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AN ANALYTICAL ASSESSMENT FOR COMPARING GENERALIZED LEAST SQUARES – QUANTILE REGRESSION AND GENERALIZED LEAST SQUARES

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Abstract: This article presents a comparative analysis of the Generalized Least Squares–Quantile Regression (GLS–QR) and Generalized Least Squares (GLS) models using macroeconomic data from the Central Bank of Nigeria (1981–2020). Focusing on poverty reduction (PR) as the dependent variable and incorporating independent variables such as money supply (MS), government expenditure, government revenue, and financial inclusion, the study evaluates the predictive performance of both models. Results indicate that GLS–QR outperforms GLS, with adjusted coefficients of determination revealing a stark contrast: GLS at 3.8% versus GLS–QR at 46%, 45%, and 47% for the 25th, 50th, and 75th percentiles respectively. These findings clearly shows the superiority of GLS–QR in providing more accurate estimations of poverty reduction dynamics, suggesting its potential utility for policymakers and economists in formulating effective poverty alleviation strategies in Nigeria.

Keywords: Generalized Least Squares–Quantile Regression (GLS–QR), Generalized Least Squares (GLS), poverty reduction (PR), money supply (MS), strategies in Nigeria

INTRODUCTION

Generalized least squares (GLS) regression was introduced by [1], it is a statistical technique that accommodates heteroscedasticity and correlation in the error terms, making it versatile for regression analysis, it minimizes the weighted sum of squared residuals, with weights drawn from the inverse of the residual's estimated covariance matrix. GLS is widely used in various fields including econometrics, biology, finance, where data often exhibit heteroscedasticity and correlated errors.

Considering the research of [2] on the adaption of akaike information criterion under least squares frameworks for comparison of stochastic models demonstrated using simulated data. [3] addressed the parameter identification challenge in a bilinear state–space system with colored noise by eliminating the state variables from the model. They developed an input–output representation of the bilinear state–space system for parameter identification and introduced a recursive generalized extended least squares method for estimating the parameters of the resulting model.

Additionally, they proposed a computationally efficient three–stage recursive generalized extended least squares technique. [4] proposed a few preconditioned generalized AOR (denoted by GAOR) approaches. A comparison was also made between the original and suggested preconditioned approaches' spectral radii for the iteration matrices. In this study, the estimator of the GLS and Quantile regression was combined to give GLS–QR for the regression parameters of the multiple regression model and subsequently subjected to the real–life data and compare the performance of GLS and GLS–QR model.

MATERIALS AND METHOD

The macroeconomic variables employs poverty reduction as the dependent variable and Money Supply, Government Expenditure, Government Revenue, and Financial Inclusion are the explanatory variables and R program was used for the analysis.

Generalized Least Squares

Generalized least squares is a statistical method used to analyze data and estimate the parameters of a linear regression model when the assumptions of ordinary least squares are violated, particularly when the errors in the data are correlated or have unequal variances.

OLS is given as:

$$g_i = \beta_0 + \beta_i s_i + u_i \tag{1}$$

Equation (1) can be reformulated as:

$$g_i = \beta_0 s_{0i} + \beta_i s_i + u_i \tag{2}$$

where $s_{0i} = 1$ for each value of i.

Assuming heteroscedastic variances σ_i^2 are known, equation (3) becomes:

$$\frac{g_i}{\sigma_i} = \beta_0\left(\frac{s_{0i}}{\sigma_i}\right) + \beta_1\left(\frac{s_i}{\sigma_i}\right) + \frac{u_i}{\sigma_i} \tag{3}$$

Thus,

$$g_i^* = \beta_0^* s_{0i}^* + \beta_1^* s_i^* + u_i^* \tag{4}$$

In order to minimize the GLS estimators. Equation (4) becomes

$$\sum u_i^{2*} = \sum(g_i^* - \beta_0^* s_{0i}^* - \beta_1^* s_i^*)^2 \tag{5}$$

Thus,

$$\sum\left(\frac{u_i}{\sigma_i}\right)^2 = \sum\left[\left(\frac{g_i}{\sigma_i}\right) - \hat{\beta}_0^*\left(\frac{s_{0i}}{\sigma_i}\right) - \beta_1^*\left(\frac{s_i}{\sigma_i}\right)\right]^2 \tag{6}$$

