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THE INFLUENCE OF LOAD TYPES ON THE ELASTIC STABILITY OF ARCHES

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Abstract: The topic of stability is an important one within engineering since buckling can lead to failure, even without exceeding the elastic limit of the material. Although the topic itself is classical, it is continuously evolving thanks to the modern materials and numerical techniques. Loaded curved members, unlike straight ones, undergo the coupled action of axial compression and bending at the same time and show nonlinear behaviour. Therefore, the pre-buckling deformations cannot be ignored for arches. This research is about the elastic stability of arches. Three load types (concentrated, vertical distributed, radial distributed) are tested on three kinds of arches (shallow, moderately shallow, deep). Commercial finite element software Abaqus is utilized, accounting for geometrical nonlinearities for a better reliability. Not only the buckling load change but also the equilibrium paths and arch shapes are assessed for multiple load domain lengths.

Keywords: stability; arch; nonlinear; finite element analysis

INTRODUCTION

The topic of stability is an important one within engineering since buckling can lead to failure, even without exceeding the elastic limit of the material [1]. Although the topic itself is classical, it is continuously evolving thanks to the modern materials and numerical techniques. Loaded curved members, unlike straight ones, undergo the coupled action of axial compression and bending at the same time and show nonlinear behaviour [2]. Therefore, the pre-buckling deformations cannot be ignored for arches. Within this topic, article [3] investigates shallow arches under concentrated and distributed forces from both theoretical and experimental aspects. In [4], an analytical solution is established for thin-walled arches under a point load. Parametric studies are performed to assess the effect of the likes of rotational support stiffness or central angle. Uniformly distributed vertical load over the entire arch span is incorporated in [5]. It is found that the parameter perturbation method is effective for shallow arches. Moreover, for arches with high rise to span ratio, the effect of the axial load component is relevant. Non-uniform shallow arches are investigated in [6]. There are three constant segment regions and imperfections are as well incorporated for a better insight.

The current article aims to broaden the knowledge about arch stability by performing finite element simulations for multiple arch geometries and load scenarios. Though, the majority of the literature tends to address symmetric arrangements, this time, the support conditions are nonsymmetric.

MODEL PROPERTIES

Pinned-fixed circular arches are considered, as shown in Figure 1. The centerline (centroidal axis)

has a constant radius R . The arch length is $S=2\theta R$. The span is l and the altitude is h . The cross-section and material are uniform. The latter is characterised by the modulus of elasticity E , and the Poisson number ν , given that the material behaviour is linearly elastic, isotropic. Loading occurs in plane xy that is a symmetry plane. Out of plane motion is restrained. The loading is generally a distributed radial or vertical one with constant f intensity, being symmetric to the centerline symmetry axis. Its domain is identified by angle coordinate $B=[0;\theta]$.

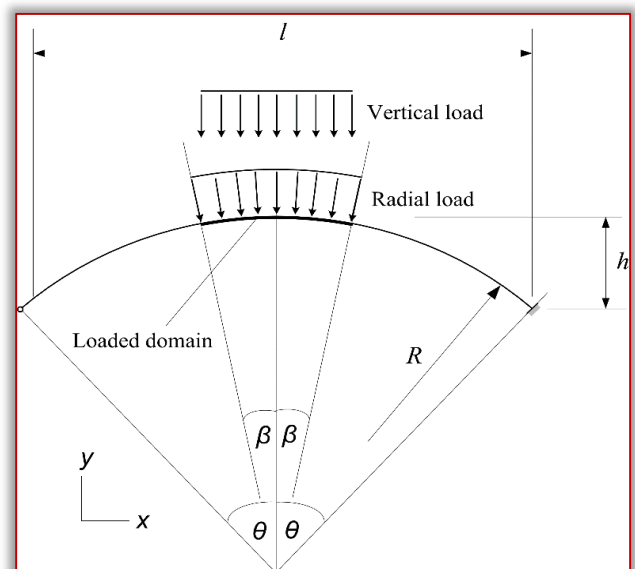


Figure 1: Arch configuration

For the sake of simplicity, the dimensionless parameter $c=B/\theta$ (in case of vertical load) and $c_r=B/\theta$ (in case of radial load) is introduced. Hence, the resultant force can be calculated by $F=2f\beta R$. Given that $c=c_r=0$, the load is a vertical concentrated force at the apex of the arch, meanwhile if $c=c_r=1$, that is a completely loaded

arch between the end supports. The arch has a uniform, doubly symmetric *I*-section [7] as per Figure 2. The bending axis *z* is a major principal axis of inertia. Further properties are listed in Table 1.

