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A SHORT REVIEW ON THE DYNAMIC INSTABILITY OF STRAIGHT BEAMS

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Abstract: This article provides a short review of the current state of research about the dynamic stability of straight beams subjected to various parametric excitations. Starting from the pioneering findings of Bolotin, the study shows how modern approaches can be used to identify stability boundaries. The review covers multiple beam issues including thin-walled, stepped, sandwich and spinning ones. This article highlights the critical influence of boundary conditions, damping, and geometric properties on stability maps, offering a practical reference for structural engineers designing slender, cost-effective members in mechanical and civil engineering systems.

Keywords: review; dynamic stability; beam; stability diagram

INTRODUCTION

Beam stability is a broad and popular topic within mechanics. Euler is known to be the pioneer of the field. About two centuries later, Bolotin [1] made a quantum leap in its extension to dynamic systems with time-dependent loading using the harmonic balance method to find the boundaries between stable and unstable parameter regions of Mathieu or Hill-type equations. It was discovered that the boundaries between different stability regions are characterised by periodic solutions. To find these, the solutions were given in Fourier series. With such solutions substituted back to the differential equations of the problem, it can be reformulated with algebraic equations. Nontrivial solution is found when the determinant of the coefficients becomes zero.

The dynamic stability analysis of beams typically involves a combination discretization techniques with analytical or numerical methods. Discretization is commonly applied to transform the resulting partial differential equations into ordinary ones. Such, like the Galerkin method (the assumed mode method) or the finite element method are common options. The Galerkin method approximates the solution using a set of predefined base functions (mode shapes). It can provide highly accurate results for the fundamental instability regions with very few degrees of freedom. Moreover, it is often combined with perturbation techniques to gain deeper insight into which vibration modes become unstable. A setback is that it is mainly applicable to simpler geometries. In the meantime, the finite element method that discretizes the structure into sub-domains uses local approximation that is not limited in the geometry. Although it is efficient, it might require more computation time.

In identifying the stability maps (region boundaries), several approaches are used. The most classical one is the Bolotin method that assumes the solution is periodic on the boundary between stable and unstable regions. With Fourier-series expansion, the problem differential equation is transformed into a set of algebraic equations. Although the method is reliable, it can become cumbersome for multi-degree-of-freedom systems. Another technique is the Multiple Scale Perturbation Method that is suitable to find higher order approximate solutions and analyzing subharmonic or combination resonances. It introduces multiple time frames. While it provides an in-depth insight into the behaviour of the system, it can become very complex for higher-order approximations. The Method of Averaging averages the rapidly oscillating terms over one period of the excitation, yielding a set of equations, but is less accurate than the previous technique. The Homotopy Perturbation Method combines the homotopy and perturbation techniques. While it does not require the introduction of a small parameter like classical perturbation techniques, apart from the embedding parameter, and converges quickly even for highly non-linear systems, it can be slow.

Given the importance of the topic of beam instability in a bunch engineering fields, the article intends to briefly review some articles that represent the major directions and most common methods that are preferred by scientists who have made a great progress in extending, generalising former results. It is crucial to continuously improve models aiming at the dynamic instability of structures and structural elements. With technology ever improving and the application of cost-effective slender members with reduced weight getting widespread, the topic is still live.

The article intends to contribute to the better design of such engineering structures.

DYNAMIC INSTABILITY FINDINGS

Article [2] addresses both transverse and axial beam vibrations given that the loading is composed of a static and a harmonic component. The cross-section is uniform and the beam can undergo infinitesimally small displacements and strains by assumption. The centroidal axis is assumed to be inextensible. Based on the modelling assumptions, a linear, second order Mathieu-Hill differential equation is established and the periodic solution is sought as a Fourier series. Ultimately, a system of linear homogeneous equations is set up where the determinant (equation of boundary frequencies) of the coefficient matrix vanishes for non-trivial solution.

