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WEIGHT OPTIMIZATION OF A SIMPLY SUPPORTED I-SECTION STEEL BEAM UNDER DEFLECTION CONSTRAINT

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Abstract: The scope of this research was to develop and study an optimization model used to minimize the weight of a simply supported I-section steel beam. The major criterion of this optimization model is the behavior of the structure under deflection as defined by BS 5950: Part1. The weight optimization is achieved by taking the geometric dimensions of the beam's cross-section as design variables. Due to the necessary idealization of the structure, unrealistic outputs had to be avoided by taking real life dimensions of UB steel sections as boundary geometric constraints. Results were derived from a sample case study and the observation of weight optimization with respect to parameters such as moment of inertia of steel sections and the span of the beam. Whilst the model produced increasing values of weight and optimized weights with respect to increasing spans, consistent values of moment of inertia was observed for optimized sections when weight optimization was performed.

Keywords: BS5950: Part 1, deflection, I-section steel beam, UB steel sections, weight optimization

INTRODUCTION

Compared to other building materials such as concrete, timber, masonry or earthworks, steel is a relatively new comer in structural construction. However, given the continuous spread and usage of industrial methods, it is a ubiquitous structural material.

The mass production of structural steel encourages the use of standard members for different structural elements (beams, columns, channels, etc.). The I-section beam focused on in this research is an example of such members. Even though the weight of an I-section steel beam is not a major criterion in the design of steel structures, its span and cross-section dimensions influence its behaviour in deflection under applied loads. However, these geometric dimensions of the structure directly determine its weight.

The optimization of steel beam design is confronted with the challenge of balancing the structural requirements, such as strength and deflection limits, with the imperative to minimize material usage and, consequently, the environmental and economic impact of construction projects. The weight optimization of steel beams is critical in this context as it directly influences the overall cost, resource consumption, and carbon footprint of a structure. However, this endeavor is complex due to the interplay of various design parameters and constraints, including safety standards and deflection limitations.

This thesis proposes to address the aforementioned challenge by developing a systematic approach for optimizing the weight of simply supported I-section steel beams while ensuring compliance with deflection constraints outlined in BS5950 Part 1. To achieve this, the study employed advanced computational methods, structural analysis techniques, and optimization algorithms to iteratively refine the beam's geometry and material distribution.

The design code defined by the research was BS 5950: Part 1. The researched literature for this was Frixos and Alan (2002). It showed the assumptions, design parameters and constraints to be made by the designer when designing the beam under deflection.

Steven and Raymond (2015) was consulted for this project. This work thoroughly discusses various optimization processes, why and when they are to be used. Here, the optimization problem of optimizing a constrained system with non-linear constraints was shown to require non-linear programming (NLP) for its solution. A list of optimization algorithms was also listed and discussed including: the simplex method, generalized reduced gradient (GRG) search method, genetic algorithms, simulated annealing and Tabu search.

Korkmaz and El-Gafy (2018) discusses the structural optimization of steel structures in the case of beam to column connections. This was done using Nonlinear Static Pushover Analysis. Despite the difference in methods used compared to this research, it demonstrated weight reduction under design constraints with each stage of analysis.

Paolo *et al.*, (2016) discusses the weight optimization of steel frames and trusses. However, genetic algorithms were used. The research included root radius in geometric parameters, resulting in a 15% weight reduction and integrating Finite Element Method simulations with a genetic algorithm.

Erkan and Aybike (2019) investigate the use of a hunting search algorithm to conduct weight optimization on steel frames. The commercial computer-aided design software ABAQUS is then used to create finite element models of each optimum frame for nonlinear analysis under loading. The study demonstrates that cellular beams can be used effectively in the design of steel frames to provide serviceability and strength while meeting design constraints such as maximum stress and buckling capacity.

Sharafi *et al*. (2014) investigated a method for optimizing the shape and size of steel sections using graph theory and the ACO algorithm. They conducted a multi-objective analysis with the goal of maximizing mass and strength. Because graph algorithms are good at finding the shortest pathways solution, the used graph theory approach was particularly suitable for optimum form analysis. In fact, because the thickness of the beam cross-section is uniform and constant, mass minimization simplifies the issue to section length minimization.