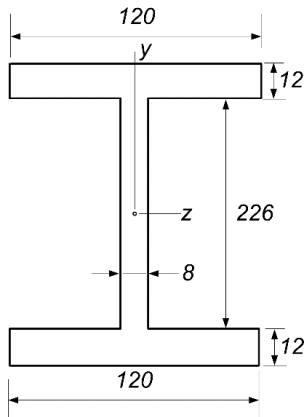


Figure 2: Doubly symmetric *I*-section and its dimensions

Table 1: Further properties of the selected cross-section

SECTION AREA	$A \text{ [mm}^2\text{]}$	4688
SECTION MOMENT OF INERTIA TO Z	$I_z \text{ [mm}^4\text{]}$	4.851×10^7
SECTION MOMENT OF INERTIA TO Y	$I_y \text{ [mm}^4\text{]}$	3.465×10^6
SECTION MODULUS TO Z	$S_z \text{ [mm}^3\text{]}$	3.881×10^5
SECTION MODULUS TO Y	$S_y \text{ [mm}^3\text{]}$	5.776×10^4
RADIUS OF GYRATION TO Z	$r_z \text{ [mm]}$	101.723
RADIUS OF GYRATION TO Y	$r_y \text{ [mm]}$	27.187

Within this article, analysis is performed for three specimen. These are distinguished by their rise to span ratio, constituting a shallow, a moderately shallow and a deep arch type. Details are provided in Table 2. The modulus of elasticity is $E=202$ GPa while the Poisson number is 0.3.

Table 2: Investigated arch data

	SYMBOL	SHALLOW ARCH	MODERATELY SHALLOW ARCH	DEEP ARCH
radius	$R \text{ [mm]}$	2000	2000	2000
arch length	$S \text{ [mm]}$	3142	6284	9426
included angle	$2\theta \text{ [rad]}$	1.571	3.142	4.713
included angle	$2\theta \text{ [}^\circ\text{]}$	90	180	270
arch rise	$h \text{ [mm]}$	585.786	2000	3414.214
arch span	$l \text{ [mm]}$	2828.427	4000	2828.427
rise to span	$h/l \text{ [–]}$	0.207	0.5	1.207

FINITE ELEMENT MODELING

The commercial finite element software Abaqus CAE 6.13 is utilised for the numerical studies. One dimensional *B21* beam elements are used throughout. The number of elements used is 32, unless indicated otherwise. Due to the unsymmetry at the supports, the whole centerline needs to be modeled. Displacement is restrained at the left end (pinned support) and displacements and rotations are both prohibited at the right (clamped) support. Vertical distributed load is assigned as line load and radial load is assigned as pressure in the software.

For the analysis, the Static, Riks step is selected to keep track of the equilibrium path. Geometric nonlinearities are incorporated for a better accuracy. The assigned, initial load magnitude is set to one unit and the load proportionality factor is incremented during analysis. The lowest buckling load is the one that corresponds to the upper limit point on the equilibrium path. Results for seven load schemes are given hereinafter: $c_r=c=[0;0.1;0.5;1]$. When relative loads are displayed in the forthcoming, such ratios are made by dividing the actual resultant buckling load by the lowest buckling load in case of $c=0$.

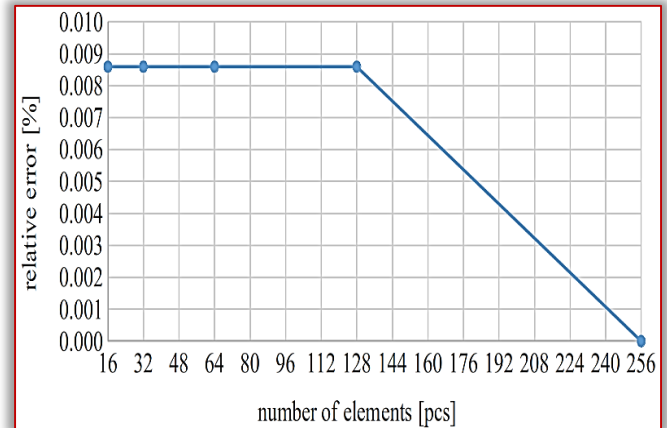


Figure 3: Relative error of the results against the number of elements

First of all, to be sure of the reliability of the findings, convergence studies are performed. The moderately deep arch geometry under concentrated load was mapped with 2-256 elements and the buckling loads were compared, considering the 'exact' or reference the one reached with 256 elements. The relative error is given in Figure 3 showing actually zero difference from 16 elements. Considering that for deep arches, the typical element length is greater, 32 elements are used throughout the study.

Rounded studies are also provided in Table 3. Obviously, 2 or 4 linear elements cannot map the curved geometry accurately, resulting in much higher errors.

