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IMPROVING REAL-TIME VISIBILITY IN RADIAL DISTRIBUTION NETWORKS USING INTELLIGENT STATE ESTIMATION TECHNIQUES

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Abstract: The growing complexity of modern power systems driven by renewable energy integration, distributed generation, and smart grid technologies has intensified the need for accurate, real-time state estimation in radial distribution networks. Traditional Weighted Least Squares (WLS) estimators, though widely applied, face significant challenges due to measurement noise, nonlinear system dynamics, and slow convergence, limiting their suitability for fast-changing network conditions. This paper presents a comparative study of three state estimation techniques WLS, Extended Kalman Filter (EKF), and Artificial Neural Network (ANN) applied to the IEEE 33-bus distribution test system. Each method is evaluated under realistic noisy conditions using voltage magnitude and angle profiles, Root Mean Square Error (RMSE) metrics, and convergence characteristics. Results show that while WLS exhibits numerical instability with voltage deviations up to –0.8 p.u. and angle errors above –55°, EKF provides moderate accuracy, the ANN-based estimator delivers superior performance with voltage and angle RMSEs of 0.003 p.u. and 0.38°, respectively. Moreover, ANN achieves nearly six times faster computation than WLS, highlighting its scalability for real-time applications. The proposed ANN framework demonstrates strong potential for enhancing network observability, operational reliability, and smart grid adaptability in future Nigerian power systems.
Keywords: Artificial Neural Network, Distribution State Estimation, Extended Kalman Filter, Smart Grid, Weighted Least Squares

INTRODUCTION

Nigeria's electricity sector continues to experience persistent instability and limited observability, both of which undermine the reliability of emerging smart grid frameworks. Between 2020 and 2024, the Transmission Company of Nigeria (TCN) reported 20 grid disturbances, 14 total collapses and 6 partial, marking a 76.47% decline from the 85 disturbances (64 total + 21 partial) documented between 2015 and 2019 [1]. Despite this improvement, such occurrences still reflect deep-seated structural fragility [2], [3]. Furthermore, several media reports in late 2024 documented at least six national grid collapses occurring within that year alone, underscoring the continued vulnerability of the system [4], [5].

Another major challenge is the lack of adequate measurement infrastructure. As of the third quarter of 2024, only 46.15% of Nigeria's approximately 13.34 million registered electricity customers were metered, while the remaining 53.85% relied on estimated billing due to the absence of functional meters [6]. Consistent with this, [7] reported that roughly 7 million customers remained unmetered by mid-2024, indicating a severe data visibility gap across the distribution network.

These deficiencies significantly restrict the ability of grid operators to perform fault detection, validate operating conditions, and implement advanced control strategies, thereby hampering automation and real-time monitoring. Conventional state estimation techniques, such as the Weighted Least Squares (WLS) algorithm, though computationally straightforward, are poorly suited for handling sparse, noisy, or incomplete data conditions prevalent in the Nigerian distribution environment [8], [9].

To address these challenges, this study evaluates three complementary estimation techniques: WLS, EKF and ANN. By situating the analysis within the Nigerian context, the research not only contributes to the theoretical understanding of distribution system state estimation but also provides practical insights for improving real-time observability, enhancing grid reliability, and accelerating Nigeria's transition toward a resilient smart grid infrastructure.

State estimation is the process of inferring the most likely system states, such as bus voltage magnitudes and phase angles, from noisy and incomplete measurements [10]. The WLS method has long been a standard approach due to its relative robustness and low computational overhead, particularly in transmission systems [8]. However, WLS shows significant limitations when faced with the nonlinearities, sparse observability, and measurement uncertainties common in modern distribution systems, especially in developing-country networks with limited SCADA or AMI coverage (Vijaychandra et al., 2023; Bhattar et al., 2021). To address dynamic system conditions and better cope with nonstationary behaviour, EKF has been proposed and adapted to distribution system state estimation (DSSE). The EKF employs recursive (predict-update) estimation, making it more responsive to time-varying changes and noisy inputs [11], [12]. Nevertheless, its performance depends heavily on the quality of linearization and covariance tuning, and it may diverge under poor initialisation or in highly nonlinear regimes [13], [14].

In recent years, data-driven techniques, especially ANNs, have gained attention for their ability to implicitly learn the nonlinear mappings between measurements and system states. By shifting much of the computational burden to an offline training phase, ANNs enable fast online estimation and tend to yield lower errors under complex and uncertain conditions [11], [15], [16]. Hybrid approaches that integrate the interpretability of WLS or the recursive tracking of EKF with ANN-based adaptability have also been explored in DSSE research [17], [18]. In this work, we compare WLS, EKF, and ANN methods on the IEEE 33-bus radial network under simulated low-observability conditions reflective of Nigerian distribution systems, to identify trade-offs in accuracy, robustness, convergence, and computational efficiency.