Searching for the minimum weight design has gained popularity over the years (Erdal , 2011; Hasancxebi and Carbas, 2014; Korouzhdeh and Eskandari-Naddaf, 2019). This search aims to detect the optimum geometry or the optimum topology and/or optimum cross-sectional dimensions for the members of a structure. To perform this, using structural optimization, a number of tools have been provided for structural designers (Carbas, 2016; Gholizadeh and Milany, 2018; Lagaros and Fragiadakis, 2007). One group of these tools, which may be categorized as traditional techniques, often faces difficulties in solving practical design optimization problems.

Kociecki and Adeli (2013) proposed a two-phase genetic algorithm for size optimization of free-form steel space-frame roof structures. They considered wind, snow and seismic loadings in linear structural simulations. The converge conditions allowed no more than 5% of overstressed beams. The achieved results provide a weight reduction of 12% using an automated design process.

Türker *et al*. studied the dynamic behavior of a two-story steel frame structure using simulations based on FEM analysis. They investigated the modal testing with and without braces. One of their remarks was that brace elements cause an increase in the natural frequencies because of the increased stiffening of the structures.

Compared to the aforementioned research works, this project is focused exclusively on a single steel member under the influence of deflection. The direct effect of certain beam properties that are geometric in nature such as length (span) and moment of inertia which were studied in this project have not been investigated to describe how they can influence the weight optimization of steel members (beam). The chosen scope is justified by the practical relevance of such beams in various structural applications, their susceptibility to weight reduction, and the critical importance of deflection control in real-world engineering projects.

The development of a computational framework for optimizing the weight of simply supported I-section steel beams was the primary objective of the research. Crucial to this was the incorporation of BS5950 Part 1 deflection constraints into the optimization process.

This research endeavors to contribute to the advancement of sustainable and cost-effective structural design practices while ensuring adherence to stringent safety and performance standards as mandated by BS5950 Part 1. Through the pursuit of these objectives, this research aims to provide valuable insights for engineers, designers, and researchers engaged in the optimization of steel structures.

MATERIALS AND METHODS

Materials

BS 5950: Part 1 is the design code for structural steel design. In this research, it was used in the development of the optimization model, specifically in the derivation of the deflection constraints and boundaries for the geometric constraints used. A personal computer with an Intel® CoreTM i3-6100U CPU @ 2.30 GHz, Windows 10 Home operating system, and 8GB RAM was used to run Microsoft Excel and execute the optimization process. An Excel spreadsheet was used to set up the model and the optimization process was executed using Excel's Solver Tool.

Optimization Method

Generalized reduced gradient (GRG) was the optimization algorithm used to carry out the weight optimization. The model in this research consisted of an objective function developed for the weight optimization, which was a non-linear equation; with deflection constraints derived from BS 5950: Part 1. The generalized reduced gradient method is a direct method available in the Solver add-in in Microsoft Excel. The optimization problem was a non-linear constrained optimization problem, which the generalized reduced gradient algorithm can resolve.

Formulation of the Optimization Model

The model created describes a mathematical relationship between the weight of the beam and the corresponding geometric and material properties of the beam, required to derive its weight. This derived weight was then optimized while under constraints for both its deflection and geometry (size) using BS 5950: Part 1 as the design guide.

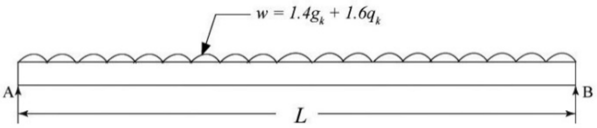

Figure 1: Loading condition and idealized structure of the model with factored loads according to BS 5950: Part 1.

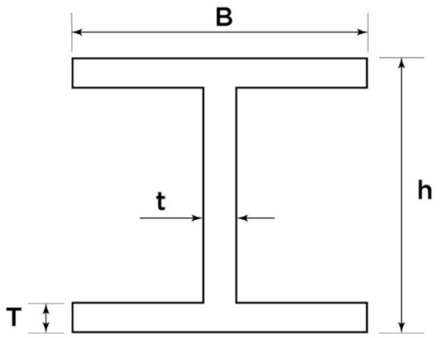

Figure 2: Cross-section of the beam with its geometric parameters used in the model.

Development of the Objective Function

The model optimizes the weight of a steel beam with a cross-section shown in Figure 2, which is a simply supported beam as shown in Figure 1. The weight of the beam is a function of the density of the steel and the beam's volume. The beam's volume is a result of the area of the beam's cross-section multiplied by its span.