The study is also extended from pinned-pinned beams to elastically restrained ones by means of the separation of variables method. The authors have demonstrated the applicability of their proposed method in some examples, where the regions of instability are drawn. It turns out that as the spring stiffness of the support is increased, the instability regions shift to higher frequencies.

Thin-walled beam instability is assessed in [3] under concentrated periodic moments at one end that cause the bending-torsion of the subject. Large rotations, small strains and cross-sectional warping are assumed through the Euler-Bernoulli and the Vlasov theories. The virtual work principle is applied to obtain the coupled equations of motion containing higher order rotational effects. It is the Ritz-Galerkin method that makes it possible to reveal the stability domains. It is found that keeping higher terms in the Ritz approximation has no real effect on the stability regions. Moreover, the dynamic stability can be improved by means of decreasing the static buckling moment percentage and applying higher values for the moment distribution factor.

Stepped cantilever beams are in the spotlight in [4] in case a periodic compressive force is exerted. Additionally, the effect of discrete elements like lumped masses are also incorporated into the model. Small strains and small rotations are assumed through the classical Euler-Bernoulli hypothesis. Nontrivial solutions to the boundary value problem are found from the transcendental equation for the natural frequencies.

The dynamic stability behaviour is characterised by a Mathieu-type equation, whose periodical solutions are the Strutt card. According to the computational results, increasing the attached mass or the inertia of the rotational element both act in a negative way for the stability, independently of their position.

The least favourable position for concentrated mass is the free end of the beam.

Within article [5], a shear deformable beam model is set up to focus on the static and dynamic stability of delaminated composite beams, accounting with non-linear strain terms.

A finite element formulation is introduced to tackle the selected issue. The equation of motion is found using Hamilton's principle. Rayleigh damping is accounted as a linear combination of the mass and stiffness matrix. Ultimately, a Mathieu-type equation is set up and solution is found using Bolotin's harmonic balance method. The boundaries between stable and unstable parts (stability maps) for multiple parameter sets are determined from the vanishing of the system matrix determinant.

It is found the applied damping does not impact significantly the size of unstable regions. Furthermore, delamination does not change the shape of unstable regions compared to intact beams. Instead, it just shifts them along the frequency axis. Furthermore, the dynamic stability of three-layered sandwich beams is in the spotlight in [6]. Beams are subjected to time-dependent axial force (with a constant and a harmonic term) through a novel non-linear theory including cross-sectional deformations, being the improvement of the 'broken line' model.

Thus, the deformed shape of the planar cross-section is a curved one. The classical 'broken line' model can be retrieved as a simplification of the newly introduced one. The mathematical model incorporates shearing effect in each layer. Hamilton's principle yields the differential equations of motion. Assuming harmonic solutions to the displacements, the governing equation of the problem is found, that is, a Mathieu-type equation. It is concluded that the ratio of the thickness of the facings to the core thickness on the fundamental natural frequency and critical force is significant.

Article [7] is about spinning, homogeneous beams of rectangular section having constant angular velocity. The Euler-Bernoulli theory is used with small deflections. There is no account for coupling between torsional and transverse vibrations. Hamilton's principle and the assumed mode method are used to find the equations of motion. After transformation, these lead to an eigenvalue problem for the critical angular velocities. Unlike some previous studies that assume identical end conditions for both principal directions, this paper focuses on cases where support conditions differ - e.g., clamped-free in one direction, simply supported-free in the other direction. Transverse deflections are coupled due to the spinning motion of the beam. The stability is significantly influenced by the flexural rigidity ratios of the rectangular cross-section. Furthermore, when end conditions are identical in both directions and

flexural rigidities are equal too, unstable zones shrink to isolated points. However, with different end conditions, these unstable zones remain broader even when the rigidities are equal. Increased divergence in flexural rigidities between the two principal directions leads to smaller stable regions. For beams with large differences in flexural rigidities, a critical speed exists above which the motion remains permanently unstable. The presented matrix formulation provides a simplified yet effective tool for engineers to analyze the stability of rotating shafts, drills, and satellite booms. Moderately thick beams are considered in [8], with transverse shear deformations and rotatory inertias being incorporated using the Timoshenko theory. The harmonic balance method and the perturbation method are both applied during the analysis. Analytical approach is used for simply supported beams and finite element method for other beams and frames. The load is a compressive, periodic force.