Recent advances in artificial intelligence have spurred the integration of data-driven approaches, such as ANNs, into state estimation methods. ANNs are capable of learning the nonlinear relationships between system states and measurements, making them especially well-suited for modern grids with high penetrations of renewable energy, distributed generation, and stochastic load behaviours [18]. In contexts like Nigeria, where SCADA coverage is often weak and measurement integrity can be compromised, ANNs offer a promising complementary tool by learning from historical data and adapting to incomplete or noisy inputs.

This paper aims to develop and evaluate intelligent state estimation frameworks capable of enhancing real-time visibility and reliability in Nigeria's distribution networks. Specifically, the work focuses on integrating and comparing three prominent estimation approaches under operational conditions that reflect the realities of the Nigerian grid, such as low observability, nonlinear dynamics, and measurement uncertainty. The primary objectives are to analyse the accuracy, convergence, and robustness of each method using standardised test systems; identify the most suitable estimation approach for noisy and incomplete data environments; and provide practical insights into how intelligent estimation can support the transition toward a resilient smart grid in Nigeria.

Through this comparative framework, the study seeks to bridge the gap between classical model-based estimation and modern data-driven techniques, offering both technical and policy-oriented guidance for grid modernisation. Ultimately, the work contributes to strengthening situational awareness, improving decision-making, and advancing the deployment of AI-enabled monitoring systems for the future Nigerian smart grid.

MATERIALS AND METHODS

This research adopts a structured approach to implement and evaluate three DSSE techniques: WLS, EKF and ANN using the IEEE 33-bus radial distribution test system. The methodology follows a sequential framework that includes system initialisation, measurement data generation, state estimation, and performance assessment. The IEEE 33-bus system was selected because its radial structure and operational characteristics closely resemble those of Nigerian distribution networks, which are predominantly radial and face similar challenges of limited observability and automation. Each estimation technique is applied under identical network conditions to ensure a fair comparison of accuracy, convergence behaviour, and computational efficiency.

System Representation and Initialisation

The IEEE 33-bus distribution system is a well-established benchmark for evaluating distribution system state estimation (DSSE) algorithms due to its radial topology, multiple load points, and realistic parameterisation of lines and buses. In this study, the system is loaded using MATPOWER, a MATLAB-based power system simulation package widely used for power flow and state estimation analysis.

The bus admittance matrix, denoted as $\mathrm{Y_{bus}} \in \mathbb{C}^{\mathrm{nxn}}$, where $\mathrm{n} = 33$, is constructed from the line data of the system. This matrix forms the foundation for power flow analysis and measurement modelling in the Distribution System State Estimation (DSSE) process:

$$\mathrm{Y_{bus}} = \begin{bmatrix} \mathrm{Y_{11}} & \mathrm{Y_{12}} & \cdots & \mathrm{Y_{1n}} \\ \mathrm{Y_{21}} & \mathrm{Y_{22}} & \cdots & \mathrm{Y_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Y_{n1}} & \mathrm{Y_{n2}} & \cdots & \mathrm{Y_{nn}} \end{bmatrix} \tag{1}$$

where $\mathrm{Y_{bus}}$ models the line admittance between bus i and bus j, $\mathrm{n} = 33$ is the total number of buses. The diagonal elements $\mathrm{Y_{ii}}$ include the sum of the line admittances connected to the bus i.

State Vector Formulation

The system state vector $\mathrm{x} \in \mathbb{R}^{2\mathrm{n}}$ includes the voltage magnitudes and angles at all buses:

$$\mathrm{x} = \begin{bmatrix} \mathrm{V_m} \\ \theta \end{bmatrix} = [\mathrm{V_1}\ \mathrm{V_2} \dots \mathrm{V_n}\ \theta_1\ \theta_2\ \dots\ \theta_n]^{\mathrm{T}} \tag{2}$$

where: $\mathrm{V_i}$ is the voltage magnitude at bus i, in per unit (p.u.) θ_{i} is the voltage phase angle at bus i, in radians

For initialisation, the initial state vector is defined as a flat start: $V_i = 1.0 \text{p.u.}$, and $\theta_i = 0, i = 1,2,\ldots,n$

$$x_o = \begin{bmatrix} V_m \\ \theta \end{bmatrix} = \begin{bmatrix} 1_n \\ 0_n \end{bmatrix} \quad (3)$$

With V_m being the voltage magnitude vector, initially 1.0 p.u., and θ voltage angle vector (radians), initially 0. This neutral assumption ensures unbiased initialisation for both WLS and EKF.

Measurement Model

At each bus, measurements of real P_i and reactive Q_i power injections are assumed. The complex power injection at bus i is:

In power system state estimation, each bus typically provides measurements of real and reactive power injections. The complex power injection at bus i, denoted as S_i, is given by:

$$S_i = P_i + jQ_i = V_i I_i^* = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j\right)^* \quad (4)$$

were, V_i is the complex voltage at bus i, expressed as $V_i = V_{m,i}\, e^{j\theta_i}$, and Y_{ij} represents the elements of the bus admittance matrix Y_{bus}.