Mathematically, the objective function is represented as:

$$\text{weight of beam} = \text{density of steel} \times \text{volume of beam} \quad (1)$$

$$M = \rho \times V \quad (2)$$

$$\text{Volume}, V = [2BT + t(h - 2T)] \times L \quad (3)$$

The objective function rewritten in full as:

$$M = \rho \times [2BT + t(h - 2T)] \times L \quad (4)$$

where:

M = Weight of the steel beam in kg.

ρ = Density of steel in kg/m^3.

L = Span of the beam in m.

T = Thickness of the beam's flange in mm.

t = Thickness of the beam's web in mm.

B = Width of the beam in mm.

h = Depth of the beam in mm.

Input Parameters

From equation 4, the weight function required certain inputs. They included both the terms defined above, as well as additional terms, which were used to compute the constraints.

The parameters for the geometric properties of the beam, as shown in Figure 1 and 2:

- Span of the beam (L).
- Width of the beam (B).
- Depth of the beam (h).
- Thickness of the beam's web (t).
- Thickness of the beam's flange (T).

All the input parameters used in the model are summarized in Table 1.

Table 1: Input parameters for the model

Parameter	Symbol	Unit
Span of the beam	L	m
Width of the beam	B	mm
Depth of the beam	h	mm
Thickness of the flange	T	mm
Thickness of the web	t	mm
Imposed load	q_k	kN/m
Dead load	g_k	kN/m
Steel density	ρ	Kg/m^3
Modulus of elasticity of steel	E	N/mm^2

Given that the nature of the research, the design constraints are for deflection. The magnitude of loads on the beam and beam structure (simply supported, with uniformly distributed load), were taken into account. The related inputs are:

- Imposed Load (q_k): These are all the other loads except that of the structure, fixtures and immovable parts loaded on the structure. It is factored according to BS 5950 by a partial factor of safety of 1.6.
- Dead Load (g_k): This is the weight of the structure itself and the weight of all loads permanently on it. It is factored according to BS 5950 by a partial factor of safety of 1.4. In this model, the derived weight from the objective function is added to the value directly inputted for the dead load when calculating the total load.

Finally, the material properties of the steel I-beam were also used to derive deflection constraints as demanded by BS-5950: Part 1. These included:

— Steel Density (ρ): This is the mass per unit volume of structural steel. From BS-5950, structural steel has a density of 7850kg/m³.

— Modulus of Elasticity of Steel (E): This is also known as Young's Modulus, and is the ratio of stress to strain of a given material. Its value for steel is taken as 205000 N/mm².

Constraints

Behavioural and geometric constraints were developed using the BS 5950: Part 1 and the British Steel's table for Universal Beam (UB) section properties respectively.

Behavioural constraints were limited to deflections. To ensure that this constraint is fulfilled, the maximum possible deflection that will occur due to loading (δ_{max}) must not exceed the recommended maximum deflection permitted by BS 5950: Part 1. This allowable defection ($\delta_{allowed}$) is dependent on the span of the beam, while the maximum deflection (δ_{max}) is dependent on the imposed load, span, moment of inertia and the modulus of elasticity of the beam. Mathematically, the behavioural constraint in the model is:

$$\delta_{max} \leq \delta_{allowed} \tag{5}$$

where:

$$\delta_{max} = \frac{5WL^3}{384EI} \tag{6}$$

Where the terms W and I which are for the imposed load and moment of inertia (i.e second moment of area about a horizontal axis midway through the height of the beam.) are derived from:

$$W = q_k \times L \tag{7}$$

$$I = \frac{[Bh^3 - (B-t)(h-2T)^3]}{12} \tag{8}$$

and

$$\delta_{allowed} = \frac{L}{360} \tag{9}$$

The geometric constraints were based on the maximum and minimum allowable steel sections from the British Steel's table for Universal Beam (UB) section properties. The minimum steel section had a designation 127 ×76×13 UB, while the maximum had a designation of 914×419×388 UB. The corresponding section properties of each both formed the lower and upper bounds of the set of geometric constraints. They are stated below:

$$127.0 \leq h \leq 921.0 \tag{10}$$

$$76.0 \leq B \leq 420.5 \tag{11}$$

$$4 \leq t \leq 21.4 \tag{12}$$

$$7.6 \leq T \leq 36.6 \tag{13}$$

All the values for the geometric constraints stated above are in mm.