Table 3: Relative error for multiple element numbers

NUMBER OF ELEMENTS	RELATIVE ERROR [%]
2	18.404
4	−4.302
8	−0.032
16	0.009
32	0.009
64	0.009
128	0.009

— MODERATELY DEEP ARCH: In Figure 4, it is shown how the length of the loaded domain (c or c_r) and the load type affect the relative resultant buckling load (maximum overall load the structural member can bear without the loss of stability divided by the buckling load in case of a point load). Buckling load is lowest for point

load and highest when load is distributed evenly on the longest distance (whole arch). Markers in the figure show the finite element results which are then connected by fitting curves. The relationship is linear in the small vicinity of origin, until about $c < 0.04$. Otherwise, the curves are nonlinear. There is a steep rise until about $c = 0.1$, then the curves becomes quite flat. The highest critical resultant force is more than 3.5 times of the point load.

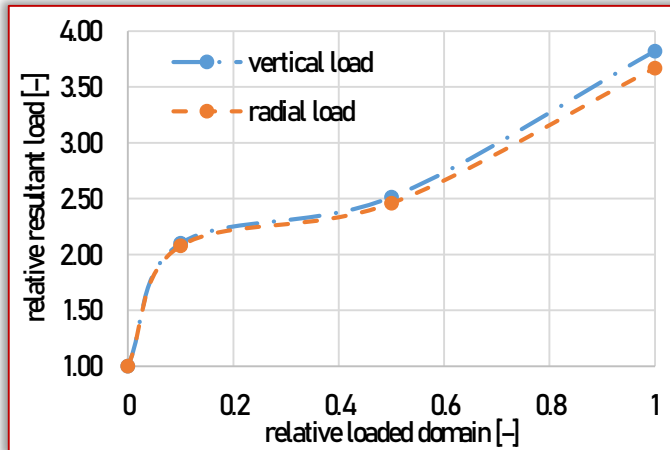


Figure 4: Relative buckling load change against the loaded domain width (moderately shallow arch)

Given that this is a moderately shallow arch, discrepancies between vertical and radial load directions are not substantial. Where the loads are not equal, the vertical load type yields slightly higher allowable forces.

Figure 5 plots the equilibrium paths. Along the horizontal axis, the relative dimensionless displacement is shown, that is reached by dividing the actual crown-point vertical displacement value by the critical crown-point displacement in case of a point load. It is clear that an evenly distributed loading holds the lowest displacement, together with the highest allowable load.

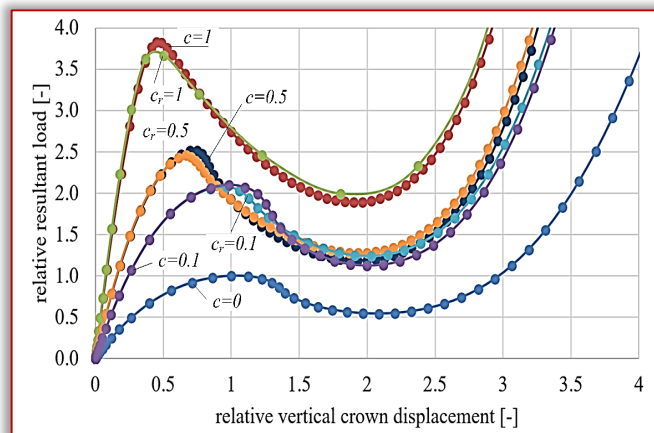


Figure 5: Equilibrium paths for various load patterns (moderately deep arch)

Upon reaching the upper limit point on the equilibrium path, a further increase in the external load triggers a snap-through phenomenon, when there is a sudden shift from the primary stable branch to the remote stable one. Its consequence

is the substantially increased displacement, something that must be avoided for structural safety considerations.

— SHALLOW ARCH: Moving on now to the shallow geometry of Table 2, it is visualized in Figure 6 how the loaded domain width and load type influence the relative buckling load. This figure can directly be compared with Figure 4. This time, however, the maximum gain in the resultant load is just about the half (1.9 times), compared to the moderately deep arch. Moreover, the discrepancies are also less between radial and vertical force that is on account of the shallow (flat) geometry.

