performed at each step. The results from the optimization are shown in the Table 2.

Table 2: Weight optimization results with varying span lengths.

| Span (m) | Original Weight (kg) | Optimized Weight (kg) | Weight Reduction (%) |
|---|---|---|---|
| 5 | 1440 | 503.115 | 65.061 |
| 6 | 1728 | 724.486 | 58.074 |
| 7 | 2016 | 986.106 | 51.086 |
| 8 | 2304 | 1440.000 | 37.500 |
| 9 | 2592 | 2050.312 | 20.898 |
| 10 | 2880 | 2812.500 | 2.344 |

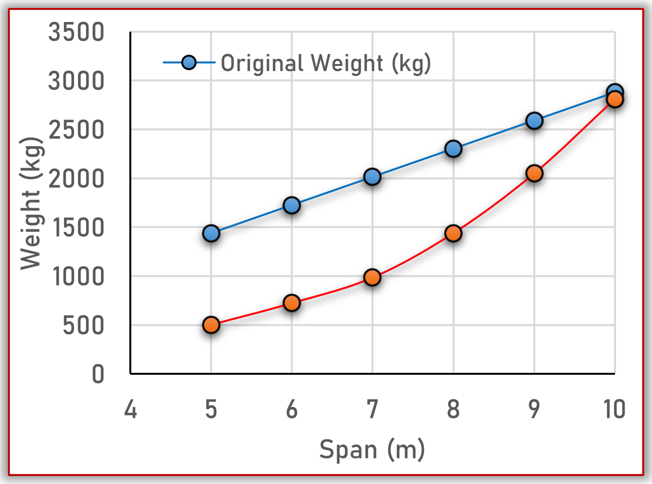


Figure 4: Graph of Span (m) against Weight (kg)

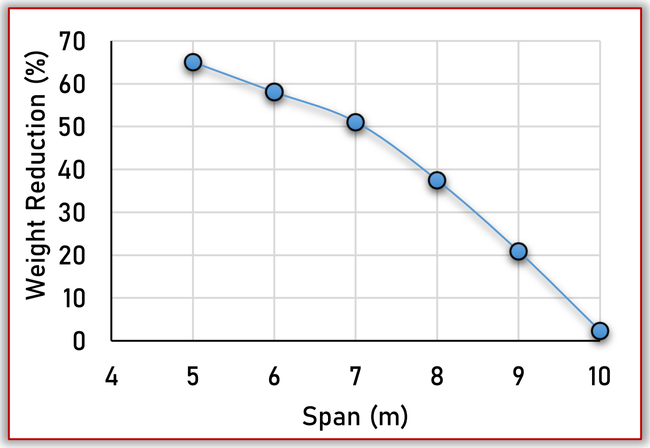


Figure 5: Graph of Span (m) against Weight Reduction (%).

### Weight Optimization with Respect to Live Load

Weight optimization was also executed at various live loads imposed of the beam. These were incrementally increased from 3 to 8 kN/m and weight optimization was performed at each step against a constant beam weight. The results from the optimization are shown in Table 3.

Table 3: Weight optimization results with varying live loads.

| Live Load (kN/m) | Original Weight (kg) | Optimized Weight (kg) | Weight Reduction (%) |
|---|---|---|---|
| 3 | 2880 | 1232.376 | 57.209 |
| 4 | 2880 | 1423.025 | 50.589 |
| 5 | 2880 | 1757.812 | 38.965 |
| 6 | 2880 | 2109.375 | 26.758 |
| 7 | 2880 | 2460.938 | 14.551 |
| 8 | 2880 | 2812.501 | 2.344 |

From Figure 6 and Figure 7 respectively, it can be seen that the weight of the beam was kept constant at 2880 kg as the live load imposed on the beam was increased from 3.0 to 8.0 kN/m. The corresponding change from the optimized weight was from 1232.376 kg to 2812.501 kg, which resulted in decrease in the weight reductions from 57.209% to 2.344%.

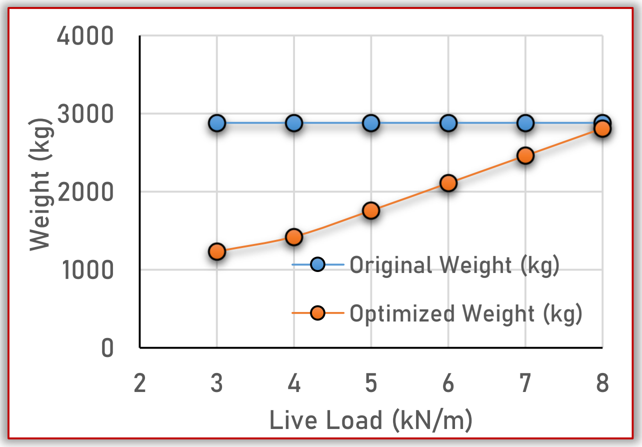


Figure 6: Graph of Live Load (kN/m) against Weight (kg).

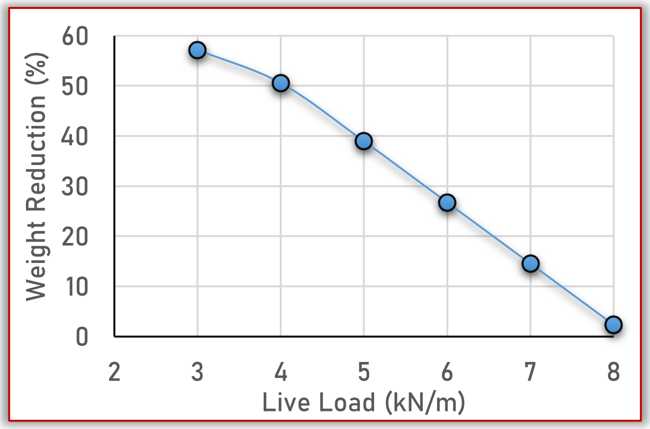


Figure 7: Graph of Live Load (kN/m) against Weight Reduction (%).

## CONCLUSIONS

Based on the results obtained from this study, the following conclusions can be drawn:

— An increase in the span length of the beam directly leads to an increase in both the original weight and the optimized weight, with the optimized weight increasing at a greater rate. The optimized weight will eventually match and exceed the original weight as the span length keeps increasing.

- The weight reduction (%) decreases with respect to increasing span length.
- Increasing values of live loads imposed on a beam with the effect of weight optimization lead to an increase in the optimal weight.
- The weight reduction (%) decreases with respect to increasing imposed live load.

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