Optimization Model and Required Outputs

Using the previously specified equations, the model was summarized as:

Minimize

$$M = \rho \times [2BT + t(h - 2T)] \times L \tag{14}$$

Subject to

$$\delta_{max} \leq \delta_{allowed} \tag{15}$$

$$127.0 \leq h \leq 921.0 \tag{16}$$

$$76.0 \leq B \leq 420.5 \tag{17}$$

$$4 \leq t \leq 21.4 \tag{18}$$

$$7.6 \leq T \leq 36.6 \tag{19}$$

where:

$$\delta_{max} = \frac{5WL}{384EI} \tag{20}$$

$$\delta_{allowed} = \frac{L}{360} \tag{21}$$

To find $X = [X_1 X_2 X_3 X_4]$ which minimizes the objective function while satisfying the constraints stated above.

Where:

$$h = X_1 \tag{22}$$

$$B = X_2 \tag{23}$$

$$t = X_3 \tag{24}$$

$$T = X_4 \tag{25}$$

The matrix X, consists of the design variables. Design variables are inputs into the model that are present in the objective function that were adjusted in order to achieve weight optimization. The outputs of the model included both the result of the optimization and the design variables.

Table 2: Outputs of the optimization model.

Parameter	Symbol	Unit
Width of the beam	B	mm
Depth of the beam	h	mm
Thickness of the flange	T	mm
Thickness of the web	t	mm
Weight of the beam	M	Kg

Model Optimization

The weight optimization of the modeled steel I-beam was performed in Microsoft Excel using the Solver add-in after the mathematical description of the model was replicated using a spreadsheet.

Development of Excel Spreadsheet

An Excel spreadsheet was used to represent the model by replicating the mathematical formulas of each of the necessary parameters in relevant cells as shown in Figure 3.

Once the spreadsheet was created, the Solver button was selected from the Data tab on the Excel interface. The Solver dialogue box displayed was then filled with pertinent data from the spreadsheet. The solving method selected in the Solver dialogue box was the

Generalized Reduced Gradient Non-linear algorithm method.

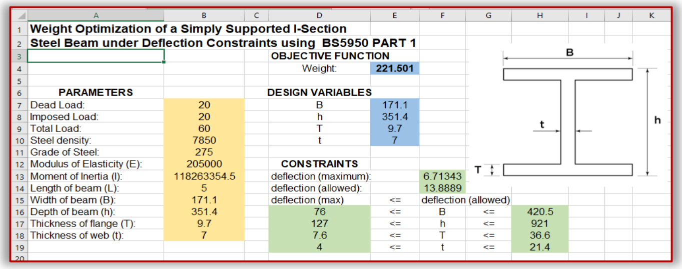

Figure 3: Excel spreadsheet set up to evaluate the weight optimization of a simply supported I-section steel beam subject to deflection constraints.

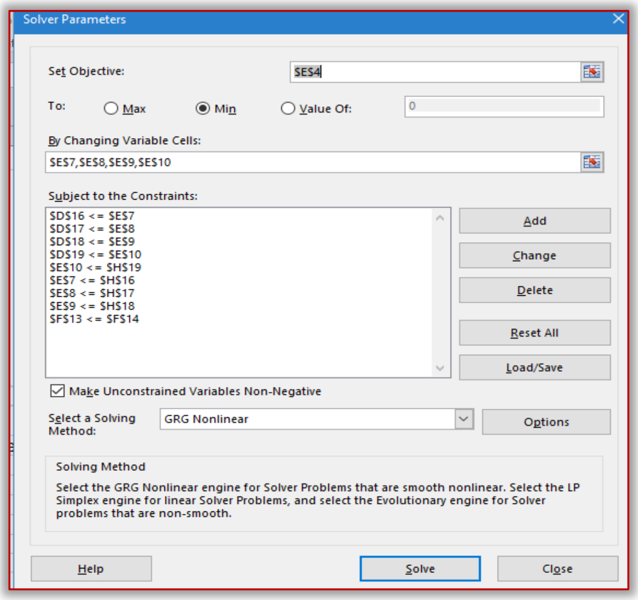

Figure 4: Solver dialogue box with relevant cells filled with information from the spreadsheet.