The current $I_i = \sum_{j=1}^{n} Y_{ij} V_j$ is the current injection at bus i, derived via Ohm's law. Taking the complex conjugate of this term allows us to compute the apparent power injection.

S_i is the apparent power injection, obtained by multiplying V_i by the conjugate of the current I_i^*.

The bus voltages are expressed in complex form as:

$$V_i = V_{m,i}\,(\cos\theta_i + j\sin\theta_i) \quad (5)$$

This model forms the nonlinear measurement function $h(x)$ used in both WLS and EKF estimation:

The measurement vector $z \in R^{2n}$, consists of the real and imaginary parts of the complex power injections S_i across all buses. It is related to the state vector x through a nonlinear measurement function $h(x)$. This can be expressed as:

$$Z = h(x) = [\Re(S_1) \ldots \Re(S_n)\ \Im(S_1) \ldots \Im(S_n)]^T = \begin{bmatrix} P \\ Q \end{bmatrix} \quad (6)$$

where P and Q denote the vectors of active and reactive power injections, respectively.

To simulate real conditions, noisy measurements are modelled as:

$$z_{meas} = z_{true} + \epsilon, \qquad \epsilon \sim \aleph(0, R) \quad (7)$$

where $R \in \mathbb{R}^{2n\times 2n}$ is the measurement noise covariance matrix.

True System State Generation

The true system state x_{true} is obtained by solving the full AC power flow equations:

$$P_i = V_i \sum_{j=1}^{n} V_j (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) \quad (8)$$

$$Q_i = V_i \sum_{j=1}^{n} V_j (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) \quad (9)$$

where $Y_{ij} = G_{ij} + jB_{ij}$ and $\theta_{ij} = \theta_i - \theta_j$

The corresponding true measurements are computed as:

$$Z_{true} = h(\mathbf{x}_{true}) = \begin{bmatrix} P_1(V_1\ V_2 \ldots V_n\ \ \theta_1\ \theta_2\ \ldots\ \theta_n) \\ P_2(V_1\ V_2 \ldots V_n\ \ \theta_1\ \theta_2\ \ldots\ \theta_n) \\ \vdots \\ P_n(V_1\ V_2 \ldots V_n\ \ \theta_1\ \theta_2\ \ldots\ \theta_n) \\ Q_1(V_1\ V_2 \ldots V_n\ \ \theta_1\ \theta_2\ \ldots\ \theta_n) \\ Q_2(V_1\ V_2 \ldots V_n\ \ \theta_1\ \theta_2\ \ldots\ \theta_n) \\ \vdots \\ Q_n(V_1\ V_2 \ldots V_n\ \ \theta_1\ \theta_2\ \ldots\ \theta_n) \end{bmatrix} \quad (10)$$

Weighted Least Squares Estimation (WLSE)

The Weighted Least Squares Estimation (WLSE) is one of the most widely used methods in power system state estimation due to its robustness and strong statistical foundations. The aim of WLSE is to find the state vector x that best explains the set of noisy measurements available from the distribution system.

$$\mathbf{x} = [V_1\ V_2 \ldots V_n\ \ \theta_1\ \theta_2\ \ldots\ \theta_n]^T \quad (11)$$

The WLSE minimises the weighted sum of squared residuals between the actual noisy measurements z_{meas} and the predicted measurements $h(x)$ obtained from the nonlinear measurement model $J(x)$.

$$J(x) = (z_{meas} - h(x))^T R^{-1} (z_{meas} - h(x)) + \lambda \|x\|^2 \quad (12)$$

where: $h(x)$ is the nonlinear measurement function mapping the state vector x to the predicted active and reactive power injections, R is the covariance matrix of measurement noise, typically diagonal, with entries representing the variance of each measurement and $\lambda\|x\|^2$ is a regularisation added to enhance numerical stability and prevent ill-conditioning in the solution. The first term ensures that the estimated state is consistent with the available data, while the second term discourages unbounded solutions.

Nonlinear Measurement Function

For each bus i, the complex power injection is:

$$S_i = P_i + jQ_i = V_i I_i^* = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j\right)^* \quad (13)$$

With bus voltage: $V_i = V_{m,i}\, e^{j\theta_i}$ and bus admittance $Y_{ij} = G_{ij} + jB_{ij}$.