Then,

$$\beta_1 = \frac{(\sum z_i)\sum(z_i x_i g_i) - (\sum z_i s_i)(\sum z_i g_i)}{(\sum z_i)(\sum z_i s_i^2) - \sum(z_i s_i)^2} \quad (7)$$

where $z_i = \frac{1}{\sigma_i}$

Quantile Regression

$$g_t = s\beta_q + u_t \quad (8)$$

$$\sum q u_t + \sum(1-q)u_t \quad (9)$$

Minimize equation (9), then it becomes

$$\sum u_t^2 = \sum(g_t - s_t'\beta_q)^2 \quad (10)$$

Thus,

$$\beta_q = \frac{\delta Q_q(g/s)}{\delta s_k} \quad (11)$$

Substituting $u_t = g_t - s_t'\beta_q$ in to equation (9), equation (12) becomes

$$q\sum(g_t - s_t'\beta_q) + 1 - q(g_t - s_t'\beta_q) \quad (12)$$

where $0 < q < 1$

Combining equation (8) and (11), gives β_{GLSQ_0} and β_{GLSQ_p}

The Proposed regression model can be written as:

$$\hat{g} = \hat{\beta}_{GLS-Q_0} + \hat{\beta}_{GLS-Q_1}s_1 + \hat{\beta}_{GLS-Q_2}s_2 + \hat{\beta}_{GLS-Q_3}s_3 + \cdots + \hat{\beta}_{GLS-Q_{p-1}}s_{p-1} \quad (13)$$

RESULTS AND DISCUSSION

GLS Regression

The association between four explanatory variables and poverty reduction was assessed using a GLS model. This equation appears in the fitted model:

$$PR = 34.025 - 0.00023\ MS - 0.00016\ GEX + 0.00032\ GRV + 3.64\ FII$$

GLS–Quantile Regression

A GLS–QR model explains the association between poverty reduction and the four explanatory factors.

The fitted models of lower, middle and upper quartiles of GLS–QR contains the equations below:

$PR = 21.84 + 0.14\ MS - 0.009\ GEX + 0.059 GRV + 0.049\ FII$ (Lower quartile GLS–QR)

$PR = 21.31 + 0.092\ MS - 0.011\ GEX + 0.15 GRV + 0.10\ FII$ (Middle quartle GLS–QR)

$PR = 30.93 + 0.036\ MS - 0.012\ GEX + 0.074\ GRV + 0.304\ FII$ (Upper quartile GLS–QR)

Comparison of GLS and GLS–QR

The GLS adjusted coefficient of determination is 3.8%, the GLS–QR (25%), GLS–QR (median), and GLS–QR (75%) all have adjusted coefficients of determination of 46%, 45%, and 47%, respectively. The adjusted coefficient of determination of GLS shows that only 3.8% of the variability in poverty reduction is collectively explained by changes in money supply, government expenditures, government revenue, and financial inclusion. The adjusted coefficient of determination of lower, median and upper quartile of GLS–QR gives a better explanatory power than GLS.

CONCLUSION

This study highlights the comparative efficacy of the GLS–QR model against the conventional Generalized Least Squares (GLS) model in analyzing the relationship between macroeconomic variables and poverty reduction in Nigeria. By utilizing extensive macroeconomic data from the Central Bank of Nigeria spanning nearly four decades, the research provides valuable insights into the predictive capabilities of these models.

The findings reveal that GLS–QR significantly outperforms GLS in estimating the dynamics of poverty reduction, as evidenced by substantially higher adjusted coefficients of determination across various percentiles. Specifically, while GLS demonstrates a modest adjusted R–squared value of 3.8%, GLS–QR yields markedly superior results, with values ranging from 46% to 47% across the lower, middle and upper quartiles

These results shows the importance of incorporating quantile regression techniques alongside traditional least squares methods in analyzing complex socio–economic phenomena like poverty reduction. The superiority of GLS–QR suggests its potential utility for policymakers and economists in devising more targeted and effective poverty alleviation strategies tailored to the diverse needs of Nigeria's population. Moving forward, further research and policy implementation efforts should consider an advanced econometric methodologies like GLS–QR to address the persistent challenge of poverty in Nigeria comprehensively.

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