A Fourier series solution is sought for the governing differential equation. Harmonic Balance Method is employed to find the boundaries of the first, second and third instability regions. The effect of linear damping is as well accounted. Alternatively, the perturbation (small parameter) method is as well tested. This consideration yields an infinite recursive sequence of differential equations. Moreover, for more complex frames and beams, a finite element technique is applied. The major conclusions include that the moderate beam thickness affects most notably the third instability region. Here the differences as per the Euler-Bernoulli and Timoshenko theories are also notable. At the same time, it is also concluded that the effect of rotatory inertia is negligible. The presence of damping shrinks the instability regions, making the first instability region the most critical one. It is also noted that the perturbation method proves to yield the same result as the harmonic balance method requiring less computational time. Therefore, it is recommended for engineering applications.

Study [9] provides a comprehensive review and development of the dynamic analysis of beam structures under moving loads. It focuses on implementing Hamilton's principle to derive governing equations for transverse vibrations and discusses the efficiency of different numerical approaches. The paper highlights that standard finite elements can induce significant errors (up to 30%) in the natural frequencies due to the polynomial shape functions. It also warns about numerical instabilities when using 'exact' shape functions in double-precision software due to the very large or small numbers in hyperbolic functions. In the article, the concept of dynamic

stiffness matrix is presented as a general principle applicable to finite, semi-infinite, and infinite beams. The study details the modal expansion method, discussing the advantages and disadvantages of using series governed by damped or undamped modes. It emphasizes that for beams on elastic foundation, a high number of modes (e.g., 50 or more) is often required for accurate results. The Hamilton principle is an efficient tool to obtain the governing equations for various beam structures. The paper concludes that proper reformulation of governing equations is essential to ensure the orthogonality of natural vibration modes, which simplifies the solution of the modal equations. While often neglected, the effect of load inertia (moving mass) should be considered for more realistic results, accordingly. At the same time, study [10] is dedicated to cantilever beams, undergoing axial oscillations without rotations. However, there is account for motion-induced stiffness variation: as the beam accelerates toward its free end, it undergoes compression which decreases its stiffness, whereas acceleration toward the fixed end results in an increased stiffness. The authors employed Kane's method to derive the linear equations of stretching and bending motion, utilizing a specific 'stretch variable' to more accurately capture the motion-induced bending stiffness variations - shearing and torsion effects are neglected.

The stability analysis is conducted using the multiple scale perturbation method, which allows for the development of both first- and second-order approximate solutions. The accuracy of the stability diagrams is subsequently verified through direct numerical integration of the equations of motion. The findings reveal that significant unstable regions emerge near the first bending natural frequency, as well as at twice the first and second bending natural frequencies. A key conclusion is the difference between the first- and second-order approximate solutions, highlighting the necessity of second-order analysis for reliable prediction.

The stability diagrams serve as a practical reference for the design of the likes of high-speed sewing machine needles and similar slender cantilever structures. Spinning pretwisted beam instability is in the spotlight in [11]. Beams are subjected to harmonic compressive axial loads. The study focuses on how rotational speed, pretwist angle, and the loading period affect the stability. The Euler-Bernoulli beam theory is employed from kinematical side. The multiple scale method is then used to determine the regions of instability caused by parametric excitations, where the axial force consists of both a steady-state value and a time-dependent harmonic component. The results reveal that flutter instability regions exist for