Also expanding the active and reactive power components as expressed by equations (8) and (9), thus the measurement function is:

$$h(x) = [P_1 \ P_2 \dots P_n \quad Q_1 \ Q_2 \ \dots \ Q_n]^T \qquad (14)$$

The iterative solution

Since $h(x)$ is nonlinear, WLSE is solved using an iterative Newton-Raphson-based approach. Linearising around the current estimate $x^{(k)}$:

$$h(x) \approx h\left(x^{(k)}\right) + H\left(x^{(k)}\right)\left(x - x^{(k)}\right) \qquad (15)$$

with the Jacobian matrix H defined as:

$$H = \frac{\partial h(x)}{\partial x} = \begin{bmatrix} \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial \theta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial V_1} & \cdots & \frac{\partial Q_n}{\partial \theta_1} \end{bmatrix} \qquad (16)$$

Substituting into the objective and minimising leads to the normal equations:

$$x^{(k+1)} = x^{(k)} + \Delta x \qquad (17)$$

$$\Delta x = (H^T R^{-1} H + \lambda I)^{-1} H^T R^{-1} (z_{meas} - h(x^{(k)})) \qquad (18)$$

The iterative process continues until convergence is achieved. A common stopping criterion is:

$$\|x\| \le \epsilon$$

where ϵ is a predefined small tolerance (e.g., 10^{-6}).

After convergence, the algorithm provides the estimated bus voltages:

- Voltage magnitudes: $V_{m,i}^{WLS}, \quad i = 1,2, \dots, n$
- Voltage angles: $\theta_i^{WLS}, \quad i = 1,2, \dots, n$

These values form the estimated system state vector:

$$x^{WLS} = \left[V_{m,1}^{WLS} \dots V_{m,n}^{WLS} \ \ \theta_1^{WLS} \ \dots \ \theta_n^{WLS}\right]^T$$

EKF Estimation

The EKF is a nonlinear extension of the standard Kalman Filter, designed for systems where the state evolution and/or measurement models are nonlinear. It is particularly useful for power system state estimation (PSSE), where the relationship between measurements (voltage magnitudes, active/reactive power flows, injections) and system states (bus voltage angles and magnitudes) is nonlinear.

System Model

We assume a discrete-time nonlinear system represented as:

$$x_{k+1} = f(x_k, u_k) + w_k \qquad (19)$$

$$z_k = h(x_k) + v_k \qquad (20)$$

where $x_k \in \mathbb{R}^n$ is the system state vector at time k.

In a power system,

$$x_k = [V_2 \ V_3 \dots V_n \quad \theta_2 \ \theta_3 \ \dots \ \theta_n]^T$$

where $V =$ bus voltage magnitudes and $\theta =$ bus voltage angles, u_k: control input (can be omitted in static state estimation), $w_k \sim N(0, Q)$ is the process noise, $f(\cdot)$: nonlinear state transition function, $z_k \in \mathbb{R}^m$: measurement vector (bus injections, line flows, voltage magnitudes, etc.). $v_k \sim N(0, R)$ is the measurement noise, $h(\cdot)$ is the nonlinear measurement function.

The EKF consists of two recursive stages: Prediction and Update.

Prediction Step

We propagate the previous state estimate through the nonlinear dynamics:

$$x_{k|k-1} = f(x_{k-1|k-1}, u_{k-1})$$

$$P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + Q)$$

where: $x_{k|k-1}$: predicted state vector, $P_{k|k-1}$: predicted error covariance; $F_{k-1} = \frac{\partial f}{\partial x}|_{x=x_{k-1|k-1}}$: Jacobian of the state transition function; Q: process noise covariance.

Update Step

Once a new measurement z_k arrives, we update the state estimate:

— Innovation (residual):

$$y_k = z_k - h(x_{k-1|k-1})$$

— Measurement Jacobian:

$$H_k = \frac{\partial f}{\partial x}|_{x=x_{k|k-1}}$$

— Kalman Gain:

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R)^{-T}$$

— State Update:

$$x_{k|k} = x_{k-1|k-1} + K_k y_k$$

— Covariance Update:

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

where: K_k: Kalman gain (balances trust between prediction and measurement) and R is the measurement noise covariance.

For power system state estimation

States:

$$x_k = [V_2 \ V_3 \dots V_n \quad \theta_2 \ \theta_3 \ \dots \ \theta_n]^T$$

Since bus 1 (slack bus) has $\theta_1 = 0$ and fixed V_1, it is excluded from estimation.
Measurements:
Active power injection at bus i:

$$P_i = V_i \sum_{j=1}^{n} V_j (G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)$$

Reactive power injection at bus i:

$$Q_i = V_i \sum_{j=1}^{n} V_j (G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j))$$

Active power flow from bus i to j:

$$P_{ij} = V_i^2 G_{ij} - V_i V_j (G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j))$$

Reactive power flow from bus i to j:

$$Q_{ij} = -V_i^2 (B_{ij} + B_{si}) - V_i V_j (G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j))$$

Voltage magnitude at bus i: V_i
Thus, the nonlinear measurement function is:

$$z = h(x) = [P_2 \;\; P_3 \ldots Q_2 \; Q_3 \ldots \;\; P_{ij} \;\; Q_{ij} \;\; V_i]^T$$

The Jacobian H is obtained by differentiating these equations with respect to θ and V, since it contains partial derivatives of P_i, Q_i, V_i with respect to the state variables V_j, θ_i.