Derivation of Results

After using the Solver dialogue box to solve the optimization model, the values in the cells containing the design variables were changed due to the success of the operation. This also led to a corresponding change in the weight of the beam. The previous values of both the objective function and design variables are recorded, as well as the corresponding values after the optimization process has been completed.

RESULTS AND DISCUSSION

An optimization study was done on a select steel I-section beam with preassigned conditions and parameters. The UB steel chosen had a designated serial size of 457 × 191 × 89. The input parameters used in the case study are shown in the table below.

In addition to those, there were initial values chosen for each design variable before the weight optimization occurred. These are presented in Table 4.

The results of the subsequent optimization are shown in Table 4. In addition, there was a comparison of the values chosen for each design variable and objective function (weight) with those values presented by the model as it completes its weight optimization process.

Table 3: Input parameters for the case study optimization.

Parameter	Value
Dead Load	30 kN/m
Imposed Load	30 kN/m
Total Load	90 kN/m
Steel density	7850 kg/m^3
Grade of Steel	275 N/mm^2
Modulus of Elasticity (E)	205000 N/mm^2
Moment of Inertia (I)	406147473.3 mm^4
Length of beam (L)	8 m

The comparison, represented by Gain (%) and was calculated using this equation:

$$\text{Gain } (\%) = \frac{\text{initial value} - \text{optimal value}}{\text{initial value}} \times 100\% \quad (25)$$

The same formula was used to derive weight reduction (%).

Table 4: Optimization of the design study, showing initial and optimal values.

Design Variables	Initial Values	Optimal Solution	Gain (%)
B	191.9	76	60.396
h	463.4	808.088	-74.382
T	17.7	7.6	57.062
t	10.5	4	61.904
Weight (kg)	708.4	271.2	61.716

As seen in the above table, all the variables had shown a considerable decrease from their initial values, with the exception of the height of the beam, h which showed a gain of -74.382%.

A weight reduction from 708.4 kg to 271.2 kg was observed from the optimization model study, which is equivalent to a 61.716 % reduction.

Weight Optimization under Varying Span Length

Weight optimization was also executed at various span lengths of the beam. Its span was incrementally increased from 1 to 10 meters and weight optimization was performed at each step. The results from the optimization are shown below.

Table 5: Weight optimization results with varying span lengths.

Span, L (m)	Weight (kg)	Optimized Weight (kg)	Weight Reduction (%)
1	88.605	12.579	85.803
2	177.210	28.010	84.194
3	265.815	53.449	79.892
4	354.420	87.716	75.251
5	443.025	131.100	70.408
6	531.630	183.739	65.439
7	620.235	245.712	60.384
8	708.840	329.194	53.559
9	797.445	454.378	43.021
10	886.050	621.472	29.860

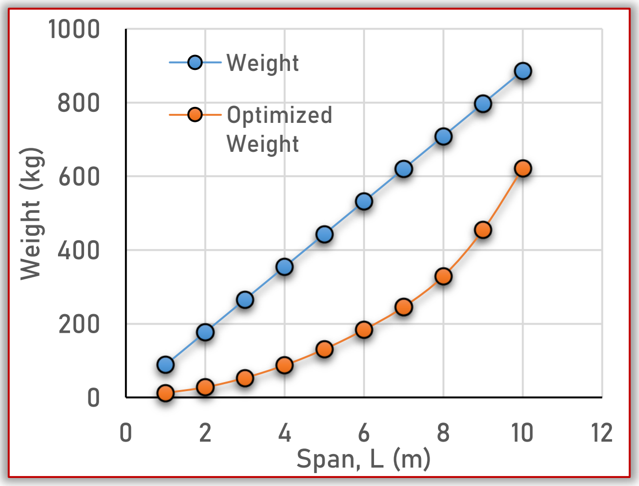

Figure 5: Graph of Span (m) against Weight (kg).

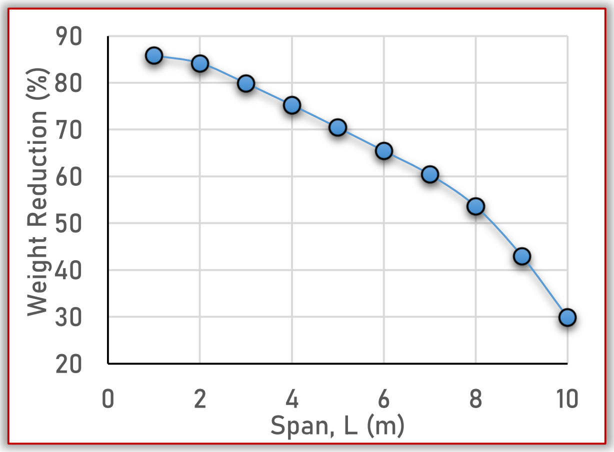

Figure 6: Graph of Span (m) against Weight Reduction (%).