beams with non-zero pretwist angles, even within otherwise stable critical speed zones. These instability regions effectively divide the stable operating zones into smaller sub-regions. Furthermore, as the rotational speed varies, consistent shifts and widening of the unstable regions occur due to axial load perturbations. The analysis also demonstrates that the width of these unstable regions decrease as the pretwist angle or compressive axial force is reduced, or as the beam's aspect ratio approaches to unit. These findings provide critical insights for the safe design and operation of rotating machinery under periodic loading conditions, like turbine blades or drill bits. Paper [12] presents a finite element solution, developed to reliably identify the zones where structural instability occurs as a function of the applied dynamic force and the excitation frequency of beams and frames. The model is capable of handling different boundary conditions, inertia effects, and different stiffness characteristics, transforming the problem into a computationally manageable eigenvalue problem based on Bolotin's method. The study includes several numerical applications to demonstrate the versatility of the proposed approach, including simple beams and more complex framed structures.

The key results highlight how variations in constraint types, damping, and structural geometry significantly influence the location and size of the instability regions. By providing a systematic way to generate stability diagrams, the paper offers a practical tool for structural engineers to ensure that buildings and mechanical systems operate safely outside of dangerous zones. The transverse parametric vibrations of an axially moving, tensioned beam is assessed in [13] based on the Timoshenko theory.

The study focuses on beams with simple supports where the axial transport speed is assumed to be a harmonic fluctuation around a constant mean value. To analyze the system, the governing equations of motion are first discretized into a set of ordinary differential equations using the Galerkin method. The method of averaging is applied to explore the instability phenomena caused by subharmonic and combination resonances.

The results of the study include the determination of instability boundaries and their verification through direct numerical integration. As it turns out, both the mean axial speed and the internal tension significantly influence the stability regions. Specifically, the paper highlights that the Timoshenko beam model with shear deformation and rotary inertia provides a more comprehensive understanding of stability boundaries. These findings are particularly relevant for the design and safety analysis of high-speed mechanical systems,

such as power transmission belts, band saws, and elevator cables, where axial acceleration can lead to dangerous parametric resonance.

Paper [14] explores the dynamic behavior and stability of a beam-moving mass system, accounting for linear and nonlinear inertia terms of the moving mass as well as for friction between the mass and the beam. The study addresses a complex nonlinear time-varying system where the moving mass significantly influences the response of the structure. To analyze the system, the governing partial differential equations are discretized using the Galerkin method.

Then the Homotopy Perturbation Method (HPM) is applied to obtain an approximate solution for the resulting nonlinear equations of motion. The key results include the determination of stability boundaries and resonance curves within the mass-velocity parametric plane. By using a semi-analytical approach, the authors identify the regions where the system becomes unstable due to the interaction between the speed of mass and the properties of the beam. The findings are validated and confirmed by numerical simulations.

The study concludes that considering friction and nonlinear inertia terms is essential for a precise assessment of the system's dynamic stability, providing valuable data for the design of transportation infrastructure and robotic systems involving moving components. Article [15] investigates uniform beams, specifically focusing on the second regions of instability, which have been less frequently reported in existing literature compared to the first regions. The governing equations for such systems are of the Mathieu-Hill type, where parametric instability occurs due to periodic axial loading. To provide an improved assessment than the classical ones, the authors utilize the finite element method. The results of the study demonstrate that the widths of the second instability regions are significant and, therefore, cannot be neglected in structural analysis. The research examines how different factors, such as the static load factor and the dynamic load factor, influence the size and position of these regions. By providing a detailed numerical analysis, the paper offers crucial insights for engineers to avoid resonance not only in the primary but also in the secondary instability zones, thereby enhancing the dynamic safety and reliability of beam-like structures under periodic excitation.

CONCLUSIONS

As it turned out, the dynamic instability of straight beams is a complex field where the choice of mathematical modelling and discretization directly impacts the accuracy of predicted stability regions (maps). Research showed that while Bolotin's method is the reliable standard, in many cases a

second-order perturbation analysis is essential for finding important instability domains. Furthermore, the stability boundaries are significantly influenced by physical factors such as support stiffness, spinning motion, and material defects like delamination. As engineering trends move toward increasingly slender and lightweight members, these advanced tools become indispensable for ensuring the safe operation of modern systems.

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