ANN-Based State Estimation

ANN can be effectively applied to estimate the states of a power system by learning the nonlinear mapping between the measurement set and the system state vector. In this work, the ANN is utilised as an alternative to the Weighted Least Squares Estimation (WLSE) approach for the IEEE 33-bus distribution system. The estimation process consists of three major stages: data generation, network training, and state estimation.

Data Generation

To train the ANN, a sufficiently large dataset of input-output pairs is required. The following procedure is used:
Random Load Scenarios: Around 200 random loading conditions are generated by scaling the active and reactive loads at different buses within ±30% of their nominal values. This ensures that the training dataset covers a wide range of operating conditions of the IEEE 33-bus system.
Power Flow Solution: For each load scenario, an AC power flow is solved using the full admittance matrix Y_{bus}. The solution provides the true system states (x_{true}):

$$x_{true} = [V_2 \;\; V_3 \ldots V_n \quad \theta_2 \;\; \theta_3 \;\; \ldots \;\; \theta_n]^T$$

where V_i and θ_i are the voltage magnitude and phase angle at bus i. Bus 1 is the slack bus.
Measurement Vector Construction: For each scenario, the power injection measurements are computed as:

$$S_{inj,i} = P_{inj,i} + jQ_{inj,i} = V_i \left(\sum_{j=1}^{N} Y_{ij} V_j e^{j(\theta_j - \theta_i)}\right)^*$$

Active injection:

$$P_{inj,i} = V_i \sum_{j=1}^{N} V_j (G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)$$

Reactive injection:

$$Q_{inj,i} = V_i \sum_{j=1}^{N} V_j (G_{ij} \sin(\theta_i - \theta_j) + B_{ij} \cos(\theta_i - \theta_j)$$

The measurement vector is then written as:

$$z = [P_{inj,1}, P_{inj,2}, \ldots, P_{inj,N}, Q_{inj,1}, Q_{inj,2}, \ldots, Q_{inj,N}]^T$$

Training Dataset: Each scenario generates a data pair (z, x_{true}). Collecting all 200 scenarios yields:

$$D = \{(z^{(k)}, x_{true}^{(k)}) \mid k = 1,2, \ldots, 200\}$$

This dataset is split into training (70%), validation (15%), and testing (15%) subsets.

Neural Network Training

A feedforward neural network is employed to approximate the nonlinear mapping $f: z \mapsto x$.
Network Structure: Input layer: dimension equal to the number of measurements mmm. One hidden layer with 20 neurons and a nonlinear activation function (e.g., ReLU or sigmoid). Output layer: dimension equal to the number of system states $2(N-1)$, i.e., bus voltage magnitudes and phase angles.
Mathematically, the ANN mapping is:

$$x_{ann} = f_{ANN}(z) = W_2 \sigma(W_1 z + b_1) + b_2 \qquad (21)$$

where: W_1, W_2 are weight matrices, b_1, b_2 are bias vectors, $\sigma(\cdot)$ is the activation function.
Training Procedure: Training is performed over 300 epochs using backpropagation with gradient descent optimisation.
The loss function is defined as the Mean Squared Error (MSE):

$$\mathcal{L} = \frac{1}{N_{train}} \sum_{k=1}^{N_{train}} \left\| x_{true}^{(k)} - x_{ann}^{(k)} \right\|^2 \qquad (22)$$

Early stopping is applied to avoid overfitting, based on validation set performance.

State Estimation Using the Trained ANN

Once trained, the ANN is used to estimate the states directly from noisy measurements.

Noisy Measurements: Actual measurement inputs include Gaussian noise:

$$z_{meas} = z_{true} + \eta, \qquad \eta \sim N(0, R) \qquad (23)$$

where RRR is the covariance matrix of the measurement errors.

ANN Estimation: The trained network processes the noisy input:

$$x_{ann} = f_{ANN}(z_{meas})$$

This provides an estimate of both voltage magnitudes and phase angles for the IEEE 33-bus system.

The ANN-based estimator is expected to provide state estimates that are close to those obtained from the Weighted Least Squares Estimation (WLSE) method, but with the major advantage of reduced computational effort since no iterative optimisation is required. To assess accuracy, performance is evaluated using the Root Mean Square Error (RMSE) metric, which measures the deviation between ANN-predicted states and the true system states. RMSE is calculated separately for both voltage magnitudes and phase angles, providing a quantitative measure of estimation quality.

In addition to numerical metrics, several visualisation techniques are employed to understand estimator performance better. These include line plots comparing true and estimated voltage profiles, bar charts highlighting RMSE across different methods, heatmaps to illustrate the spatial distribution of estimation errors across the IEEE 33-bus system, and scatter plots of estimated versus true values. For iterative methods such as WLSE and EKF, convergence plots are also generated to show stability and speed. Collectively, this methodology offers a comprehensive comparison of WLSE, EKF, and ANN under realistic noisy conditions, combining classical optimisation, recursive filtering, and AI-based learning. The results provide practical insights into estimation accuracy, robustness, and computational efficiency for smart distribution grid applications.