There is a steady increase in the values of the beams weight as the span increases in length. This matching increase also occurs for the optimized weight even though its values are lower than those of the original weights.

However, given the nature of the graph for the optimized weights, and the steadily decreasing values of the weight reduction, the optimized weight at a given span would match the values of the original weight.

The weight of the beam showed an increase from 88.605 kg to 886.05 kg as the span of the beam was increased from 1.0 to 10.0 meters. The corresponding change from the optimized weight was from 12.579 kg to 621.472 kg, which resulted in a decrease of the weight reductions from 88.803% to 29.86%.

Weight optimization with varying steel sections

The model executes weight optimization on a simply supported I-section steel beam, where the steel beams according to BS 5950: Part 1 are of different designated sections. Each of these steel beams have different geometric properties (i.e values for h, B, t and T) and correspondingly have different values of moment of inertia (I) and weight.

Table 6: Steel sections with varying geometric properties, moment of inertias and weights.

Section	h (mm)	B (mm)	t (mm)	T (mm)	Moment of Inertia (mm^4)	Weight (kg)
127 x 76 x 13	127	76	4	7.6	4588602.14	62.8942
254 x 146 x 37	256	146.4	6.3	10.9	54707487.3	183.179
356 x 171 x 45	351.4	171.1	7	9.7	118263355	221.501
457 x 152 x 60	454.6	152.9	8.1	13.3	250996307	295.707
533 x 210 x 12	544.5	211.9	12.7	21.3	751901483	604.492
686 x 254 x 12	677.9	253	11.7	16.2	1159695535	618.167
762 x 267 x 14	754	265.2	12.8	17.5	1655425543	725.544
838 x 292 x 22	850.9	293.8	16.1	26.8	3354667170	1121.930
914 x 419 x 38	921	420.5	21.4	36.6	7109008167	1920.248

Weight optimization was performed on the above sections with respect to the moment of inertia and weights respectively.

Weight Optimization with Varying Moment of Inertias for Steel Sections

Weight optimization was performed on the steel sections with respect to the moment of inertia and the moment of inertia of the optimized section was observed and recorded.

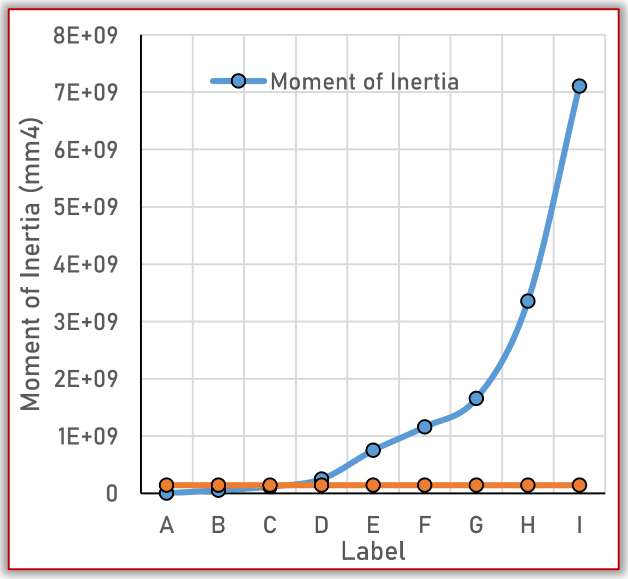

Figure 7: Graph of moment of inertia of different sections underweight optimization.

Table 7: Weight optimization on designated steel sections with respect to moment of inertia.