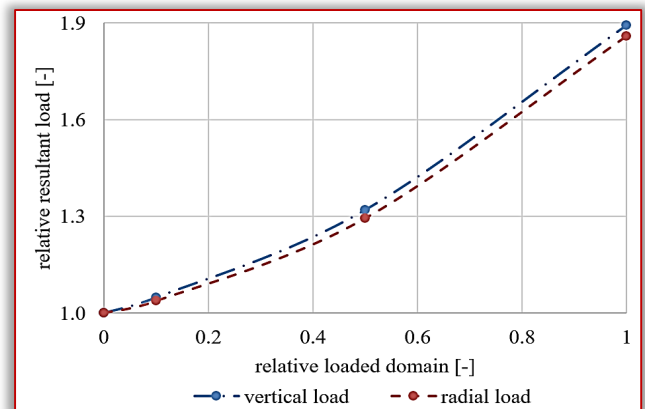


Figure 6: Relative buckling load change against the loaded domain width (shallow arch)

The equilibrium paths - with relative values - in case of a concentrated load and a fully distributed radial load are compared in Figure 7. Increasing the force beyond the upper limit point (see the hollow rectangular and triangular symbols on peak of the primary equilibrium path), makes the arch snap to the remote branch, causing a sudden increment in the displacement. This instantaneous remote equilibrium is again depicted by the same hollow symbols.

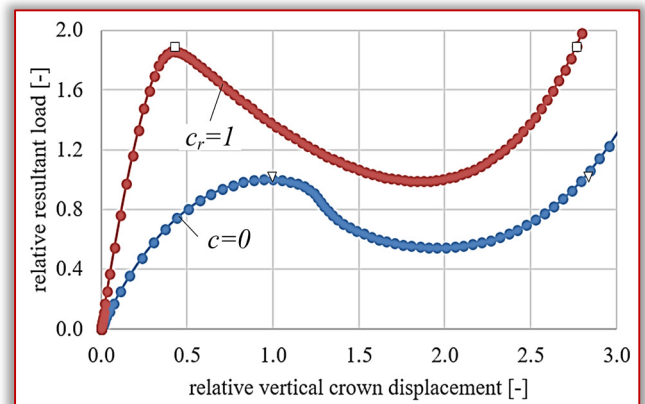


Figure 7: Equilibrium paths for various load patterns (shallow arch)

The arch shapes at the upper limit point on the primary equilibrium path and at the buckled configuration are shown in Figure 8 for both the previously mentioned load types. Greater discrepancies are found in the pre-buckling shapes, while the buckled shapes are very similar. Note

that the deformations are magnified for a better visibility.

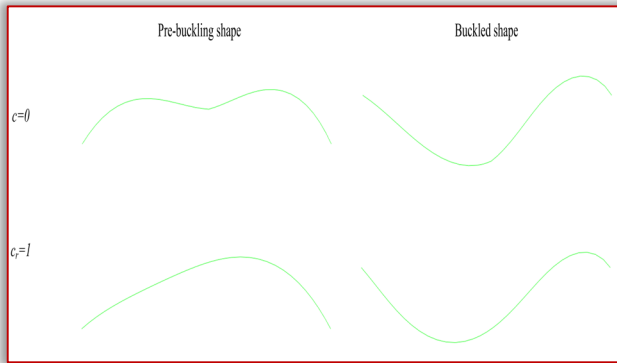


Figure 8: Pre- and post-buckling shapes for shallow arches

— **DEEP ARCH:** Finally, when the deep arch geometry of Table 2 is considered, the relative buckling load deviation is given in Figure 9. This time, it can happen in a small range that the resultant load for vertically aligned forces becomes lower than for a concentrated force. Meanwhile, in case the radial load is distributed over the whole arch, the structural member might not undergo instability. For this geometry, the highest discrepancies are found between the case of radial and vertical load.

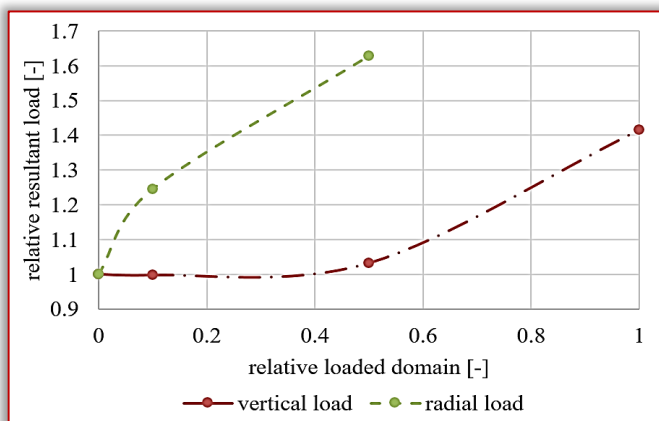


Figure 9: Relative buckling load change against the loaded domain width (deep arch)

CONCLUSIONS

The article is dedicated to the in-plane elastic stability of pinned-fixed arches. Computations were performed with Abaqus. The buckling loads and behaviours were compared for concentrated and evenly distributed loads. The evenly distributed load along the whole centerline resulted in the highest allowable loads and lowest peak displacement. Generally, a point load triggers instability first. Greater loaded domain width helps in improving the structural resistance. The only exception is the selected deep arch geometry, where a local vertical distributed load in the Crown Point vicinity can affect negatively the behaviour.

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