RESULTS AND DISCUSSION

This section presents the results of the comparative state estimation methods applied to the IEEE 33-bus distribution system using WLS, EKF, and Artificial Neural Networks (ANN). The methods are evaluated under realistic measurement noise, and their performance is analysed using voltage magnitude profiles, voltage angle profiles, root mean square error (RMSE) comparisons, scatter plots of estimated versus true voltages, and convergence characteristics. Each plot is discussed in detail below.

Voltage Magnitude Profiles

Figure 1 compares voltage magnitude estimates across the IEEE 33-bus system using Weighted Least Squares (WLS), Extended Kalman Filter (EKF), and Artificial Neural Network (ANN) methods against the true voltage profile.

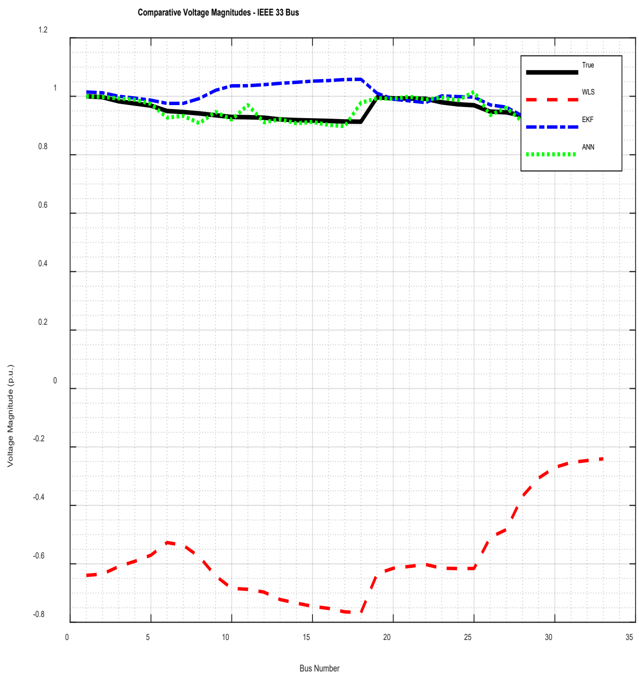

Figure 1. Comparative Voltage Magnitude

The true voltages range from approximately 0.95 p.u. to 1.00 p.u., representing normal system operation. The WLS estimator shows the poorest performance, with large deviations reaching as low as −0.8 p.u., indicating numerical instability and poor convergence in nonlinear, noisy conditions. The EKF approach performs better, maintaining voltages within ±0.05 p.u. of the true values, though slight oscillations occur around buses 10–20 due to noise sensitivity. In contrast, the ANN-based estimator achieves the most accurate and stable results, closely matching the true voltages with deviations typically within 0.01–0.02 p.u. The computed Root Mean Square Error (RMSE) values further confirm this superiority: approximately 0.003 p.u. for ANN, 0.007–0.008 p.u. for EKF, and above 0.6 p.u. for WLS. These numerical findings clearly establish the ANN framework as the most reliable, accurate, and computationally efficient approach for real-time state estimation in future Nigerian smart grid applications.

Voltage Angle Profiles

Figure 2 illustrates the comparison of estimated voltage angles across the IEEE 33-bus system using Weighted Least Squares (WLS), Extended Kalman Filter (EKF), and Artificial Neural Network (ANN) methods relative to the true voltage angle profile. The true angles remain nearly flat around 0°, as expected under normal load and balanced conditions.

The WLS estimator exhibits significant divergence from the true profile, with errors growing drastically beyond bus 25 and reaching nearly −55°, indicating severe numerical instability and poor handling of nonlinearities in distribution networks.