Section	Label	Moment of Inertia (mm^4)	Optimal Moment of Inertia (mm^4)
127 x 76 x 13	A	4588602.141	142915528.5
254 x 146 x 37	B	54707487.29	142911590.6
356 x 171 x 45	C	118263354.5	142911604.6
457 x 152 x 60	D	250996307	142911579.7
533 x 210 x 122	E	751901482.9	142911584.6
686 x 254 x 125	F	1159695535	142911434.4
762 x 267 x 147	G	1655425543	142911585
838 x 292 x 226	H	3354667170	142911586
914 x 419 x 388	I	7109008167	142911585.7

As the moment of inertial increases for each respective section, the moment of inertia for the optimized section generally decreases by relatively small amounts. This is seen from Figure 7 and Table 7, where the optimal moment of inertia appears to have a singular value with very minute variations regardless of the values of the original moment of inertia.

Weight Optimization with Varying Weights for Steel Sections

Weight optimization was performed on the steel sections with respect to the weight and the weight of the optimized section was observed and recorded.

Table 8: Weight optimization on designated steel sections.

Section	Label	Weight (kg)	Optimal Weight (kg)	Weight Reduction (%)
127 x 76 x 13	A	62.894	131.100	-108.445
254 x 146 x 37	B	183.179	131.099	28.431
356 x 171 x 45	C	221.501	131.099	40.813
457 x 152 x 60	D	295.707	131.099	55.666
533 x 210 x 122	E	604.491	131.099	78.312
686 x 254 x 125	F	618.169	131.099	78.792
762 x 267 x 147	G	725.544	131.099	81.931
838 x 292 x 226	H	1121.930	131.099	88.315
914 x 419 x 388	I	1920.248	131.099	93.173

Similar to the weight optimization with respect to moment of inertia, the optimization study of the section weights results in the optimized weights similarly converging to a singular value with very minute deviations as seen in Table 8.

This value of the optimized weights is approximately 131.1 kg is maintained regardless of the variance of the original section weights. This leads to an initial weight reduction of approximately -108.445% which increases as the original weights increases to approximately 93.173% on the final section.

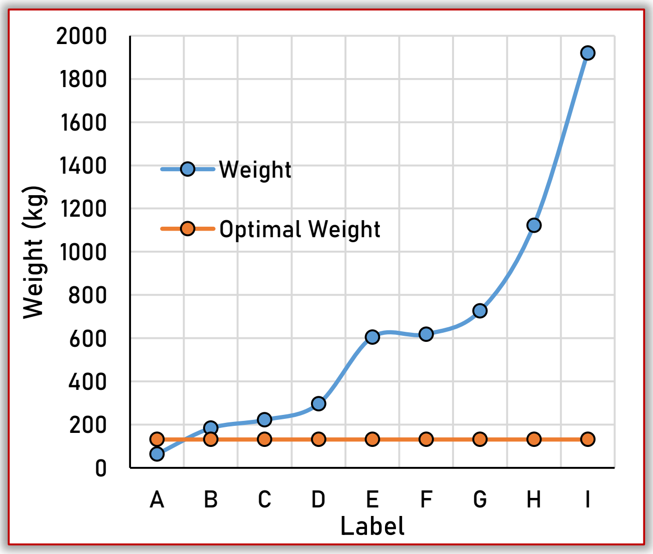

Figure 8: Graph of weight of different sections underweight optimization.

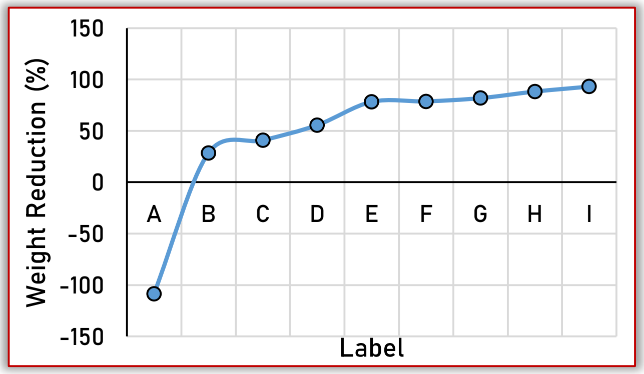

Figure 9: Graph of weight reduction of different sections under weight optimization.

CONCLUSION

The following conclusions can be made based on the results obtained from the study:

— An increase in the span length of the beam directly led to an increase in the original weight and also the optimized weight.

— The weight reduction (%) reduces as the span increased.

— Increase in a sections moment of inertia increases the weight of the beam. However, the optimized weight and moment of inertia show almost no variation from a single value.

— Weight reduction (%) increases with respect to the moment of inertia and the resulting weights of sections.

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