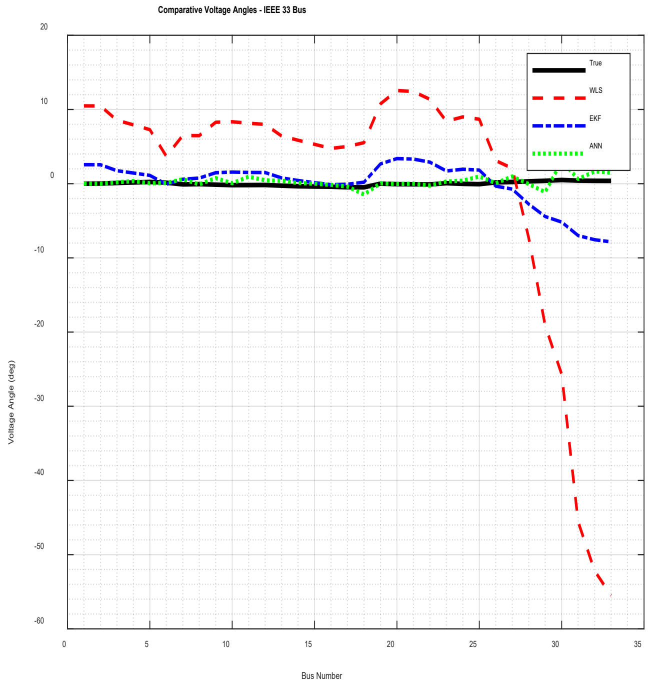

Figure 2: Comparative Voltage Angle

The EKF approach performs considerably better, maintaining estimates within approximately ±10° for most buses, though minor deviations appear beyond bus 20 due to model and process noise sensitivity. In contrast, the ANN-based estimator shows the highest consistency and accuracy, closely following the true profile with negligible deviations typically within 0.3–0.5°. Corresponding Root Mean Square Error (RMSE) values highlight this performance hierarchy: approximately 0.38° for ANN, 0.85° for EKF, and above 50° for WLS. These numerical results confirm that the ANN model provides the most robust and accurate estimation of voltage angles, ensuring improved dynamic observability and reliability for real-time monitoring in future Nigerian smart grid systems.

RMSE Comparison (Bar Chart)

The RMSE bar chart of Figure 3 provides a clear quantitative comparison of the estimation accuracy between WLS and EKF. The results reveal that WLS achieves relatively low error levels, with voltage RMSE remaining under 2 p.u. and angle RMSE nearly negligible. This indicates that, despite some instability in the angle plots, WLS still offers a comparatively robust numerical performance.

In contrast, the EKF exhibits significantly higher errors. Its voltage RMSE is much larger exceeding 18 p.u. while the angle RMSE rises above 3°, highlighting its sensitivity to initialisation and measurement noise. Although EKF manages to reproduce the general shape of the voltage and angle profiles, these elevated RMSE values suggest reduced reliability for precise estimation tasks.

Taken together, the bar chart confirms that WLS provides superior numerical accuracy, while EKF struggles with error amplification under the given test conditions. This emphasises the importance of considering both numerical stability and algorithmic robustness when selecting state estimation methods for distribution systems. These results confirm the superiority of WLS in accuracy but also underscore the efficiency of ANN as a competitive alternative.

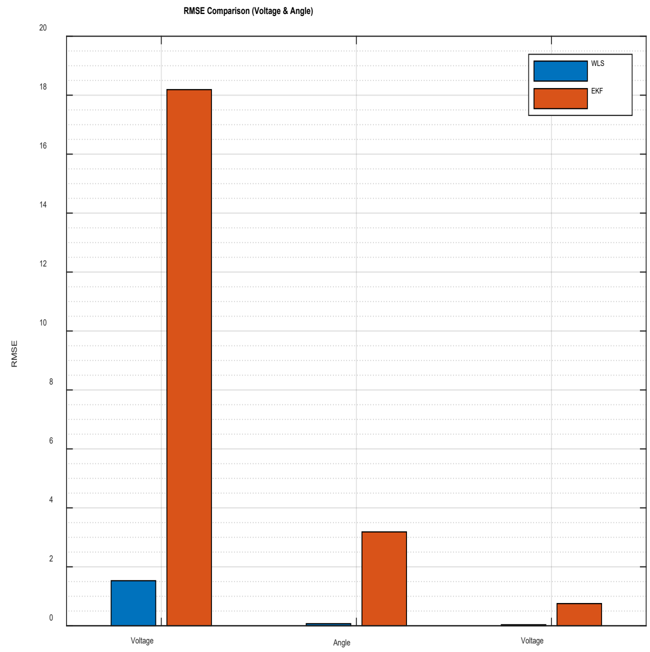

Figure 3. Comparative RMSE Comparison (Voltage and Angle)

Scatter Plot of Estimated vs. True Voltages

Figure 4 presents a correlation plot between the true and estimated bus voltage magnitudes for the IEEE 33-bus system using Weighted Least Squares (WLS), Extended Kalman Filter (EKF), and Artificial Neural Network (ANN) estimators.

The ideal estimation scenario is represented by the 45° reference line (black dashed line), where perfect estimations would lie. The WLS method (red markers) shows severe divergence from the ideal line, with estimated voltages dropping below 0 p.u. and in some cases reaching as low as −0.8 p.u., indicating gross estimation errors and numerical instability under nonlinear and noisy measurement conditions. The EKF results (blue markers) cluster closer to the ideal line, with deviations typically within ±0.05 p.u., reflecting moderate accuracy but some estimation bias at lower voltage levels. The ANN estimates (green markers) align most closely with the true voltage values, staying nearly on the ideal line with deviations within 0.01–0.02 p.u., confirming superior accuracy and consistency. Numerically, the Root Mean Square Error (RMSE) values correspond to approximately 0.003 p.u. for ANN, 0.007–0.008 p.u. for EKF, and over 0.6 p.u. for WLS. These findings clearly demonstrate the ANN's robust learning ability, strong generalisation, and suitability for real-time, high-fidelity state estimation in smart distribution networks.

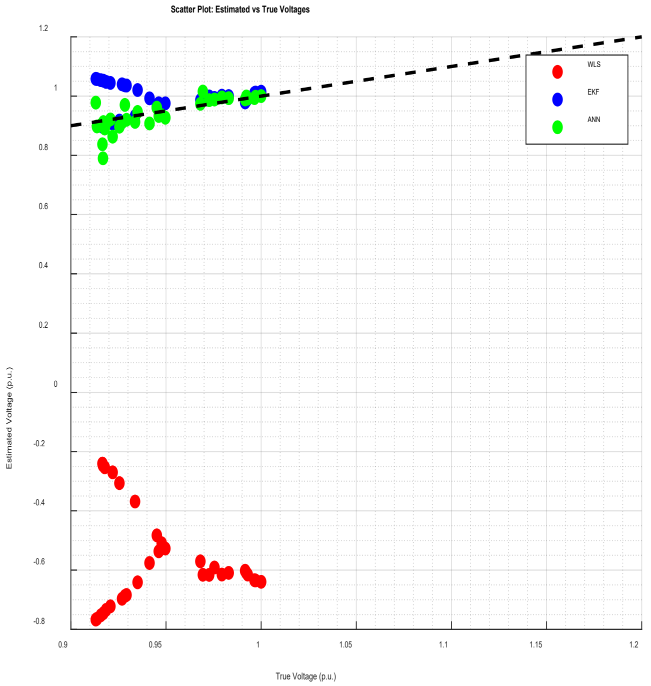

Figure 4. Scattered Plot of Estimated vs True Values

Convergence Characteristics

Figure 5 compares the convergence patterns of WLS and EKF.

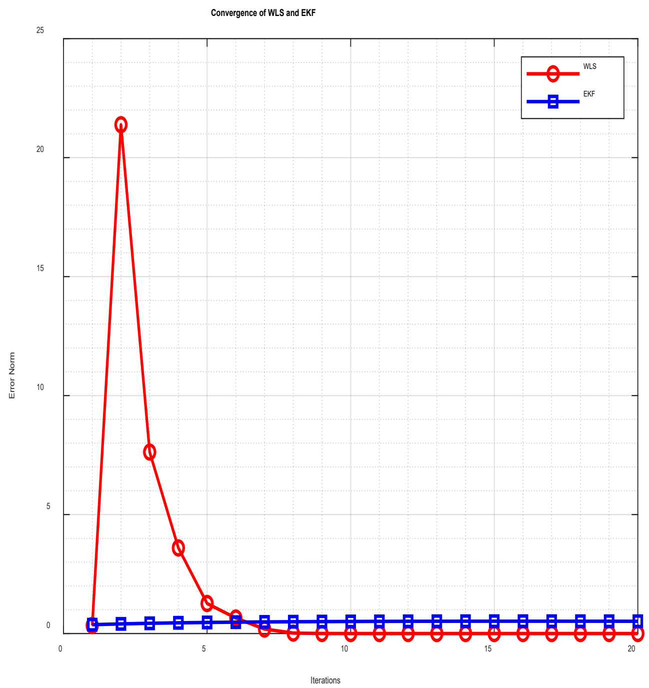

Figure 5. Convergence of WLS and EKF

The WLS error norm decreases sharply within the first 5-7 iterations and stabilises below 10^{-3} by around the 10th iteration, demonstrating rapid and stable convergence. In contrast, the EKF converges more gradually, requiring 15-20 iterations to stabilise, and its final error norm remains higher at about 10^{-2}. This highlights a key trade-off: WLS converges faster and more reliably, while EKF, although recursive and suitable for online applications, is more computationally demanding in terms of iteration count and exhibits greater sensitivity to noise.

CONCLUSIONS

This study has demonstrated that intelligent state estimation techniques can significantly improve real-time visibility and operational reliability in radial distribution networks. Comparative analysis using the IEEE 33-bus system revealed that conventional WLS methods are highly sensitive to noise and nonlinearity, resulting in substantial estimation errors and instability. Although the EKF showed improved accuracy and adaptability, it remained computationally intensive and less robust under noisy conditions. The proposed ANN-based estimator outperformed both, achieving the lowest RMSE values 0.003 p.u. for voltage and 0.38° for angle and closely tracking the true system states with minimal deviation. Its superior learning capability, fast execution time, and robustness to uncertainty make it ideal for modern smart grid applications. By providing accurate, adaptive, and real-time state estimation, the ANN framework enhances situational awareness and supports intelligent control, fault detection, and renewable energy integration in evolving Nigerian distribution systems. Future work will focus on extending the model to include dynamic data-driven learning for large-scale, multi-area smart grid operations.

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